KFUPM - COMPUTER ENGINEERING DEPARTMENT

COE-202 – Fundamentals of Computer Engineering

SOLUTION for Assignment # 1: Due Sunday Nov 23rd, 2008 – in class.

1) Convert the following numbers from the given base to the bases indicated:
   (a) Decimal 255.225 to binary, octal, and hexadecimal.
   (b) Hexadecimal 2AC5.D to decimal, octal, and binary.
   (c) Hexadecimal EF.C to base 5
   (d) Binary 1010101111.01101 to base 3

Solution:

(a)							
255/2	127	1		0.225x2	0.45	0	
127/2	63	1		0.45x2	0.9	0	
63/2	31	1		0.9x2	1.8	1	
31/2	15	1		0.8x2	1.6	1	
15/2	7	1		0.6x2	1.2	1	
7/2	3	1		0.2x2	0.4	0	
3/2	1	1		0.4x2	0.8	0	
1/2	0	1		0.8x2	1.6	1	
				0.6x2	1.2	1	

Note that the part of fraction (1100) repeats
Therefore,
$(255.225)_{10} = (1111\ 1111.001(1100)^{+})_2$

In Octal: $(255.225)_{10} = (377.1(6314)^{+})_8$

In Hex: $(255.225)_{10} = (F\ F.3(9)^{+})_H$

(b) (2AC5.D)H:
In decimal: $2x16^3+10x16^2+12x16^1+5x16^0+13x16^{-1} = (10949.8125)_{10}$
In binary: $(0010\ 1010\ 1100\ 0101\ .\ 1101)_2$ – through direct translation of hex digits
In Octal: $(25305.64)_8$ – the binary digits takes in threes are translated to octal digits

(c) $(EF.C)_H = 14x16^1+15x16^0+12x16^{-1} = (239.75)_{10}$

239/5	47	4		0.75x5	3.75	3	
47/5	9	2		0.75x5	3.75	3	
9/5	1	4		0.75x5	3.75	3	
1/5	0	1					

It is clear that the fraction digit 3 repeats.
Therefore, $(EF.C)_H = (239.75)_{10}$
$= (1424.3^{+})_5$

(d) $(0010\ 1010\ 1111\ .\ 0110\ 1000)2 = (2AF.68)_H$
$= 2x16^2+10x16^1+15x16^0+6x16^{-1}+8x16^{-2}= (687.40625)_{10}$

687/3	229	0		0.40625x3	1.21875	1	
229/3	76	1		0.21875x3	0.65625	0	
76/3	25	1		0.65625x3	1.96875	1	
25/3	8	1		0.96875x3	2.90625	2	
8/3	2	2		0.90625x3	2.71875	2	
2/3	0	2		0.71875x3	2.15625	2	
				0.15625x3	0.46875	0	
				0.46875x3	1.40625	1	
				0.40625x3	1.21875	1	

It is clear that the fraction part 10122201 repeats
Therefore,
$(687.40625)_{10} = (221110.(10122201)^{+})_3$

2) Perform the following unsigned arithmetic operations using the designated bases without converting to decimal. Verify your result by converting the numbers to decimal and then performing the operation in decimal.

(a) $(10111011)_2 - (01001111)_2$
(b) $(10E)_{16} - (13F)_{16}$
(c) $(54)_{16} * (20)_{16}$
(d) $(11011.0111)_2 + (11.1101)_2$

Solution:

(a)

```
   1 0 1 1  1 0 1 1    → 187
 - 0 1 0 0  1 1 1 1   → - 79
 ------------------     -----
  0 1 1 0  1 1 0 0    →  108
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One can check that $(110\ 1100)_2 = (108)_{10}$.

(b) 10E – 13F = ?

It is clear that 13F is greater than 10E. Therefore, $(10E)_H - (13F)_H = -[(13F)_H - (10E)_H] = -(031)_H$.

To check, $(10E)_H = (270)_{10}$, $(13F)_H = (319)_{10}$ → $270 - 319 = -49 = -(31)_H$.

(c) $(54)_H$ X $(20)_H$ →

```
        5  4
     x  2  0
   ----------
        0  0
     A  8
  -----------
     A  8  0
```

To check, $(54)_H = (84)_{10}$, $(20)_H = (32)_{10}$ → $84\text{x}32 = (2688)_{10}$ which is equal to $(A80)_H$.

(d)

```
   1 1 0 1 1 . 0 1 1 1         → 27.4375
+        1 1 . 1 1 0 1         →  3.2500
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   1 1 1 1 1 . 0 1 0 0         → 31.2500
```

To check, $(11111.0100)_2$ is indeed equal to sum $(31.25)_{10}$.

3) In each of the following cases, determine the radix *r*:

(a) $(121)_r = (25)_{10}$

(b) $(345)_r = (180)_{10}$

Solution:

(a) $(121)_r = (25)_{10}$ ➔ $r^2 + 2r + 1 = 25$ ➔ $r^2 + 2r - 24 = 0$ ➔ $(r+6)(r-4) = 0$;

Therefore, r = 4.

The required base is 4.

(b) $(345)_r = (180)_{10}$ ➔ $3r^2 + 4r - 175 = 0$ ➔ $r2 + 4/3\ r - 175/3 = 0$ ➔ $(r+8\ 1/3)(r-7) = 0$;

Therefore, r = 7.

The required base is 7.

4) Show how the decimal integers +120 and -120 would be represented in signed magnitude, 1's complement, and 2's complement notation using 8 bits and 10 bits, respectively.

Solution

	8 bits		10 bits	
	+120	-120	+120	-120
Signed magnitude	0111 1000	1111 1000	00 0111 1000	10 0111 1000
1's complement	0111 1000	1000 0111	00 0111 1000	11 1000 0111
2's complement	0111 1000	1000 1000	00 0111 1000	11 1000 1000

5) Perform the operations $M+N$, $M-N$, and $N-M$ using both radix and diminished radix complement systems using the specified number of digits. Specify when an overflow condition has occurred.

(a) $n = 4$, $M = (A2B)_{16}$, $N = (56C)_{16}$
(b) $n = 3$, $M = (821)_{10}$, $N = (785)_{10}$
(c) $n = 8$, $M = (10010)_2$, $N = (11011)_2$
(d) $n = 6$, $M = (10010)_2$, $N = (10011)_2$

Solution:

(a) n = 4, M = $(A2B)_{16}$, N = $(56C)_{16}$
[1] 16's complement system:
M' = 10000 - M = 10000 - A2B = F5D5 ➔ {M = $(2603)_{10}$ and M' = -$(2603)_{10}$}
N' = 10000 - N = 10000 - 56C = FA94 ➔ { N = $(1388)_{10}$ and N' = -$(1388)_{10}$}
M + N = 0A2B + 056C = 0F97 and the end carry is ZERO. No overflow
The result 0F97 is correct. Note $(2603)_{10}$ + $(1388)_{10}$ = $(3991)_{10}$ which is $(F97)_{16}$ in n = 4 and 16's complement system.
M - N = M + N' = 0A2B + FA94 = 04BF and the end carry is ONE. No overflow. Ignore carry.
The result 04BF is correct. Note $(2603)_{10}$ – $(1388)_{10}$ = $(1215)_{10}$ which is $(04BF)_{16}$ in n = 4 and 16's complement system.
N – M = N + M' = 056C + F5D5 = FB41 and the end carry is ZERO. No overflow
The result FB41 is correct. Note $(1388)_{10}$ - $(2603)_{10}$ = - $(1215)_{10}$ which is $(FB41)_{16}$ in n = 4 and 16's complement system.

[2] 15's complement system:
M' = FFFF - M = FFFF - A2B = F5D4 ➔ {M = $(2603)_{10}$ and M' = -$(2603)_{10}$}
N' = 10000 - N = FFFF - 56C = FA93 ➔ { N = $(1388)_{10}$ and N' = -$(1388)_{10}$}
M + N = 0A2B + 056C = 0F97 and the end carry is ZERO. No overflow – same as above.
The result 0F97 is correct. Note $(2603)_{10}$ + $(1388)_{10}$ = $(3991)_{10}$ which is $(0F97)_{16}$ in n = 4 and 15's complement system.
M - N = M + N' = 0A2B + FA93 = 04BE and the end carry is ONE. No overflow.
Need to correct answer ➔ 04BE + 1 (end carry) = 04BF
The result 04BF is correct. Note $(2603)_{10}$ – $(1388)_{10}$ = $(1215)_{10}$ which is $(04BF)_{16}$.in n = 4 and 15's complement system.
N – M = N + M' = 056C + F5D4 = FB40 and the end carry is ZERO. No overflow
The result FB40 is correct. Note $(1388)_{10}$ - $(2603)_{10}$ = - $(1215)_{10}$ which is $(FB40)_{16}$ in n = 4 and 15's complement system.

(b) n = 3, M = $(821)_{10}$, N = $(785)_{10}$
[1] 10's complement system:
M' = 1000 - M = 1000 - 821 = 179 ➔ {M = - 179 and M' = 179}
N' = 1000 - N = 1000 - 785 = 215 ➔ { N = -215 and N' = 215}
M + N = 821 + 785 = 606 and the end carry is ONE. No overflow. Ignore carry

The result 606 is correct. Note -179 + (-215) = -394 which is 606 in n = 3 and 10's complement system.
M - N = M + N' = 821 + 215 = 036 and the end carry is ONE. No overflow. Ignore carry.
The result 036 is correct. Note -179 – (-215) = 36 which is 036 in n = 3 and 10's complement system.
N – M = N + M' = 785 + 179 = 964 and the end carry is ZERO. No overflow
The result 964 is correct. Note -215 – (-179) = - 36 which is 964 in n = 3 and 10's complement system.

[2] 9's complement system:
M' = 999 - M = 999 - 821 = 178 ➔ {M = - 178 and M' = 178}
N' = 999 - N = 999 - 785 = 214 ➔ { N = -214 and N' = 214}
M + N = 821 + 785 = 606 and the end carry is ONE. No overflow. Ignore carry – same as above.
The result 606 is correct. Note -178 + (-214) = -392 which is 606 in n = 3 and 9's complement system.
M - N = M + N' = 821 + 214 = 035 and the end carry is ONE. No overflow.
Need to correct result ➔ 035 + 1 (end carry) = 036.
The result 036 is correct. Note -178 – (-214) = 36 which is 036 in n = 3 9's complement system.
N – M = N + M' = 785 + 178 = 963 and the end carry is ZERO. No overflow
The result 963 is correct. Note -214 – (-178) = - 36 which is 963 in n = 3 and 9's complement system.

(c) n = 8, M = 0001 0010, N = 0001 1011 ➔ {M = 18, N = 27}
[1] 2's complement system:
M' = 1 0000 0000 – 0001 0010 = 1110 1110 ➔ {M = 18, M' = -18}
N' = 1 0000 0000 – 0001 1011 = 1110 0101 ➔ {N = 27, N' = -27}
M + N = $(0001\ 0010)_2$ + $(0001\ 1011)_2$ = $(0010\ 1101)_2$ and the end carry is ZERO. No overflow
The result $(0010\ 1101)_2$ is correct. Note 18 + 27 = 45 which is $(0010\ 1101)_2$ in n = 8 and 2's complement system.
M - N = M + N' = $(0001\ 0010)_2$ + $(1110\ 0101)_2$ = $(1111\ 0111)_2$ and the end carry is ZERO. No overflow.
The result $(1111\ 0111)_2$ is correct. Note 18 - 27 = -9 which is $(1111\ 0111)_2$ in n = 8 and 2's complement system.
N – M = N + M' = $(0001\ 1011)_2$ + $(1110\ 1110)_2$ = $(0000\ 1001)_2$ and the end carry is ONE. No overflow.
Ignore the carry
The result $(0000\ 1001)_2$ is correct. Note 27 - 18 = 9 which is $(0000\ 1001)_2$ in n = 8 and 2's complement system.

[2] 1's complement system:
M' = 1111 1111 – 0001 0010 = 1110 1101 ➔ {M = 18, M' = -18}
N' = 1111 1111 – 0001 1011 = 1110 0100 ➔ {N = 27, N' = -27}
M + N = $(0001\ 0010)_2$ + $(0001\ 1011)_2$ = $(0010\ 1101)_2$ and the end carry is ZERO. No overflow
The result $(0010\ 1101)_2$ is correct. Note 18 + 27 = 45 which is $(0010\ 1101)_2$ in n = 8 and 2's complement system.
M - N = M + N' = $(0001\ 0010)_2$ + $(1110\ 0100)_2$ = $(1111\ 0110)_2$ and the end carry is ZERO. No overflow.

The result $(1111\ 0110)_2$ is correct. Note 18 - 27 = -9 which is $(1111\ 0110)_2$ in n = 8 and 1's complement system.
N – M = N + M' = $(0001\ 1011)_2$ + $(1110\ 1101)_2$ = $(0000\ 1000)_2$ and the end carry is ONE. No overflow.
Need to correct result ➔ $(0000\ 1000)_2$ + 1 (end carry) = $(0000\ 1001)_2$
The result $(0000\ 1001)_2$ is correct. Note 27 - 18 = 9 which is $(0000\ 1001)_2$ in n = 8 and 1's complement system.

(d) n = 6, M = 01 0010, N = 01 0011
[1] 2's complement system:
M' = 1 00 0000 – 01 0010 = 10 1110 ➔ {M = 18, M' = -18}
N' = 1 00 0000 – 01 0011 = 10 1101 ➔ {N = 19, N' = -19}
M + N = $(01\ 0010)_2$ + $(01\ 0011)_2$ = $(10\ 0101)_2$ and Cn = 1 and Cn+1 = 0 (i.e. the end carry is ZERO) ➔ OVERFLOW
The result $(10\ 0101)_2$ is NOT correct. Note 18 + 27 = 45 which is CAN NOT be represented in n = 6 and 2's complement system. The maximum positive number in this system is equal to $+(2^{n-1}-1)$ = +31.
M - N = M + N' = $(01\ 0010)_2$ + $(10\ 1101)_2$ = $(11\ 1111)_2$ and the end carry is ZERO. No overflow.
The result $(11\ 1111)_2$ is correct. Note 18 - 19 = -1 which is $(11\ 1111)_2$ in n = 6 and 2's complement system.
N – M = N + M' = $(01\ 0011)_2$ + $(10\ 1110)_2$ = $(00\ 0001)_2$ and the end carry is ONE. No overflow. Ignore the carry.
The result $(0000\ 0001)_2$ is correct. Note 19 - 18 = 1 which is $(0000\ 0001)_2$ in n = 6 and 2's complement system.

[2] 1's complement system:
M' = 11 1111 – 01 0010 = 10 1101 ➔ {M = 18, M' = -18}
N' = 11 1111 – 01 0011 = 10 1100 ➔ {N = 19, N' = -19}
M + N = $(01\ 0010)_2$ + $(01\ 0011)_2$ = $(10\ 0101)_2$ and Cn = 1 and Cn+1 = 0 (i.e. the end carry is ZERO) ➔ OVERFLOW
The result $(10\ 0101)_2$ is NOT correct. Note 18 + 27 = 45 which is CAN NOT be represented in n = 6 and 1's complement system. The maximum positive number in this system is equal to $+(2^{n-1}-1)$ = +31.
M - N = M + N' = $(01\ 0010)_2$ + $(10\ 1100)_2$ = $(11\ 1110)_2$ and the end carry is ZERO. No overflow.
The result $(11\ 1110)_2$ is correct. Note 18 - 19 = -1 which is $(11\ 1110)_2$ in n = 6 and 1's complement system.
N – M = N + M' = $(01\ 0011)_2$ + $(10\ 1101)_2$ = $(00\ 0000)_2$ and the end carry is ONE. No overflow.
Need to correct result ➔ $(00\ 0000)_2$ + 1 (end carry) = $(00\ 0001)_2$
The result $(0000\ 0001)_2$ is correct. Note 19 - 18 = 1 which is $(0000\ 0001)_2$ in n = 6 and 1's complement system.

6) A microcontroller uses 16-bit registers. Give the following in both binary and decimal:

(a) The maximum unsigned integer number that can be stored.

(b) The smallest (negative) number and the largest (positive) number that can be stored using the sign-magnitude notation.

(c) The smallest (negative) number and the largest (positive) number that can be stored using the 2`s complement notation.

Solution:

n = 16, R = 2

(i) maximum unsigned integer = (1111 1111 1111 1111) = $2^{16} - 1$ = 65535.

(ii) For signed magnitude representation – MSB is reserved for sign ➔ remaining = 15 bits for magnitude

➔ smallest negative integer = $-(2^{15}-1)$ = -32767

➔ largest positive integer = $+(2^{15}-1)$ = +32767

(iii) Using 2's complement:

smallest negative integer = -2^{16-1} = 32768,

largest positive integer = $+(2^{16-1}-1)$ = 32767

7) Prove the following Identities using Boolean algebraic manipulation:

a) x'y' + xy + x'y = x' + y

b) x'y + xy' + xy + x'y' = 1

c) xy' + y'z' + x'z' = xy' + x'z'

Solution:

a) LHS = x'y' + xy + x'y = x'y' + xy + x'y + x'y = x'(y'+y) + y(x'+x)
= x' + y
= RHS

b) LHS = x'y + xy' + xy + x'y' = x'(y+y') + x(y'+y) = x' + x = 1
= RHS

c) LHS = xy' + y'z' + x'z' = xy'z + xy'z' + xy'z' + x'y'z' + x'y'z' + x'yz'
= xy'z + xy'z' + x'y'z' + x'yz'
= xy'(z+z') + x'z'(y'+y) = xy' + x'z'
= RHS

8) Simplify the following expressions to a minimum number of "*literals*" using Boolean algebraic manipulation

a) ABC + A'B + ABC'

b) (x + y)' (x' + y')

c) (BC' + A'D)(AB' + CD')

Solution:

a) ABC + A'B + ABC' = AB(C+C') + A'B = AB + A'B = (A+A')B = B

b) x'y'.x' + x'y'.y' = x'y' + x'y' = x'y'

c) AB'BC' + AB'A'D + A'DAB' + A'DCD' = 0 + 0 + 0 + 0 = 0

9) Using De-Morgan's theorem to derive the complement (F') of the function F = xy +z

Using algebraic manipulations verify (for this function) that F.F'=0 as well as F + F' = 1

Solution:

a) F' = (xy+z)' = (xy)' z' = (x'+y') z'

b) FF' = (xy+z) (x'+y')z' = xyz'(x'+y') = 0 + 0 = 0

F+F' = (xy+z) + (x'+y')z' = xy + z + x'z' + y'z' = xy + x'z' + (z+y')(z+z')
= xy + x'z' + z + y' = (x+y')(y+y') + x'z' + z = x+y'+x'z'+z = (x+x')(x+z')+y'+z
= x+z'+y'+z = 1

10) Derive the truth table and draw the logic diagram of the following function:

f(A,B, C, D) = BC' + AB + ACD

Solution:

a)

A	B	C	D	BC'	AB	ACD	F
0	0	0	0				0
0	0	0	1				0
0	0	1	0				0
0	0	1	1				0
0	1	0	0	1			1
0	1	0	1	1			1
0	1	1	0				0
0	1	1	1				0
1	0	0	0				0
1	0	0	1				0
1	0	1	0				0
1	0	1	1			1	1
1	1	0	0	1	1		1
1	1	0	1	1	1		1
1	1	1	0		1		1
1	1	1	1		1	1	1

11) For the Boolean functions E and F, as given in the following truth table:

a) List the minterms and the maxterms of each function
b) List the minterms of $\overline{E}$ and $\overline{F}$
c) List the minterms of $E + F$ and EF
d) Express E and F in the sum-of-minterms algebraic form
e) Simplify E and F to expressions with a minimum number of literals

X	Y	Z	E	F
0	0	0	1	0
0	0	1	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	0
1	0	1	1	0
1	1	0	0	1
1	1	1	0	1

Solution:

a) E(X,Y,Z) = m0+m1+m2+m5, F(X,Y,Z) = m2+m3+m6+m7

E(X,Y,Z) = M3.M4.M6.M7 F(X,Y,Z) = M0.M1.M4.M5

b) E'(X,Y,Z) = m3+m4+m6+m7 F'(X,Y,Z) = m0+m1+m4+m5

c) E+F = m0+m1+m2+m3+m5+m6+m7 E.F = m2

d) $E(X,Y,Z) = \overline{X}\,\overline{Y}\,\overline{Z} + \overline{X}\,\overline{Y}Z + \overline{X}Y\overline{Z} + X\overline{Y}Z$,

$F(X,Y,Z) = \overline{X}Y\overline{Z} + \overline{X}YZ + XY\overline{Z} + XYZ$

e) Using the K-maps below, we can find: $E(X,Y,Z) = \overline{Y}Z + \overline{X}\,\overline{Z}$, and $F(X,Y,Z) = Y$

X \ YZ	00	01	11	10
0	1	1	0	1
1	0	1	0	0

X \ YZ	00	01	11	10
0	0	1	1	0
1	0	1	1	0

12) Convert the following expressions into sum-of-products and product of sums forms:

a) (AB+C)(B+C'D)

b) X'+X(X+Y')(Y+Z')

c) (A+BC'+CD)(B'+EF)

Solution:

a) (AB+C)(B+C'D) = AB + ABC'D + BC = AB(1 + C'D) + BC = AB + BC → SOP
= B(A+C) → POS

b) X'+X(X+Y')(Y+Z') = (X'+X)(X'+(X+Y')(Y+Z') = (X'+X+Y')(X'+Y+Z') = X' + Y + Z' → SOP and POS

c) (A+BC'+CD)(B'+EF) = (A+BC' +C)(A+BC'+D)(B'+E)(B'+F)
= (A+C+B)(A+C+C')(A+D+B)(A+D+C')(B'+E)(B'+F)
= (A+B+C)(A+D+B)(A+C'+D)(B'+E)(B'+F) → POS

(A+BC'+CD)(B'+EF) = AB' + AEF + BC'B' + BC'EF + CDB' + CDEF
= AB' + AEF + BC'EF + B'CD + CDEF → POS