KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

COLLEGE OF COMPUTER SCIENCES & ENGINEERING

COMPUTER ENGINEERING DEPARTMENT

COE 540 – Computer Networks

Assignment 2 – Due Date Dec 14th, 2010

Problem	Points	
1	20	
2	40	
3	20	
Total		

Problem 1 (20 points – Discrete Markov Chains and Random Variables):

On day 0 a house has two new light bulbs in reserve. The probability that the house will need a single new light bulb during day n is p, and the probability that it will not need any is $q = 1 - p$. Let Y_n be the number of NEW light bulbs left in the house at the end of day n.

a) Draw the state transition diagram for Y_n and specify the 1-step transition probability matrix.

b) What is the probability that ONE new light bulb is needed in n days?

c) Compute the n-step transition probability matrix.

d) Show that the house will run out of new light bulbs with probability one as n tends to infinity for any $p < 1$.

Problem 2 (40 points - Markov Chains):

Data in the form of fixed-length packets arrive in slots on the FOUR input lines of a multiplexer. A slot contains a packet with probability *p*, independent of the arrivals during other slots or on the other line. The multiplexer transmits one packet per time slot and has the capacity to store THREE packets only. If no room for a packet is found, the packet is dropped.

a) COMPUTE the probability of *j* (for all possible *j* values) packets arriving on the four input lines during any given time slot.
b) Compute average and variance of total number of arriving packets in a time slot.
c) DRAW the state transition diagram and SPECIFY the transition matrix **P** – The state is taken to be the number of packets in the multiplexer.
d) If *p* is equal to 0.5, what is the probability that the MUX will contain 3 packets after 8 time slot (i.e. at the start of the 9^{th} time slot)? Assume that we start with an empty MUX.
e) Provide a graph of throughput (for $p = 0.5$) as a function of time. Plot only for the first 10 time slots.
f) At steady state and for a given *p*, what is the probability of dropping a packet in a given time slot? Specify this quantity in terms of the steady state distribution.

Problem 3 (20 points – Queueing Systems):

Consider a system with Poisson arrivals of rate λ, and exponential service times of parameter μ. Suppose that the number of servers is so large that arriving customers always find a server available. This system can be modeled by an M/M/∞ where the number of servers is equal to infinity. For this system:

a) Draw the transition rate diagram for the system depicting the states and specifying the transition rates.

b) What is the distribution of the number of customers in the system? Show that.

c) If the arrival rate λ is equal to 2 customers per time unit, while the service rate for each of the servers, μ, is equal to 1 customer per time unit, compute the mean number of customers in this system.