King Fahd University of Petroleum & Minerals
Computer Engineering Dept

COE 543 – Mobile and Wireless Networks

Term 072

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Material for This Topic

In addition to the Pahlavan’s Chapter of RF propagation, the following paper is a must read:

B. Skar, “Rayleigh fading channels in mobile digital communication systems .I. Characterization,” IEEE Communications Magazine, Vol. 35, Issue 7, pp. 90-100, 1997.

Lecture Contents

1. Free-Space Propagation
2. Two-Ray Model for Mobile Radio Environment
3. Path-Loss Models:
    a. Path-Loss Models for Macrocellular Areas
    b. Path-Loss Models for Microcellular Areas
    c. Path Models for Picocellular Indoor Areas
    d. Path Models for Femtocellular Areas
4. Multi-path Propagation
    a. Rayleigh fading
    b. Doppler
    c. Delay Spread
    d. Coherence Bandwidth

Received Signal Strength

- Metric: Received Power / Transmitted Power
- Factors:
    - Distance (Path Loss Model)
    - Slow fading or Shadowing (Obstacles)
    - Fast fading (mobility, scatterers, etc.)
        - Doppler spread
        - Multipath

Free-Space Propagation

- For free-space, the ratio of received power, P_r, to transmitted power, P_t, is given by

$$\frac{P_r}{P_t} = G_t G_r \times \left(\frac{\lambda}{4\pi d} \right)^2$$

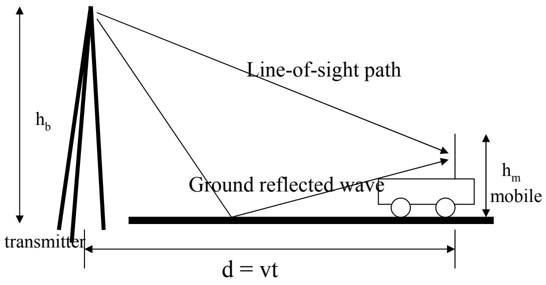
- Where: G_t and G_r are the transmit and receive antenna gains respectively – λ is the wavelength of the radio signal, while d is the distance between the transmitted and the receiver
- Define P_0 as the relative signal strength at one meter distance or $P_0 = P_t G_t G_r (\lambda/4\pi)^2$ ➔ $P_r = P_0/d^2$
- Note that received signal strength is inversely proportional to the square of the distance

Two-Ray Model for Mobile Radio Environments

- The ratio of received power, P_r, to transmitted power, P_t, is given by

$$\frac{P_r}{P_t} = G_t G_r \times \left(\frac{h_b h_m}{d^2} \right)^2$$

- Note the received power relative to transmitted power falls 40 dB per decade ($\alpha\ d^{-4}$)
- Signal strength increases as antenna heights are increased



Distance-Power Gradient Relation

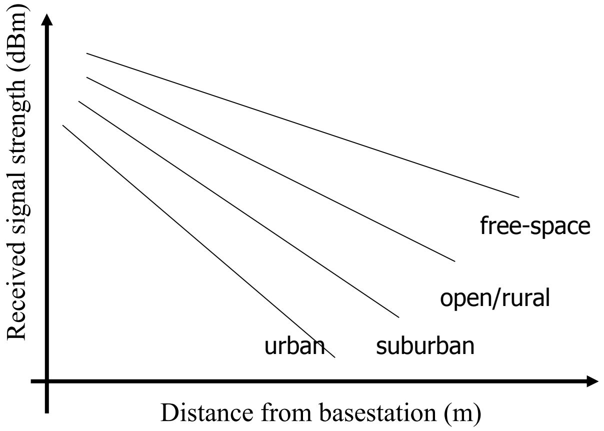
- The ratio of received power, P_r, to transmitted power, P_t, is given by

$$P_r = P_0 d^{-\alpha}$$

- For free-space α = 2
- For the simplified two-ray model, α = 4
- For indoor and urban radio channels ~ depending on obstructions and environment:
  - Indoor corridors or open areas α < 2
  - Metallic buildings α > 4 (~ 6)

Distance-Power Gradient Relation (2)

- Typical outdoor received-signal strength vs. distance curves



Distance-Power Gradient Relation (3)

- Writing the previous relation in dBs

$$10\log(P_r) = 10\log(P_0) - 10\alpha\log(d)$$

- The 10log(P_0) ~ represents the power loss in dB with respect to received power at one meter
- Loss of 10α dB per decade of distance
- Let

$$L_0 = 10\log(P_t) - 10\log(P_0)$$

be the path-loss in dB at a distance of one meter, then the total path-loss, L_p, at distance d, is given by

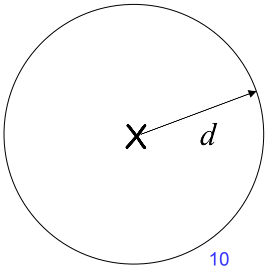
$$L_p = L_0 + 10\alpha\log(d)$$

Shadow Fading

- The received signal strength at the same distance d from transmitted is NOT exactly fixed
  - Due to obstructions (e.g. buildings for outdoor, walls for indoor, etc.)
- The variation is slow (compared to other yet to be introduced variations) – frequency independent (almost)
- Measurements have shown that when the received signal strength is reported in decibels, the variation (or random component) follows the normal distribution
- Modifying the path-loss equation to include this variation, yields

$$L_p = L_0 + 10\alpha\log(d) + X_{dB}$$

Where X_{dB} follows the normal distribution with mean zero and standard deviation of σ_{dB}



Shadow Fading (2)

- σ_{dB} is a function of the propagation environment – typical value ~ 8 dB
- The signal variation on the linear scale – follows the log-normal distribution:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma_{dB} x} \exp\left(\frac{-(\ln x - \mu)^2}{2\sigma_{dB}^2} \right)$$

- Note that from the above formula, the distribution of the ln$x$ is normal with mean μ and standard deviation of σ_{dB}
- Fade Margin (F_σ): When computing signal coverage a margin of power dBs are added to compensate for the slow fading parameter

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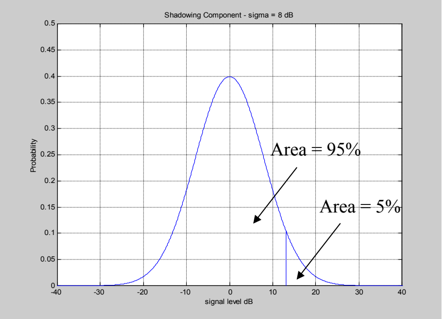
Example: Fade Margin Calculation

- A mobile system is to provide 95% successful communication at the fringe of coverage with a shadow fading component having a zero mean Gaussian distribution with standard deviation of 8 dBs. What is the required fade margin?

Example: Fade Margin Calculation

- Solution:

Let the local mean = M,
Therefore, the overall signal level,
$S = M + X$
where X is a normal r.v. specifying the shadowing process (i.e. $X \sim N(0, \sigma=8\ \text{dB})$)
Note that Prob[S< M] = Prob[S>M] = 0.5
It is desired to add a fade margin F such that
Prob[M+X<M+F] = 0.95, or
Prob[X > F] = 0.05, or
Prob[X/σ > F/σ] = 0.05
→ 0.05 = 0.5 erfc((F/σ)/sqrt(2))
→ F/σ = 1.163Xsqrt(2)
→ F = 13.16 dB



Recall: for a Gaussian variable X with mean m and standard deviation of σ
Prob[X > F] = Prob[(X-m)/ σ > (F-m)/ σ] = Prob[X_σ > F_σ]
where X_σ is the zero-mean Gaussian r.v. with unity standard deviation.

$\Pr[X_\sigma > F_\sigma] = \frac{1}{\sqrt{2\pi}} \int_{F_\sigma}^{\infty} e^{-t^2/2}\, dt$ which is tabulated as Q(F_σ)

One can also use the erfc as well: since erfc is defined as $erfc(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^2}\, dt$

therefore, Q(F_σ) = 0.5 erfc(F_σ/sqrt(2)),

Path-Loss Models for Megacellular Areas

- Span 100s of kilometers
- Served mostly by LEO Satellites

- Model ~ free space

Path-Loss Models for Macrocellular Areas – Okumura-Hata Model

- Empirical formula calculating the median path-loss for a quasismooth terrain in an urban area

$$L_p = 69.55 + 26.16\log f_c - 13.83\log h_b - a(h_m) + \left[44.9 - 6.55\log h_m\right]\log d$$

- Where
  - f_c in MHz - 150 < f_c < 1500 MHz
  - h_b in meters – basestation antenna height – 30 < h_b < 200 m
  - h_m in meters – mobile antenna height - 1 < h_m < 10 m
  - d in kilometers – distance - 1 < d < 20 km

Path-Loss Models for Macrocellular Areas – Okumura-Hata Model (2)

- The correction factor for the mobile antenna height is given by:
- Small-medium city:

$$a(h_m) = (1.1\log f_c - 0.7)h_m - (1.56\log f_c - 0.8)$$

- Large city:

$$a(h_m) = 8.29(\log 1.54h_m)^2 - 1.1, \qquad fc \le 200MHz$$

$$a(h_m) = 3.2(\log 11.75h_m)^2 - 4.97, \qquad fc \le 400MHz$$

- For a suburban area:

$$L_p = L_p(urban) - \left[2\log\left[\frac{f_c}{28}\right]^2 - 5.4\right]$$

- For an open area:

$$L_p = L_p(urban) - 4.78(\log f_c)^2 + 18.33\log f_c - 40.94$$

Path-Loss Models for Macrocellular Areas – Okumura-Hata Model (3)

- COST-231: For PCS operating at 1,800 – 2,000 MHz, the European Co-operative for Scientific and Technical Research extended the previous model
  - See parameters in Appendix 2C in Pahlavan's book
- Joint Technical Committee (JTC) of Telecommunication Industry Association (TIA) has models for PCS at 1,800 MHz

A good summary of Hata / COST 231 models can be found at http://www.comappls.com/tonyt/Applets/Propagation/PropEqns1.doc

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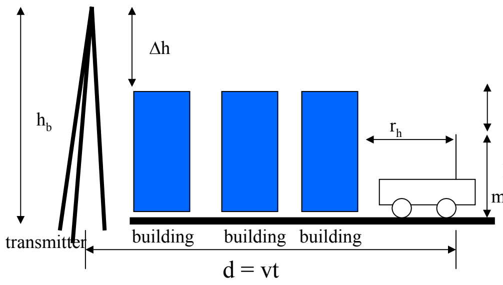
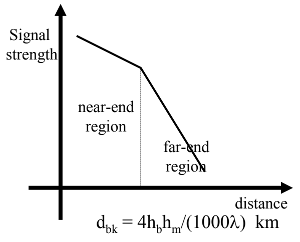
Path-Loss Models for Microcelluar Areas

- Spanning: 100 m ~ few kilometers
- Basestation height ~ roof tops or lampposts
- See Table 2.2 (of Pahlavan's book) for detailed model



Typical microcellular environment



$d_{bk} = 4h_b h_m/(1000\lambda)$ km

Typical path-loss model

Path-Loss Models for Picocelluar Indoor Areas

- Covering buildings or parts of buildings
- Spanning: 30 ~ 100 m
- Applications: WLANs, wireless PBX, PCS
- Multifloor Attenuation Model

$$L_p = L_0 + nF + 10\alpha \log(d)$$

Where F is signal attenuation per floor
L_0 is the path-loss at first meter
d is distance in meters
n is number of floors

- Typical values for F = 10 dB and 16 dB for measurements at 900 MHz and 1.7 GHz, respectively
- Furniture objects cause shadowing ~ σ_{dB} = 4 dB

Path-Loss Models for Picocelluar Indoor Areas – JTC Model (@ 1.8 GHz)

- The previous formula modified to

$$L_p = L_0 + L_f(n) + 10\alpha \log(d) + X$$

Where L_f is power loss due to floors
L_0 is the path-loss at first meter
d is distance in meters
α is the path-loss exponent
n is number of floors
X is log-normally distributed (σ_{dB})

Environment	Residential	Office	Commercial
L_0	38	38	38
10α	28	30	22
$L_f(n)$	4n	15+4(n-1)	6+3(n-1)
σ_{dB}	8	10	10

Path-Loss Models for Picocelluar Indoor Areas – Partition Dependent Model

- Fixed path-loss exponent α = 2; introduces loss for each partition encountered by a straight line connecting the transmitter and receiver

$$L_p = L_0 + 20\log(d) + \sum m_{type} w_{type}$$

Where m_{type} refers to # of partition of that type
w_{type} the loss in dBs for that type
d is distance in meters

- Partitions: Soft ~ 1.4 dB – Hard ~ 2.4 dB
- Walls: dry plywood ~ 1 dB – concrete ~ 20 dB

Signal Attenuation of 2.4 GHz through	**dB**
Window in brick wall	2
Metal frame, glass wall into building	6
Office wall	6
Metal door in office wall	6
Cinder wall	4
Metal door in brick wall	12.4
Brick wall next to metal door	3

Path-Loss Models for Femtocelluar

- Span: 2 ~ 10s of meters
- Individual residences
- Applications: bluetooth, home RF
- Previous JTC model may apply at 1.8 GHz
- For operation at 2.4 GHz and 5 GHz (unlicensed bands)

$$L_p = L_0 + 10\alpha\log(d)$$

Where L_0 is the path-loss at first meter
d is distance in meters
α is the path-loss exponent

Path-Loss Models for Femtocelluar – Model Parameters

f_c (GHz)	Environment	Scenario	Path Loss at d = 1 m (dB)	Path Loss Gradient α
2.4	Indoor office	LOS	41.5	1.9
		NLOS	37.7	3.3
5.1	Meeting room	LOS	46.6	2.22
		NLOS	61.6	2.22
5.2	Suburban residences	LOS and same floor	47	2 to 3
		NLOS and same floor		4 to 5
		NLOS and room in the higher floor directly above Tx		4 to 6
		NLOS and room in the higher floor not directly above the Tx		6 to 7

Multi-path Characteristic of a Radio Wave

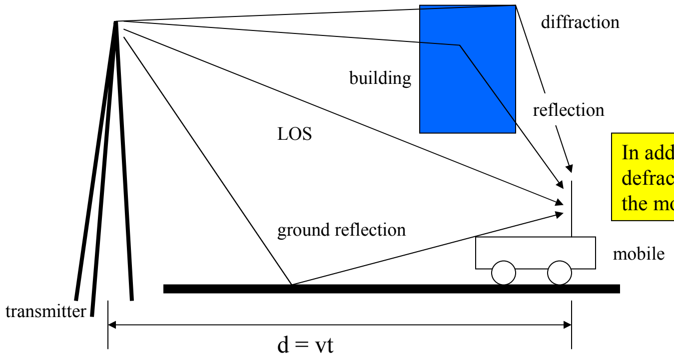
- Radio waves arrive at the receiver from different directions with different delays
- At the receiver antenna they combine via vector addition

⇓

- Received signal level varies (10s of dBs):
  - Short-term (rapid) variations
  - Long-term (slow) variations

Statistical Propagation Model

- Random number of paths, each with
  - Random amplitude and phase
  - Random Doppler
  - Random delay



In addition to local scatterers, defractors, and reflectors around the mobile itself

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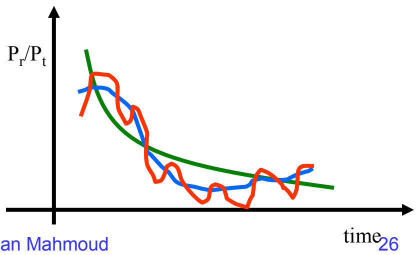
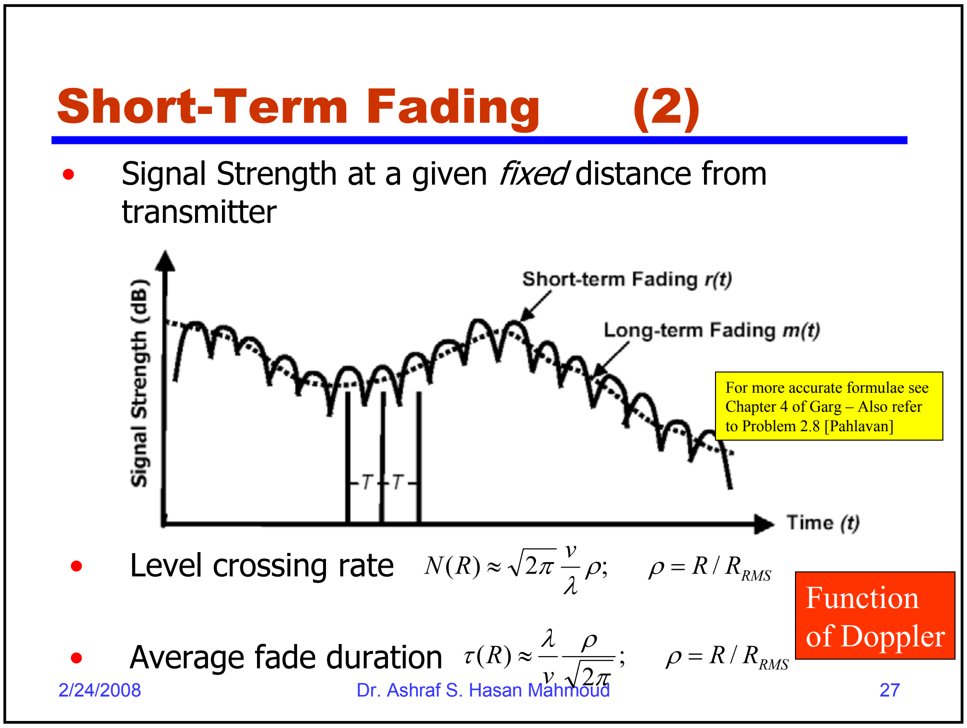
Short-Term Fading

- Three contributors to final signal level:
  - Path loss (function of distance and path loss exponent – covered earlier)
  - Long-term or slow fading – function of obstructions and environment (covered earlier too)
  - Short-term or fast fading – function of scattering environment and speed of mobile

Received power relative to transmitted power as the mobile moves away from transmitter – Of course the net or observed level is the red one





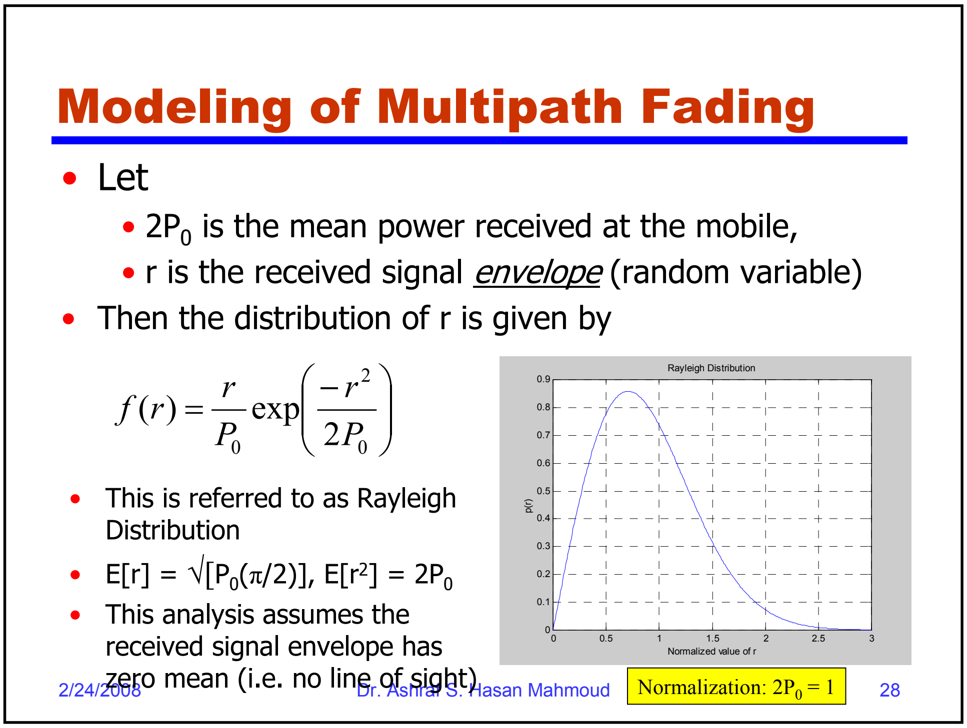
Short-Term Fading (2)

- Signal Strength at a given *fixed* distance from transmitter

For more accurate formulae see Chapter 4 of Garg – Also refer to Problem 2.8 [Pahlavan]

- Level crossing rate $N(R) \approx \sqrt{2\pi}\frac{v}{\lambda}\rho; \quad \rho = R/R_{RMS}$
- Average fade duration $\tau(R) \approx \frac{\lambda}{v}\frac{\rho}{\sqrt{2\pi}}; \quad \rho = R/R_{RMS}$

Function of Doppler



Modeling of Multipath Fading

- Let
  - $2P_0$ is the mean power received at the mobile,
  - r is the received signal *envelope* (random variable)
- Then the distribution of r is given by

$$f(r) = \frac{r}{P_0}\exp\left(\frac{-r^2}{2P_0}\right)$$

- This is referred to as Rayleigh Distribution
- $E[r] = \sqrt{[P_0(\pi/2)]}$, $E[r^2] = 2P_0$
- This analysis assumes the received signal envelope has zero mean (i.e. no line of sight)

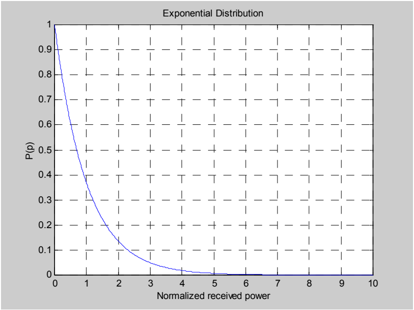
Normalization: $2P_0 = 1$

Modeling of Multipath Fading (2)

- What is the instantaneous power?
- Instantaneous power = r^2
- The distribution of the instantaneous power is exponential, as in

$$f(p) = \frac{1}{2P_0}\exp\left(\frac{-p}{2P_0}\right)$$

- You can note that the mean received power is $2P_0$



Normalization: $2P_0 = 1$

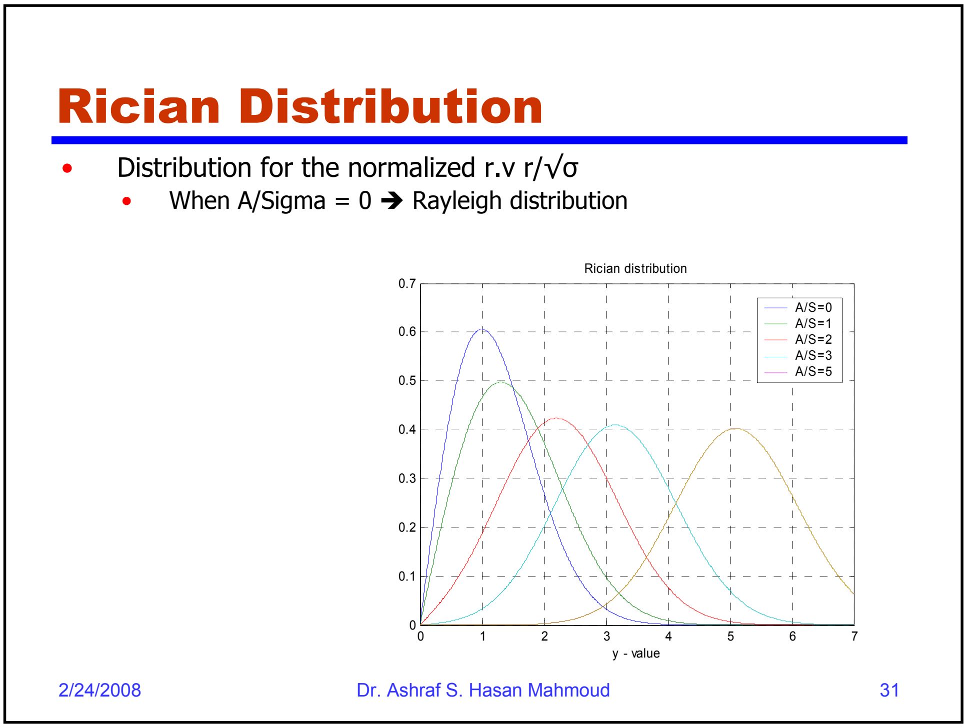
What if There is a Line-of-Sight Component?

- When there is a LOS component, the signal envelope follows the Rician distribution:

$$f(r) = \frac{r}{P_0}\exp\left(\frac{-\left(r^2+K\right)}{2P_0}\right) I_0\left(\frac{Kr}{P_0}\right) \qquad r \geq 0; K \geq 0$$

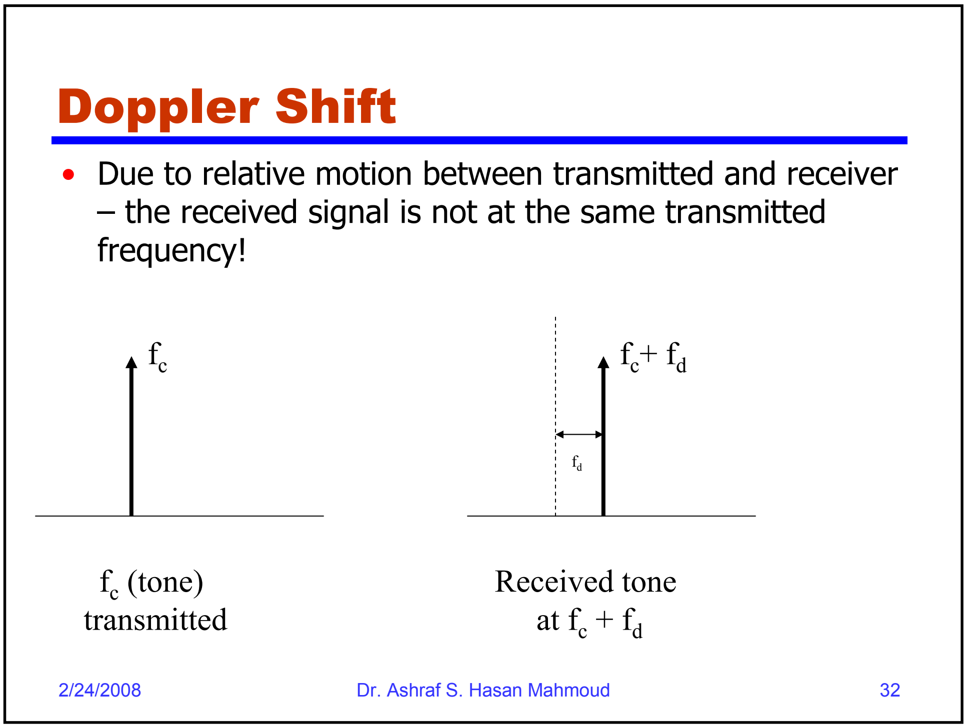
- **Exercise**:

1. Derive the Rician probability distribution for the case where the LOS component is not equal to 0?
2. Show that this reduces to the Rayleigh distribution when the line-of-sight component is set to zero.



Rician Distribution

- Distribution for the normalized r.v r/√σ
  - When A/Sigma = 0 ➔ Rayleigh distribution



Doppler Shift

- Due to relative motion between transmitted and receiver – the received signal is not at the same transmitted frequency!

f_c (tone)
transmitted

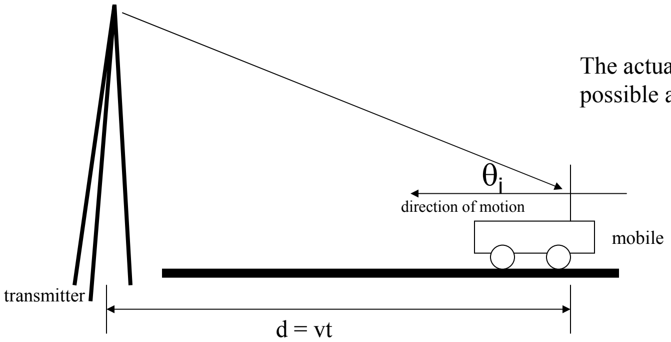
Received tone
at $f_c + f_d$

Doppler Shift (2)

- Due to relative motion between transmitted and receiver – the received signal is not at the same transmitted frequency!

θ_i : Arrival angle for the ith ray

The actual picture includes all possible arrivals



Doppler Shift (3)

- Speed of light, $C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times10^8$

Where μ_0 and ε_0 are the free-space magnetic permeability ($4\pi \times 10^{-7}$ H/m), and the free-space electric permitivity (8.854×10^{-12} F/m)

- For a vehicle moving in a direction with a constant velocity, *v*, the received carrier is Doppler shifted by

$$f_d = f_m \times \cos(\theta) = \frac{v\cos(\theta)}{\lambda} = \frac{v_{eff}}{\lambda} = \frac{v_{eff} \times f_c}{c}$$

Where $f_m = v/\lambda$ = maximum value of Doppler

v_{eff} is the effective velocity

f_c is the carrier frequency

θ is the path angle

Example:

- A vehicle is traveling at speed of 100 km/h receiving a signal at a carrier frequency of 880 MHz ➔ maximum Doppler shift, $f_m = v/\lambda = 100X(1000/3600)X880X10^6/3X10^8 = 81.5$ Hz

Is Doppler Shift Bad?

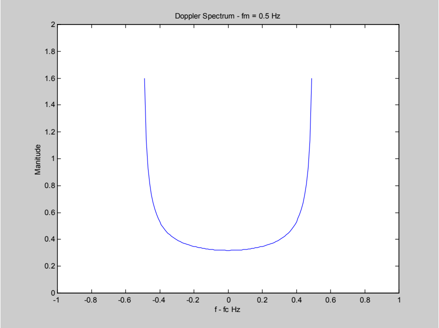
- It is not so much the frequency shift that is harmful, but rather the fact that a large number of rays with different amplitudes and phases add vector-wise to form the resultant signals
- This resultant signal varies greatly in amplitude and phase
- The GREATER the Doppler shift the greater this variation
- Verify the above statement by comparing the expression for the level crossing rate with the Doppler shift expression

Doppler Spectrum

- The Doppler spectrum for a Rayleigh fading channel is modeled by

$$D(f) = \frac{1}{2\pi f_m} \times 1 \Big/ \sqrt{1 - \left((f - f_c) \big/ f_m \right)^2}$$

- The uniform model is also used for indoor channels (see problem 2.10 [Pahlavan])



How Do Systems Overcome the Doppler Effect?

- Analog Technology:
  - Use low carrier frequency → f_m is low
- GSM
  - Channel bit rate well above Doppler spread
  - TDMA during each bit / burst transmission the channel is fairly constant.
  - Receiver training/updating during each transmission burst
  - Feedback frequency correction
- IS-95
  - Downlink: Pilot signal for synchronization and channel estimation
  - Uplink: Continuous tracking of each signal

Source: http://www.wireless.per.nl:202/multimed/cdrom97/doppler.htm

Delay Spread

- Due to multipath, a transmitted narrow pulse arrives as multiple added copies with different strengths and different delays
- Wideband channel impulse response

$$h(t) = \sum_{i=0}^{L} \alpha_i \delta(t - \tau_i) \exp(j\phi_i)$$

- Where: α_i ~ Rayleigh distributed amplitude
  ϕ_i ~ uniform r.v. $(0, 2\pi)$
  τ_i ~ multipath arrival time
- $E[\alpha_i^2] = 2\sigma_i^2$ – power associated with the ith path
- Intersymbol Interference (ISI) → irreducible errors

See tutorial at: http://users.ece.gatech.edu/~mai/tutorial_multipath.htm

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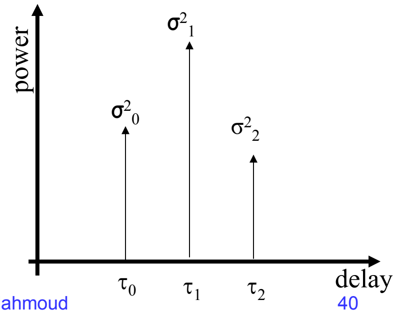
RMS Delay Spread

- For a discrete wideband channel impulse response as the one depicted, the rms delay spread parameter is computer by

$$\tau_{rms} = \sqrt{\frac{\sum_{k=0}^{N} \tau_k^2 \sigma_k^2}{\sum_{k=0}^{N} \sigma_k^2} - \left(\frac{\sum_{k=0}^{N} \tau_k \sigma_k^2}{\sum_{k=0}^{N} \sigma_k^2} \right)^2}$$

- Delay spread limits the maximum data rate supported by the channel – successive symbols interfere with each other (ISI) and can not be detected properly – refer to coherence bandwidth slides



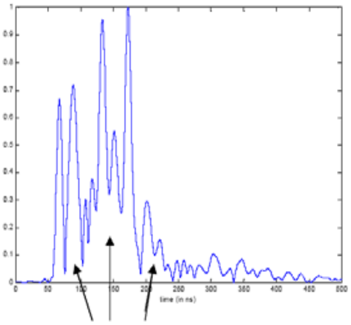
Typical Delay Spread Figures

- **Indoor: @910 MHz: 50~250 ns; @2.4GHz: 10~40 ns**
- **Urban areas**
  - **RMS delay spread: 2 μsec**
  - **Min 1 μsec to max 3 μsec**
- **Suburban areas**
  - **RMS delay: 0.25 μsec to 2 μsec**
- **Rural areas**
  - **RMS delay: up to 12 μsec**
- **GSM example**
  - **Bit period 3.69 μsec**
  - **Uses adaptive equalization to tolerate up to 15 μsec of delay spread (26-bit Viterbi equalizer training sequence)**

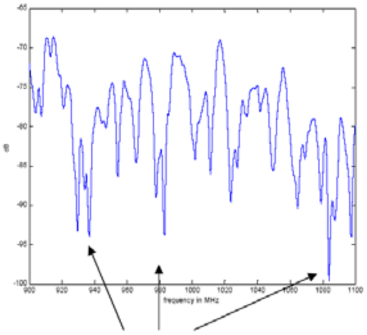
Coherence Bandwidth

- Coherence Bandwidth of a Channel, Bc: The range of frequencies that experience the same (flat) fading
- $Bc \sim 1/(5\tau_{rms})$
- Signals with bandwidth less than Bc are said to experience *flat* fading
- Signals with bandwidth greater than Bc are said to experience *selective* fading
- Transmission rate, R is typically equal to Bc (without resorting to advanced transmission techniques)
  - Example: For indoor areas $\tau_{rms} \sim$ 30ns to about 300 ns → Maximum data rate ~ 6.7 Mb/s to about 0.67 Mb/s at most
  - Example: For outdoor urban $\tau_{rms} \sim 4\mu s$ → Maximum data rate ~ 50 kb/s

Frequency Selective Fading



(a) Multipath arrival



(b) Frequency selective fading

Mitigation Methods

Issue	Performance Affected	Mitigation Techniques
Shadow fading	Received signal strength	Fade margin – Increase transmit power or decrease cell size
Fast fading	Bit error rate	Error control coding
	Packet error rate	Interleaving, Frequency hopping, Diversity
Multipath delay spread	ISI and irreducible error rates	Equalization, DS-spread spectrum, OFDM, Directional antennas

References

- Chapter 2: "Principles of Wireless Networks – A Unified Approach," Kaveh Pahlavan and Prashant Krishnamurthy
- Chapter 4: "Wireless sand Personal Communications Systems," Vijay Garg and Joseph Wilkes
- Chapters 3 and 4 "Probability and Random Processes for Electrical Engineering," Alberto Leon-Garcia Addison Wesley, 1989 (TK153.L425 1989)

☞

A *necessary* background info for the course

Example 1:

- **Problem 2.8 [Pahlavan]**: The modulation technique used in the existing AMPS is analog FM. The transmission bandwidth is 30 kHz per channel and the maximum transmitted power from a mobile use is 3 W. The acceptable quality of the input SNR is 18 dB, and the background noise in the bandwidth of the system is -120 dBm (120 dB below the 1mW reference power). In the cellular operation we may assume the strength of the signal drops 30 dB for the first meter of distance from the transmitter antenna and 40 dB per decade of distance for distances beyond 1 meter.
  a. What is the maximum distance between the mobile station and the base station at which we have an acceptable quality of signal?
  b. Repeat (a) for digital cellular systems for which the acceptable SNR is 14 dB

Example 1: cont'd

- **Solution:**

(a) Maximum distance for an SNR of 18 dB.

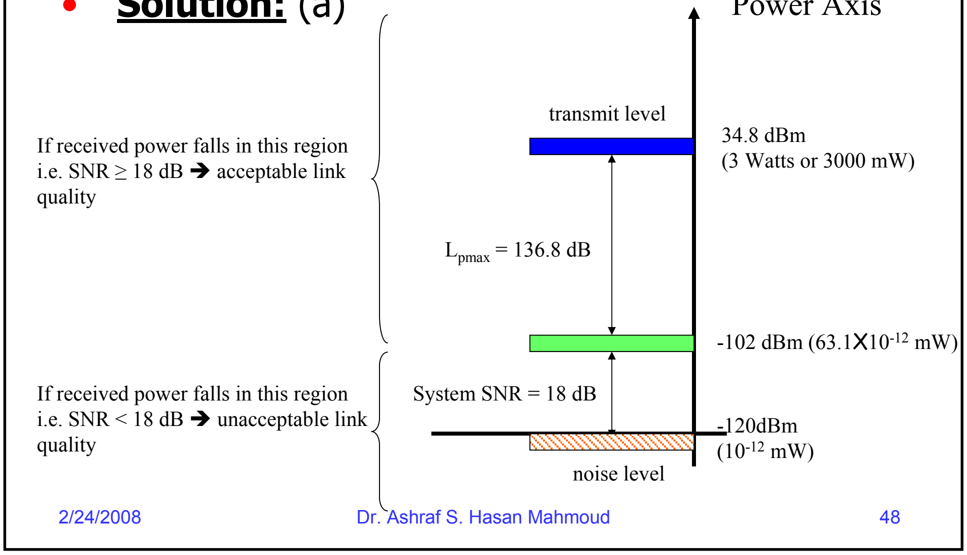
- The transmitter power is $P_t = 10\log(3\,W/1\,mW)$ = 34.8 dBm
- The minimum acceptable received power is P_{rmin} = -120 dBm + 18 dB = -102 dBm
- The maximum allowable path loss is $L_{p\max} = P_t - P_{r\min}$ = 34.8 dBm - (-102 dBm) = 136.8 dB
- The path loss model based on 30 dB in the first meter and 40 dB per decade of distance is

$$L_p = 30 + 40\log(d) \quad \text{so that} \quad d = 10^{\frac{L_p - 30}{40}} \quad \text{and}$$

$$d_{\max} = 10^{\frac{L_{pmx} - 30}{40}} = 10^{\frac{136.8 - 30}{40}} = 468 \text{ m}$$

Example 1: cont'd

- **Solution:** (a)



Power Axis

If received power falls in this region i.e. SNR ≥ 18 dB ➔ acceptable link quality

If received power falls in this region i.e. SNR < 18 dB ➔ unacceptable link quality

Example 1: cont'd

- **Solution:**

(b) Maximum distance for an SNR of 14 dB.

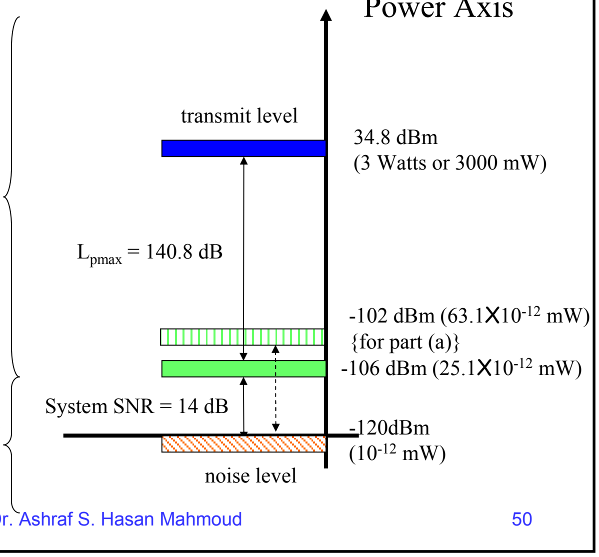
- The transmitter power is $P_t = 10\log(3\,W/1\,mW) = 34.8$ dBm
- The minimum acceptable received power is P_{rmin} = -120 dBm + 14 dB = -106 dBm
- The maximum allowable path loss is $L_{p\max} = P_t - P_{r\min}$ = 34.8 dBm - (-106 dBm) = 140.8 dB
- The path loss model based on 30 dB in the first meter and 40 dB per decade of distance is

$L_p = 30 + 40\log(d)$ so that $d = 10^{\frac{L_p - 30}{40}}$ and

$$d_{\max} = 10^{\frac{L_{pmx} - 30}{40}} = 10^{\frac{140.8 - 30}{40}} = 589 \text{ m}$$

Example 1: cont'd

- **Solution:** (b)



If received power falls in this region i.e. SNR ≥ 14 dB ➔ acceptable link quality

If received power falls in this region i.e. SNR < 14 dB ➔ unacceptable link quality

Example 1: cont’d

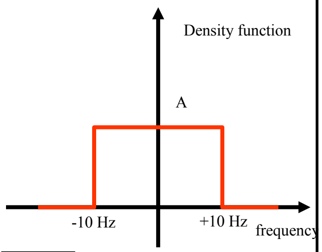
- **Solution:**
- A reduced SNR requirement at the mobile receiver translates to greater coverage distance
- Digital modulation and advance communication techniques (equalization, diversity, CDMA, etc) allow the reduction of minimum acceptable SNR

Example 2:

- **Problem 2.10 [Pahlavan]:** The Doppler spectrum of the indoor radio is often assumed to have uniform distribution with a maximum Doppler shift of 10 Hz
  a. Determine the rms Doppler spread of the channel
  b. Determine the average number of fades per second and the average fade duration, assuming that the threshold for fading is chosen 10 dB below the average rms value of the signal

Example 2:

- **Solution:** (a)
- The maximum Doppler frequency is ±10 Hz, and for a uniform distribution the area should be equal to 1, so that *A* [10 - (-10)] = 20 *A* = 1, or *A* = 0.05 ➔
  D(λ) = 0.05 [U(λ + 10 Hz) - U(λ - 10 Hz)]



- The RMS Doppler spread is given by $\lambda_{rms} = \sqrt{\overline{\lambda^2} - \left(\overline{\lambda}\right)^2}$, but

$$\overline{\lambda} = \int_{-\infty}^{\infty} \lambda D(\lambda) d\lambda = \int_{-10}^{10} 0.05\, \lambda\, d\lambda = 0.05 \left[\frac{\lambda^2}{2}\right]_{-10}^{10} = \text{0 Hz, and}$$

$$\overline{\lambda^2} = \int_{-\infty}^{\infty} \lambda^2 D(\lambda) d\lambda = \int_{-10}^{10} 0.05\, \lambda^2\, d\lambda = 0.05 \left[\frac{\lambda^3}{3}\right]_{-10}^{10} = 33.33 \text{ Hz}^2\text{, therefore}$$

$$\lambda_{rms} = \sqrt{\overline{\lambda^2} - \overline{\lambda}^2} = \sqrt{33.33 - (0)^2} = 5.77 \text{ Hz}$$

Example 2:

- **Solution:** (b)

For a Raleigh fading envelope distribution, the average number of downward crossings of a level *A* per second, *N*, is given by:

$$N(\rho) = \sqrt{2\pi}\, B_{D-rms}\, \rho\, e^{-\rho^2}$$

where ρ = A/A_{rms} is the ratio of the threshold level to the RMS amplitude of the fading envelope, and $B_{D\text{-}rms}$ is the RMS Doppler shift of the signal.

The average fade duration for a given threshold ρ is given by

$$\tau(\rho) = \frac{\Pr ob[\alpha < \rho]}{N(\rho)} = \frac{e^{\rho^2} - 1}{\sqrt{2\pi}\, B_{D-rms}\, \rho}$$

Example 2: cont'd

- **Solution:** (b)

In our case, ρ = -10 dB or 0.316 (This is an amplitude so use 20log(x)), and $B_{D\text{-}rms}$ = 5.77 Hz

and

$$N(\rho) = \sqrt{2\pi}\, B_{D-rms}\, \rho\, e^{-\rho^2} = \sqrt{2\pi}\,(5.77\ Hz)\, 0.316\, e^{-0.316^2} =$$ 4.14 fades / sec

The average fade duration $\tau(\rho) = \dfrac{e^{\rho^2} - 1}{\sqrt{2\pi}\, B_{D-rms}\, \rho} = \dfrac{e^{0.316^2} - 1}{\sqrt{2\pi}\, 5.77 Hz\, 0.316}$ =23 msec

---

Example 3:

- **Problem 2.14 [Pahlavan]:** Use a software tool like Matlab^{TM} or Mathcad to generate 1000 impulse responses of the JTC indoor residential radio channel (for case A). Determine the RMS multi-path delay spread for each sample and plot the cumulative distribution function

Example 3: cont’d

- **Solution:** (code)

```
%############## Chapter 2, Problem 2.14 #########
% dependents: ftrms.m
% Generate 1000 cirs of JTC indoor residential
% RMS delay spread for each sample
% CDF of the RMS delay spread
%###############################################
close all;
clear;
clc;
akA_db = [0 -13.8];      % avg tap power in db
tkA    = [0 100];        % relative delay in ns
```

channel definition

```
%---------------------------------------------
ak_square = 10.^(akA_db/10);% avg tap amplitude linear
tk        = 1e-9*tkA;    % relative delay in seconds
Ntaps     = length(tk);
for i=1:1000
   for j=1:Ntaps
      x_nor_1     = randn(1);
      x_nor_2     = randn(1);
      x_rayleigh  = sqrt(x_nor_1^2 + x_nor_2^2);
      ak_square_rayleigh(j) = ak_square(j) +
   x_rayleigh^2;
   end;
   trmsA(i) = ftrms(ak_square_rayleigh, tk);
end;
%---------------------------------------------
[fA x] = hist(trmsA * 1e9,[0:2:300]);
pA     = fA./sum(fA);
cdfA   = cumsum(pA);
sfA    = 1 - cdfA;
figure(1)
plot(x, sfA)
title('JTC indoor residential wideband models - Model
   A')
xlabel('RMS delay spread (ns)')
ylabel('probability signal level > abscissa')
```

creating random channel samples

calculating the survivor function and plotting the results

Example 3: cont’d

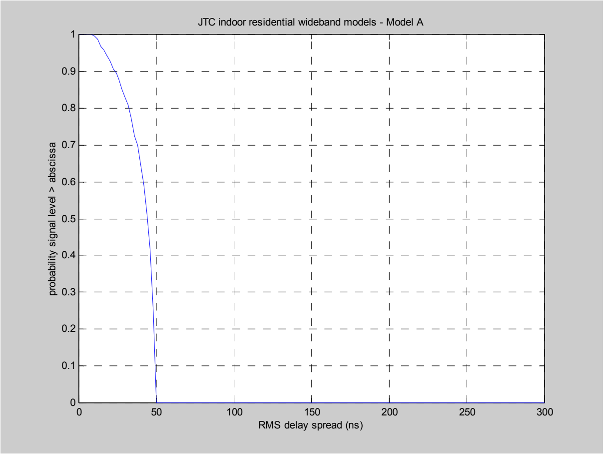
- **Solution:** (code)

```
function y=ftrms(sig,tau);
% function y =ftrms(sig,tau) to calculate the RMS delay
   spread
% sig is the local mean strengths of the discrete
   multipath components
% tau is the relative delays
% Note you should supply sig = E(alph^2)
den = sum(sig);
tbar=sum(sig.*tau)./den;
tbar2=sum(sig.*tau.^2)./den;
y=sqrt(tbar2-tbar^2);
```

Function to calculate τ_{rms}

Example 3: cont'd

- **Solution:**



Problems