King Fahd University of Petroleum & Minerals
Computer Engineering Dept

COE 402 – Example of a Queueing System

Term 043

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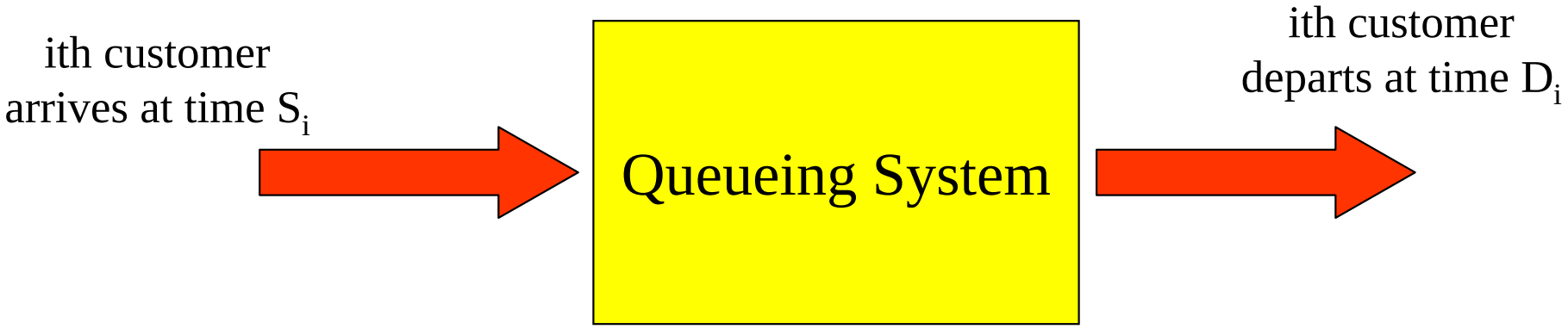
Queuing Model

- **Consider the following system:**

$A(t)$ $N(t) = A(t) - D(t)$ $D(t)$



$T_i = D_i - A_i$ $W_i = T_i - S_i$
$= D_i - A_i - S_i$

$A(t)$ – number of arrivals in (0, t]
$D(t)$ – number of departures in (0, t]
$N(t)$ – number of customers in system in (0,t]
T_i – duration of time spent in system for ith customer
W_i – duration of time spent waiting for service for ith customer

Example: Queueing System

Problem: **A data communication line delivers a block of information every 10 microseconds. A decoder check each block for errors and corrects the errors if necessary. It takes 1 microsecond to determine whether the block has any errors. If the block has one error it takes 5 microseconds to correct it and it has more than 1 error it takes 20 microseconds to correct the error. Blocks wait in the queue when the decoder falls behind. Suppose that the decoder is initially empty and that the number of errors in the first 10 blocks are: 0, 1, 3, 1, 0, 4, 0, 1, 0, 0.**

a) Plot the number of blocks in the decoder as a function of time.

b) Find the mean number of blocks in the decoder

c) What percent of the time is the decoder empty?

Example: Queueing System – cont'd

Solution:

Interarrival time = 10 μsec

Service time = 1 if no errors

1+5 if 1 error

1+20 if more than 1 error

The queue parameters (A, D, S, and W) are shown below:

Block #:	1	2	3	4	5	6	7	8	9	10
Arrivals:	10	20	30	40	50	60	70	80	90	100
Errors:	0	1	3	1	0	4	0	1	0	0
Service:	1	6	21	6	1	21	1	6	1	1
Departs:	11	26	51	57	58	81	82	88	91	101
Waiting:	0	0	0	11	7	0	11	2	0	0

Example: Queueing System – cont'd

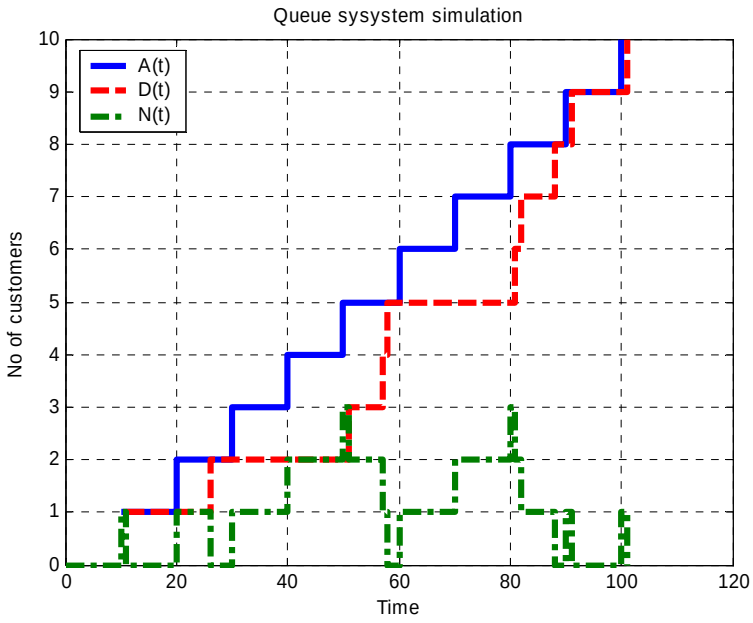
Solution:

Using the previous results and knowing that

$$N(t) = A(t) - D(t)$$

One can produce the following results

```
Average no of customers in system =   0.950
Average customer waiting time     =   3.100 microsec
Maximum simulation time           = 101.000 microsec
Duration server busy              =  65.000
Server utilization                =  0.6436
Server idle                       =  0.3564
```



The following Matlab code can be used to solve this queue system (Note the code is general – it solves any system provided The Arrivals vector A, and the service vector S)

Example: Queueing System – cont'd

```
0001 %
0002 % Problem 9.3 - Leon Garcia's book
0003 clear all
0004 A = [10:10:100];
0005 Errors = [0 1 3 1 0 4 0 1 0 0];
0006 S = zeros(size(A));
0007 D = zeros(size(A));
0008 %
0009 % this loop to computes service times
0010 for i=1:length(A);
0011     if (Errors(i)==0)  S(i) = 1;
0012     else
0013         if (Errors(i)==1) S(i) = 6;
0014         else
0015             S(i) = 21;
0016         end
0017     end
0018     %
0019     % this section computes the departure time for
the ith user
0020     if (i>1) % this is not the first user
0021         if (D(i-1) < A(i)) D(i) = A(i) + S(i);
0022         else
0023             D(i) = D(i-1) + S(i);
0024         end
0025     else
0026         D(i) = A(i)+S(i);
0027     end
0028     %
0029     % compute waiting time
0030     W(i) = D(i) - A(i) - S(i);
0031 end
0032 %
```

```
0033 % Compute N(t)
0034 T    = [];  % time axis
0035 T(1) = 0;   % time origin
0036 N    = [];  % number of cutomers
0037 N(1) = 0;   % initial condition
0038 k    = 2;   % place for next insert
0039 A_max = A(length(A)); % last arrival instant
0040 i = 1;      % index for arrivals
0041 j = 1;      % index for departures
0042 t = 0;      % system time
0043
0044 while (t < A_max)
0045     t = min(A(i), D(j));
0046     if (t == A(i))
0047         N(k) = N(k-1) + 1;
0048         T(k) = t;
0049         k    = k + 1;
0050         i    = i + 1;  % get next arrival
0051     else % departure occurs
0052         N(k) = N(k-1) - 1;
0053         T(k) = t;
0054         k    = k + 1;
0055         j    = j + 1; % get next departure
0056     end
0057 end
0058 %
0059 % record remaining departure instants
0060 for i=j:1:length(D)
0061     t    = D(i);
0062     N(k) = N(k-1) - 1;
0063     T(k) = t;
0064     k    = k + 1;
0065 end
0066
0067 k = k - 1; % decrement k to get real size of N and T
0068 %
0069 % compute means
0070 MeanW              = mean(W);
0071 T_Intervales       = T(2:k)-T(1:k-1);
0072 MeanN              = sum(N(1:k-1).*T_Intervales) / T(k);
0073 IdleDurationsIndex = find(N(1:k-1) ~= 0);
0074 Utilization        = sum(T_Intervales(IdleDurationsIndex))/T(k);
0075 %
```

Example: Queueing System – cont'd

```
0076 % Display results
0077 fprintf('Block #:  '); fprintf('%3d  ', [1:1:length(A)]); fprintf('\n');
0078 fprintf('Arrivals: '); fprintf('%3d  ', A); fprintf('\n');
0079 fprintf('Errors:   '); fprintf('%3d  ', Errors); fprintf('\n');
0080 fprintf('Service:  '); fprintf('%3d  ', S); fprintf('\n');
0081 fprintf('Departs:  '); fprintf('%3d  ', D); fprintf('\n');
0082 fprintf('Waiting:  '); fprintf('%3d  ', W); fprintf('\n');
0083 fprintf('\n\n');
0084 fprintf('Average no of customers in system = %7.3f\n', MeanN);
0085 fprintf('Average customer waiting time     = %7.3f microsec\n', MeanW);
0086 fprintf('Maximum simulation time           = %7.3f microsec\n', T(k));
0087 fprintf('Duration server busy              = %7.3f microsec\n', ...
0088          sum(T_Intervales(IdleDurationsIndex)));
0089 fprintf('Server utilization                = %7.4f\n', Utilization);
0090 fprintf('Server idle                       = %7.4f\n',1.0-Utilization);
0091 %
0092 % Plot results
0093 figure(1)
0094 h = stairs(T, N); grid
0095 set(h, 'LineWidth', 3);
0096 xlabel('Time');
0097 ylabel('No of customers in system, N(t)');
0098
0099 figure(2);
0100 [AT, AA] = stairs(A, cumsum(ones(size(A))));
0101 [DT, DD] = stairs(D, cumsum(ones(size(D))));
0102 [NT, NN] = stairs(T, N);
0103 h = plot(AT, AA, '-', DT, DD,'--r', NT, NN,'-.'); grid
0104 set(h, 'LineWidth', 3);
0105 title('Queue sysystem simulation');
0106 ylabel('No of customers');
0107 xlabel('Time');
0108 legend('A(t)', 'D(t)', 'N(t)', 0);
0109
0110 figure(3);
0111 h = stem(W); grid
0112 set(h, 'LineWidth', 3);
0113 ylabel('Waiting time');
0114 xlabel('Customer index');
0115 LegendStr = ['MeanW = ' num2str(MeanW)];
0116 legend(LegendStr, 0);
```