# KFUPM - COMPUTER ENGINEERING DEPARTMENT <br> COE-543 - Mobile Computing and Wireless Networking <br> Assignment 1 - Due May $3^{\text {rd }}, 2009$. 

Problem 1: Describe briefly each of the following mobile systems: UMTS, LTE, and UMB. The description should include the main characteristics and the base technology and service features of the system. Not more than one page per system is required.

Problem 2: Rayleigh fading channels
The envelope of the received signal in a multipath channel can be modeled using a Rayleigh or Rician distributions. Specify each of the distributions and identify the corresponding parameters and their physical meaning. Show that the Rayleigh distribution is a special case of the Rician distribution.

Textbook Problems: 2.2, 2.4 (but for $90 \%$ coverage and 12 dB standard deviation), 2.7, 2.13, and 3.3.

For Problem 1 - refer to the introductory material covered at the beginning of the course and the slides package by Brough Turner from NMS Communications covered in class.

For Problem 2 - was covered in class and in notes - should discuss it from RF point of view.

## Problem: 2.2 ( 30 points):

At 900 MHz we use the Okumura - Hata model:
$\mathrm{P}_{\mathrm{t}}-\mathrm{P}_{\mathrm{r}}(\mathrm{dB})=69.55+26.16 \log \left(\mathrm{f}_{\mathrm{c}}\right)-13.82 \log \left(\mathrm{~h}_{\mathrm{b}}\right)-\mathrm{a}\left(\mathrm{h}_{\mathrm{m}}\right)+\left(44.9-6.55 \log \left(\mathrm{~h}_{\mathrm{b}}\right)\right) \log (\mathrm{d})$
At 1900 MHz we use the COST-231 model:
$\mathrm{P}_{\mathrm{t}}-\mathrm{P}_{\mathrm{r}}(\mathrm{dB})=46.3+33.9 \log \left(\mathrm{f}_{\mathrm{c}}\right)-13.82 \log \left(\mathrm{~h}_{\mathrm{b}}\right)-\mathrm{a}\left(\mathrm{h}_{\mathrm{m}}\right)+\left(44.9-6.55 \log \left(\mathrm{~h}_{\mathrm{b}}\right)\right) \log (\mathrm{d})+\mathrm{C}_{\mathrm{m}}$
Where $a\left(h_{m}\right)=3.2\left(\log 11.75 h_{m}\right)^{2}-4.97 d B$ for $f_{c}>300 \mathrm{MHz}$
Plug in the following parameters in above models:
$\mathrm{h}_{\mathrm{b}}=30 \mathrm{~m}, \mathrm{~h}_{\mathrm{m}}=2 \mathrm{~m}$ and $\mathrm{C}_{\mathrm{M}}=0$.
Then $\mathrm{a}\left(\mathrm{h}_{\mathrm{m}}\right)=1.045 \mathrm{~dB} . \mathrm{P}_{\mathrm{t}}-\mathrm{P}_{\mathrm{r}}(\mathrm{dB})=130 \mathrm{~dB}$.
@ 900 MHz , Okumura - Hata model (designed for frequencies less than 1500 MHz ):
$(44.9-6.55 \log (30)) \log (\mathrm{d})=130-(69.55+26.16 \log (900)-13.82 \log (30)-1.045)$
$35.225 \log (\mathrm{~d})=4.626$
$\mathrm{d}=10^{(4.626 / 35.225)}=10^{0.131}=\mathbf{1 . 3 5 3} \mathbf{K m}$
@ 1900 MHz, COST-231 model (designed for frequencies more than $\mathbf{1 8 0 0 M H z}$ ):
$(44.9-6.55 \log (30)) \log (\mathrm{d})=130-(46.3+33.9 \log (1900)-13.82 \log (30)-1.045)$
$35.225 \log (\mathrm{~d})=-5.991$
$\mathrm{d}=10^{(-5.991 / 35.225)}=10^{-0.170}=\mathbf{0 . 6 7 6} \mathbf{K m}$

## Problem: 2.4 ( $\mathbf{3 0}$ points):

The solution to the fading margin is obtained by equating the tail probability to 0.1 ( $90 \%$ of the cell edge is adequately covered). If $F_{L S}$ is the shadow fading component and $F$ is the fading margin, this means:
$\int_{F}^{\infty} F_{L S}(x) \cdot d x=0.1$
The shadow fading component is normally distributed with mean zero and standard deviation $\sigma=12 \mathrm{~dB}$. We can thus rewrite the equation as:
$\int_{F}^{\infty} \frac{1}{\sigma \sqrt{2 \pi}} \exp \left(\frac{-x^{2}}{2 \sigma^{2}}\right) \cdot d x=0.1$
We make a substitution of variables, and let $y=x / \sigma$. This results in the equation:
$\int_{\frac{F}{\sigma}}^{\infty} \frac{1}{\sigma \sqrt{2 \pi}} \exp \left(\frac{-y^{2} \sigma^{2}}{2 \sigma^{2}}\right) \cdot \sigma \cdot d y=0.1 \rightarrow \int_{\frac{F}{\sigma}}^{\infty} \frac{1}{\sqrt{2 \pi}} \exp \left(\frac{-y^{2}}{2}\right) \cdot d y=0.1$
This is nothing but the Q function with the argument of the function being $F / \sigma$. This equates to $Q\left(\frac{F}{\sigma}\right)=0.1$. Since $\sigma=12, Q\left(\frac{F}{12}\right)=0.1$ and from the Q-table, this gives $\frac{F}{12}=1.2816$. So the fade margin, is $F=15.3786 \mathrm{~dB}$.

## Problem 2.7 ( 20 points):

The delay profile for the channel is given by the following table:

| Relative <br> delay ( $\boldsymbol{\mu} \mathbf{s e c}$ ) | Average <br> relative <br> power, $\boldsymbol{\sigma}^{\mathbf{2}}$ <br> $(\mathbf{d B})$ | Average <br> relative <br> power $\boldsymbol{\sigma}^{\mathbf{2}}$ |
| :---: | :---: | :---: |
| 0.0 | -1.0 | 0.7943 |
| 0.5 | 0.0 | 1.0000 |
| 0.7 | -3.0 | 0.5012 |
| 1.5 | -6.0 | 0.2512 |
| 2.1 | -7.0 | 0.1995 |
| 4.7 | -11.0 | 0.0794 |



The channel multipath profile is as shown in figure above.
Total excess delay spread $=\tau_{L}-\tau_{1}=4.7-0=4.7 \mu \mathrm{~s}$
$E[\tau]=\bar{\tau}=\frac{\sum_{\forall i} \tau_{i} \sigma_{i}^{2}}{\sum_{\forall i} \sigma_{i}^{2}}=0.7149 \mu \mathrm{sec}, E\left[\tau^{2}\right]=\frac{\sum_{\forall i} \tau_{i}^{2} \sigma_{i}^{2}}{\sum_{\forall i} \sigma_{i}^{2}}=1.3078 \quad(\mu \mathrm{~s})^{2}$, and therefore,
$\tau_{r m s}=\sqrt{E\left[\tau^{2}\right]-E[\tau]^{2}}=0.8926 \mu \mathrm{~s}$
Coherence bandwidth $=1 /\left(5^{\star} \tau_{\text {rms }}\right)=224 \mathrm{kHz}$
Because the 25 kbps data rate is less than 224 kHz , this channel is NOT considered wideband

## Problem 2.13 ( 20 points):

(a) RMS delay spread
$\bar{\tau}=\frac{\sum_{1}^{N} \tau_{i}\left|\beta_{i}\right|^{2}}{\sum_{1}^{N}\left|\beta_{i}\right|^{2}}=\frac{50 \times 0.4+100 \times 0.4+200 \times 0.2}{0.4+0.4+0.2}=100 \mathrm{~ns}$
$\overline{\tau^{2}}=\frac{\sum_{1}^{N} \tau_{i}^{2}\left|\beta_{i}\right|^{2}}{\sum_{1}^{N}\left|\beta_{i}\right|^{2}}=\frac{50^{2} \times 0.4+100^{2} \times 0.4+200^{2} \times 0.2}{0.4+0.4+0.2}=13,000 \mathrm{~ns}^{2}$
$\tau_{r m s}=\sqrt{\overline{\tau^{2}}-\bar{\tau}^{2}}=\sqrt{13,000-(100)^{2}}=54.8 \mathrm{~ns}$
(b) RMS Doppler spread

$$
\begin{aligned}
& \bar{\lambda}=\frac{\int_{-\infty}^{\infty} \lambda D(\lambda) d \lambda}{\int_{-\infty}^{\infty} D(\lambda) d \lambda}=\frac{\int_{-5}^{5} 0.1 \lambda d \lambda}{\int_{-5}^{5} 0.1 d \lambda}=\frac{0.1\left[\frac{\lambda^{2}}{2}\right]_{-5}^{5}}{1}=0 \mathrm{~Hz} \\
& \overline{\lambda^{2}}=\frac{\int_{-\infty}^{\infty} \lambda^{2} D(\lambda) d \lambda}{\int_{-\infty}^{\infty} D(\lambda) d \lambda}=\frac{\int_{-5}^{5} 0.1 \lambda^{2} d \lambda}{\int_{-5}^{5} 0.1 d \lambda}=\frac{0.1\left[\frac{\lambda^{3}}{3}\right]_{-5}^{5}}{1}=25 \mathrm{~Hz}^{2} \\
& \lambda_{\text {rms }}=\sqrt{\lambda^{2}-\bar{\lambda}^{2}}=\sqrt{25-(0)^{2}}=5 \mathrm{~Hz}
\end{aligned}
$$

(c) Coherence bandwidth of the channel
$\frac{1}{5 \tau_{r m s}}=\frac{1}{5 \times 54.8 \mathrm{~ns}}=3.7 \mathrm{MHz}$

## Problem 3.3 ( $\mathbf{3 0}$ points):

(a) From Table 3A. 1 in the case of QPSK $(M=4, m=2)$ in Flat Rayleigh Fading channels, we have
$P e=\frac{2^{m-1}}{M-1}\left(1-\sqrt{\frac{\sin ^{2}\left(\frac{\pi}{m}\right) m \gamma_{b}}{1+\sin ^{2}\left(\frac{\pi}{m}\right) m \gamma_{b}}}\right)=\frac{2}{3}\left(1-\sqrt{\frac{\sin ^{2}\left(\frac{\pi}{2}\right) 2 \gamma_{b}}{1+\sin ^{2}\left(\frac{\pi}{2}\right) 2 \gamma_{b}}}\right)=\frac{2}{3}\left(1-\sqrt{\frac{2 \gamma_{b}}{1+2 \gamma_{b}}}\right)$
and for a BER of $10^{-3}$, we have $\gamma_{b}=\frac{1}{2} \frac{\left[1-\frac{3}{2} \mathrm{Pe}\right]^{2}}{1-\left[1-\frac{3}{2} \mathrm{Pe}\right]^{2}}=\frac{1}{2} \frac{\left[1-\frac{3}{2} \mathrm{Pe}\right]^{2}}{1-\left[1-\frac{3}{2} \mathrm{Pe}\right]^{2}}=331.13=25.2 \mathrm{~dB}$
(b) From Table 3A. 1 in the case of QPSK $(M=4, m=2)$ in AWGN channels, we have $P e=\frac{2^{m-1}}{M-1} \operatorname{erfc}\left(\sqrt{\sin ^{2}\left(\frac{\pi}{M}\right) m \gamma_{b}}\right)=\frac{2}{3} \operatorname{erfc}\left(\sqrt{\sin ^{2}\left(\frac{\pi}{4}\right) 2 \gamma_{b}}\right)=\frac{2}{3} \operatorname{erfc}\left(\sqrt{\gamma_{b}}\right) \quad$ and $\quad \gamma_{b}=\operatorname{erfc} c^{-1}\left(\frac{3}{2} P e\right)$
and for a BER of $10^{-3}$, we have $\gamma_{b}=\operatorname{erfc}^{-1}\left(\frac{3}{2} 10^{-3}\right)=5.01=7.0 \mathrm{~dB}$
(c) Probability of outage

Therefore for an average $\gamma_{\mathrm{b}}=331.13=25.2 \mathrm{~dB}$, and a threshold $\gamma_{\mathrm{th}}=5.01=7.0 \mathrm{~dB}$
$P_{\text {out }}=1-e^{-\frac{\gamma_{\text {lt }}}{\gamma_{b}}}=1-e^{-\frac{5.01}{331.13}}=1.5 \times 10^{-2}$

