King Fahd University of Petroleum & Minerals Computer Engineering Dept

COE 543 – Mobile and Wireless Networks

Term 061

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Lecture Contents

1. Traffic Engineering - Erlang C and Erlang B models

This material is found in section 4.2.5 of Pahlavan's book (page 176)

Performance of Fixed-Assignment Access Methods

- FDMA/TDMA provide a hard capacity limit (number of channels)
  - FDMA – maximum number of carriers per cell
  - TDMA – maximum number of slots per frame X number of carriers per cell
- CDMA-based also has a hard capacity limit dictated by the number of Walsh codes for example, but usually practical capacity is lower
  - Soft-capacity figure: Near the capacity boundary, the addition of one extra user degrades the link quality for all
  - Call admission control mechanism attempt to limit maximum number of ongoing calls before link quality degrades for all
- If you operate a maximum no of channels, then call blocking and call delay are the two important measures!

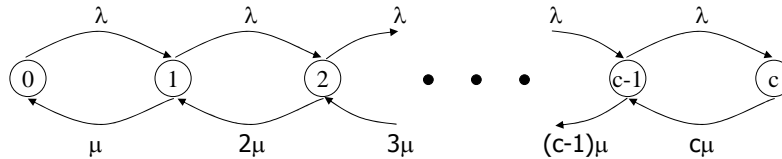## Erlang-B and Erlang-C Models

- More details to be provided in COE540
- Model designed to predict blocking probability (Erlang-B) and average call delay (Erlang-C) for a given number of channels and traffic intensity
- Valid for voice and traffic models conforming to the basic assumption (usually not applicable to data)
- Assumptions, Terminology and Parameters:
  - Channels $\leftrightarrow$ Servers: c servers
  - Users $\leftrightarrow$ Calls
  - Calls arrive according to a Poisson process with rate = λ
    - Inter-call arrival is an exponentially distributed r.v. with mean $1/\lambda$
  - Call duration is exponentially distributed r.v. with mean = $1/\mu$
  - Traffic intensity, $\rho = \lambda/\mu$

Erlang-B (M/M/c/c) Model – Call Blocking

- As was shown earlier – review previous notes, the call blocking probability is given by

$$B(c,\rho) = \frac{\rho^c / c!}{\sum_{i=0}^{c} \rho^i / i!}$$

- ρ – is referred to as the offered load, while $\rho \times [1-B(c,\rho)]$ is referred to as the carried load
- Note in this model – calls arriving while there are c calls are blocked – no buffering is employed

Truncking Efficiency

- Efficiency = ρ / C for a given blocking rate, PB
- Example: for PB = 1% (typical for cellular networks)

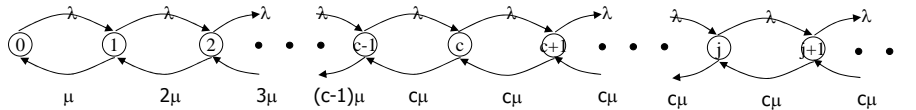| No of channels, C	Offered Load, ρ	Efficiency
5	1.361	0.272
10	4.462	0.446
50	37.900	0.785
100	84.050	0.841
180	160.400	0.891

3

Erlang-C (M/M/c) Model – Call Delay

- The probability that an arriving call having to wait is given by

$$\Pr(delay > 0) = \frac{\rho^c}{\rho^c + c!\left(1 - \dfrac{\rho}{c}\right)\displaystyle\sum_{k=0}^{c-1}\frac{\rho^k}{k!}}$$

- The average delay is given by

$$D = \Pr(delay > 0) \times \frac{1}{\mu(c - \rho)}$$

- The probability of the delay exceeding t time units is given by

$$\Pr(delay > t) = \Pr(delay > 0)e^{-(c-\rho)\mu t}$$

Example 1: Erlang B

- An IS-136 cellular provider owns 50 cell sites and 19 traffic carriers per carrier per cell each with bandwidth of 30 kHz. Assuming each user makes three calls per hour and the average holding time per call is 5 minutes. Determine the total number of subscribers that the service provider can support with a blocking rate less than 2%
- Solution:

c = 19X3 = 57 per cell

B(57,ρ) = 0.02 ➔ ρ = 45 Erlangs per cell

$(\lambda/\mu)_{sub}$ = 3/60*5 = 0.25 Erlangs per sub

Number of subs = total traffic / traffic per user

$\qquad\qquad$ = 45 / 0.25

$\qquad\qquad$ = 180 per cell

Number of subs for all sites = 180 X 50 = 9,000 subs

> Note that $\rho_{all\_subs} = (\lambda/\mu)_{all\_subs}$
> $= \lambda_{all\_subs}/\mu$
> whereas, $\rho_{sub} = (\lambda/\mu)_{sub}$
> $= \lambda_{sub}/\mu$
> λ_{all_subs} = no of subs X λ_{sub}

Example 2: Erlang C

- **A company has a system with five private telephone lines connecting two of its sites. The number of employees is 100 where on average each employee makes a three-minute telephone call every two hours.**

  a) **What is the average delay for an employee to get access to the telephone?**

  b) **What is the probability of an employee waiting more than a minute for the access to the telephone?**

Example 2: cont'd

- **Solution:**

a) c = 5, ρ = 100 users X 1/120 call/min X 3 min/call
$$= 2.5 \text{ Erlangs}$$

Prob[Delay > 0] = ρ^c /[ρ^c+c!(1-ρ/c)Σ (ρ^j/j!)]
$$= 0.13$$
The average delay = Prob[Delay > 0]/[μ(c-ρ)]
$$= 0.13/[1/3(5-2.5)]$$
$$= 0.17 \text{ minutes}$$

b) Prob(Delay > 1 min) = Prob(Delay > 0)e$^{-\mu(c-\rho)t}$
$$= 0.13 \text{ X e}^{-(5-2.5)*1/3}$$
$$= 0.0565$$