

King Fahd University of Petroleum & Minerals Computer Engineering Dept

COE 541 – Design and Analysis of
Local Area Networks

Term 041

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Revision – Fourier Transform

- A “transformation” between the time domain and the frequency domain

Time (t) **Frequency (f)**
s(t) **↔** **S(f)**

$$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi ft} dt$$

Fourier Transform

$$s(t) = \int_{-\infty}^{\infty} S(f) e^{+j2\pi ft} df$$

Inverse Fourier Transform

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Revision – Fourier Transform (2)

- **F.T. can be used to find the BANDWIDTH of a signal or system**
 - **Bandwidth - system: range of frequencies passed (perhaps scaled) by system**
 - **Bandwidth – signal: range of (+ve) frequencies contained in the signal**

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Revision – Fourier Transform (3)

- **Remember for periodic signals (i.e. $s(t) = s(t+T)$ where T is the period) → Fourier Series expansion:**

$$s(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} [A_n \cos(2\pi n f_0 t) + B_n \sin(2\pi n f_0 t)]$$

$$A_0 = \frac{2}{T} \int_0^T s(t) dt \quad B_n = \frac{2}{T} \int_0^T s(t) \sin(2\pi n f_0 t) dt$$

$$A_n = \frac{2}{T} \int_0^T s(t) \cos(2\pi n f_0 t) dt$$

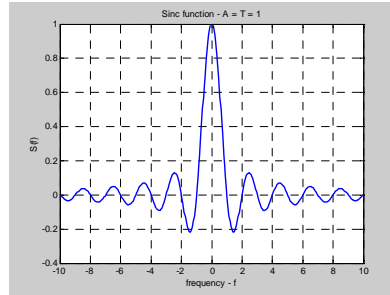
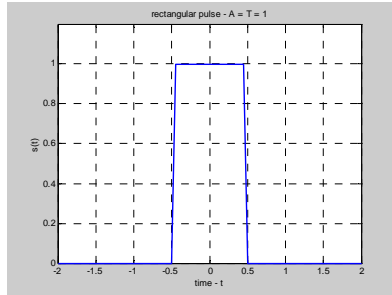
f_0 is the fundamental frequency and is equal to $1/T$

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Revision – Fourier Transform (4)

- **Famous pairs – rectangular pulse (A = T = 1)**



$$s(t) = \Pi(t/T)$$

$$S(f) = AT \frac{\sin(\pi f T)}{\pi f T}$$

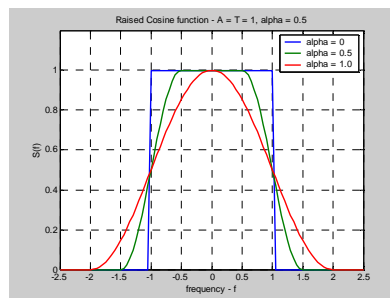
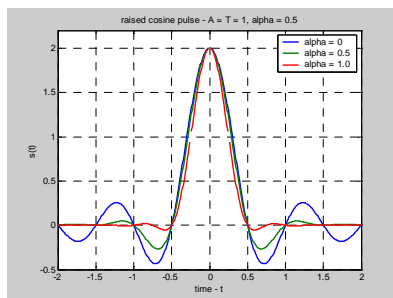
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$$S(f) = AT \text{ for } f = 0 \\ = 0 \text{ for } f = n/T; n = \pm 1, 2, \dots$$

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Revision – Fourier Transform (5)

- **Famous pairs – Raised Cosine pulse (A = T = 1), as a function of α**



$$s(t) = \frac{(2A)}{T} \frac{\cos(2\pi\alpha t)}{1 - (4\alpha t)^2} \frac{\sin(2\pi/T)}{2\pi/T}$$

$$S(f) = \begin{cases} A & |f| > \frac{1}{T} - \alpha \\ A \cos^2\left(\frac{\pi}{4\alpha}\left(|f| - \frac{1}{T} + \alpha\right)\right) & \frac{1}{T} - \alpha < |f| < \frac{1}{T} + \alpha \\ 0 & |f| > \frac{1}{T} + \alpha \end{cases}$$

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$\frac{1}{T_6}$

Revision – Fourier Transform (6)

- **Raised Cosine Pulse: $0 < \alpha < 1/T$**
- **Note that $s(t) = 0$ for $t = nT/2$ where $n = +/- 1, 2, \dots$**
 - **Very good for forming pulses**
 - **ZERO ISI for ideal situation**
- **BW for $s(t) = 1/T + \alpha$**
 - **Maximum = $2 \times 1/T$ (for $\alpha = 1/T$)**
 - **Minimum = $1/T$ (for $\alpha = 0$)**

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Revision – Fourier Transform (7)

- **Matlab code:**

```
clear all % clear all variables

A = 1;
T = 1;
alphas = [0 0.5 1];

for k = 1:length(alphas)
    alpha = alphas(k);

    t = -2:0.01:2; % define the time axis
    s_t(k,:) = ((2*A)/T) * (cos(2*pi*alpha*t) ./ ...
        (1-(4*alpha*t).^2)) .* (sin(2*pi*t/T) ./ ...
        (2*pi*t/T)); % define s(t)

    f = -2.5:0.05:2.5; % define the freq axis
    S_f(k,:) = zeros(size(f));
    i = find(abs(f) <= (1/T-alpha));
    S_f(k,i) = A;
    i = find((abs(f) <= (1/T+alpha)) & ...
        (abs(f) > (1/T-alpha)));
    S_f(k,i) = A*(cos(pi/(4*alpha)* ...
        (abs(f(i))-1/T+alpha))).^2;% define S(f)
end

figure(1);
plot(t, s_t); % plot s(t)
title('raised cosine pulse - A = T = 1');
xlabel('time - t');
ylabel('s(t)');
legend('alpha = 0', 'alpha = 0.5', 'alpha = 1.0');
axis([-2 2 -0.5 2.2]);
grid

figure(2);
plot(f, S_f); % plot S(f)
title('Raised Cosine function - A = T = 1');
xlabel('frequency - f');
ylabel('S(f)');
legend('alpha = 0', 'alpha = 0.5', 'alpha = 1.0');
axis([-2.5 2.5 0 1.2]);
grid
```

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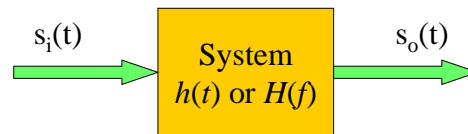
Signals and Systems

- **For linear Systems:**
 - **$h(t)$ is the system's impulse response – i.e. $s_o(t) = h(t)$ when $s_i(t) = \delta(t)$**
 - **$S_i(t)$ is system input signal**
 - **$S_o(t)$ is system output signal**

$$s_o(t) = \int_{-\infty}^{\infty} s_i(\tau)h(t - \tau)d\tau$$

$$s_o(t) = s_i(t) * h(t)$$

$$S_o(f) = S_i(f)H(f)$$



A good introduction into linear systems is found at http://www.ece.utexas.edu/~bevans/courses/ee313/lectures/04_Convolution/lecture4.pdf

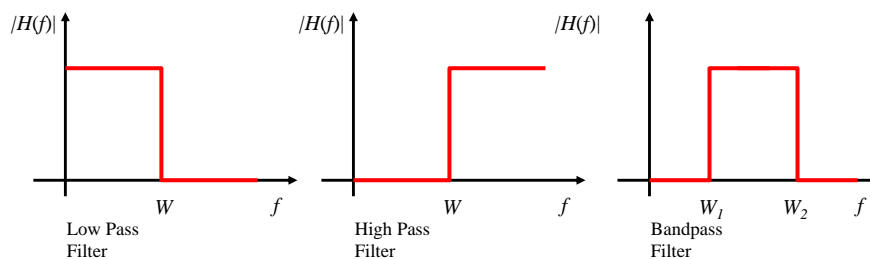
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Signals and Systems (2)

- **System bandwidth is determined by examining the Fourier transfer of the system function $h(t)$, $H(f)$**
- **Example (transmission) systems:**



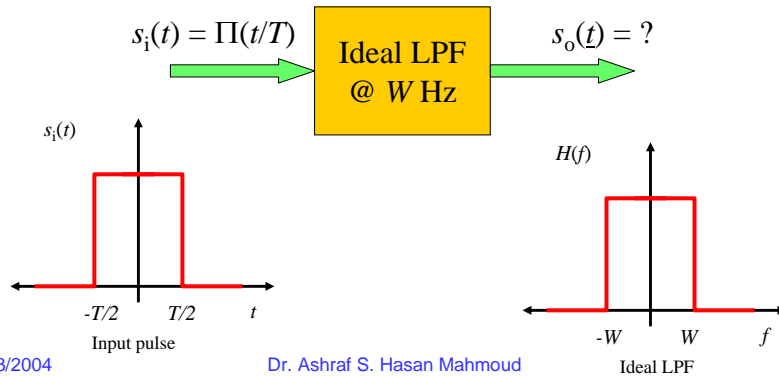
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Signals and Systems - Example

- **Ideal Low Pass Filter – find the output signal for rectangular input pulse?**



Signals and Systems - Example

- The input signal $s_i(t)$ is given by:

$$s_i(t) = \begin{cases} A & |t| \leq T/2 \\ 0 & \text{otherwise} \end{cases}$$

- Where as its Fourier transform $S_i(f)$ is given by (note that $s_i(t)$ contains all frequencies from 0 till ∞ - refer to Fourier transform of rectangular pulse):

$$S_i(f) = AT \frac{\sin(\pi fT)}{\pi fT} \quad \text{for all } f$$

- The Fourier transform of the system impulse response, $H(f)$ is given by (note this transmission system limits frequencies to at most W Hz):

$$H(f) = \begin{cases} 1 & |f| \leq W \\ 0 & \text{otherwise} \end{cases}$$

Signals and Systems - Example

- Therefore the Fourier transform of the output signal is given by:
- $|S_o(f)| = |S_i(f)| \times |H(f)|$

$$= AT \frac{\sin(\pi fT)}{\pi fT} \quad \text{for } |f| < W$$

$$= 0 \quad \text{otherwise}$$

(note the output signal has frequencies up to W Hz only)

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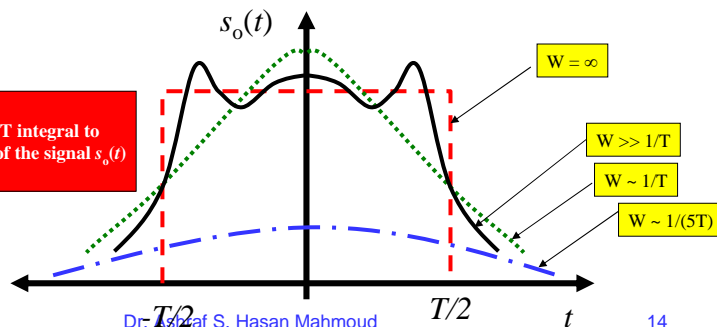
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Signals and Systems - Example

- To find the output signal $s_o(t)$, one has to use the inverse Fourier transform on $S_o(f)$
- As the BW of the system is increased, the output signal approaches a rectangular pulse (copy of input)

Exercise:

Try to apply the inverse F.T integral to obtain an analytical form of the signal $s_o(t)$ as a function of W and T.



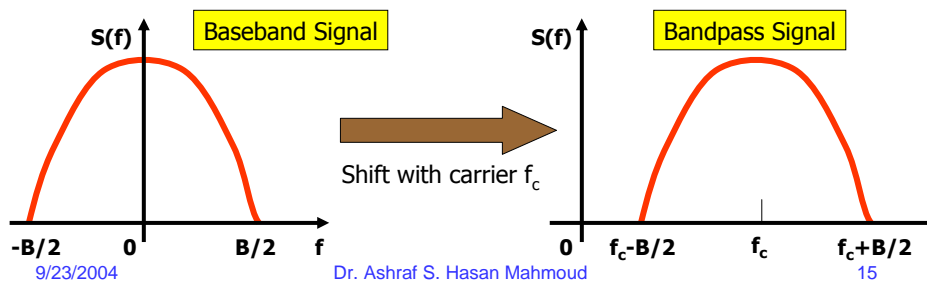
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Baseband vs. Bandband

- **Baseband Signal:**
 - Spectrum not centered around non zero frequency
 - May have a DC component
- **Bandpass Signal:**
 - Does not have a DC component
 - Finite bandwidth around or at f_c



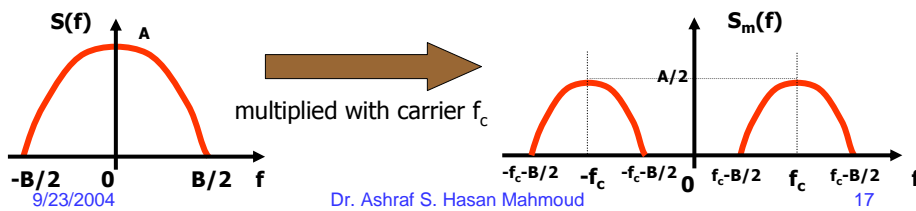
Modulation

- **Is used to shift the frequency content of a baseband signal**
 - **Basis for AM modulation**
 - **Basis for Frequency Division Multiplexing (FDM)**

Modulation

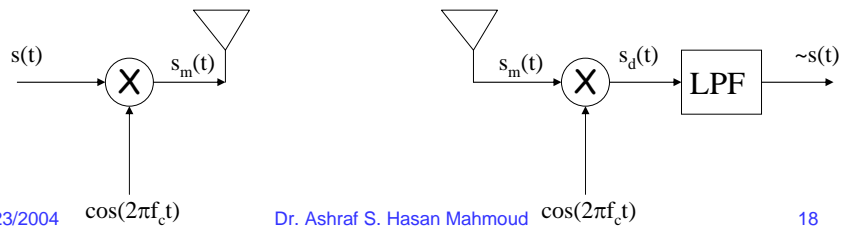
- Consider the signal $s(t)$,
 $s_m(t) = s(t) \times \cos(2\pi f_c t)$
 The spectrum for $s_m(t)$ is given by

$$S_m(f) = \frac{1}{2} \times \{S(f-f_c) + S(f+f_c)\}$$



Modulation – Txer/Rxer

- At the receiver side:
 $s_d(t) = s_m(t) \times \cos(2\pi f_c t)$
 $= s(t) \times \cos(2\pi f_c t) \times \cos(2\pi f_c t)$
 $= \underbrace{\frac{1}{2} s(t)}_{\text{desired term}} + \underbrace{\frac{1}{2} s(t) \times \cos(2\pi 2f_c t)}_{\text{undesired term – signal centered around } 2f_c \text{ filtered out using the LPF}}$



Nyquist Bandwidth

- **For a noiseless channels of bandwidth B , the maximum attainable bit rate (or capacity) is given by**

$$C = 2B \log_2(M)$$

Where M is the size of the signaling set

Shannon Capacity

- **Capacity of a channel of bandwidth B , in the presence of noise is given by**

$$C = B \log_2(1 + \text{SNR})$$

where SNR is the ratio of signal power to noise power – a measure of the signal quality

Example: Shannon Capacity

- Consider a GSM system with BW = 200 kHz. If SNR is equal to 15 dB, find the channel capacity?

- Solution:

$$\text{SNR} = 15 \text{ dB} = 10^{(15/10)} = 31.6$$

$$\begin{aligned} C &= 200 \times 10^3 \times \log_2(1+31.6) \\ &= 1005.6 \text{ kb/s} \end{aligned}$$

Note GSM operates at 273 kb/s which is ~27% of maximum capacity at SNR = 30 dB.

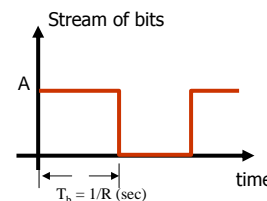
E_b/N₀ Expression

- An alternative representation of SNR
- Consider the bit stream shown in figure – for bit of rate R, then each bit duration is equal to T_b = 1/R seconds
- Energy of signal for the bit duration is equal to A² X T_b, where its power is equal to bit energy / T_b or A².
- Noise power is equal to N₀ X B (refer to thermal noise section)
- Hence, SNR is given by signal power / noise power or

$$\text{SNR} = \frac{\text{signalpower}}{N_0 B} = \frac{E_b}{N_0} \times \frac{R}{B}$$

- One can also write

$$\left(\frac{E_b}{N_0} \right)_{dB} = \text{SignalPower}(dBW) - 10 \log R - 10 \log k - 10 \log T$$



Signal Elements or Pulses

- **Unit of transmission** – repeated to form the overall signal
- **Shape of pulse** determines the bandwidth of the transmitted signal
- **Digital data is mapped or encoded to the different pulses or units of transmission**
- **Baud/Modulation or Symbol Rate (R_s)**
 - The bit rate $R_b = R_s \log_2(M)$
- **Please refer to earlier examples of pulses and the corresponding BW**

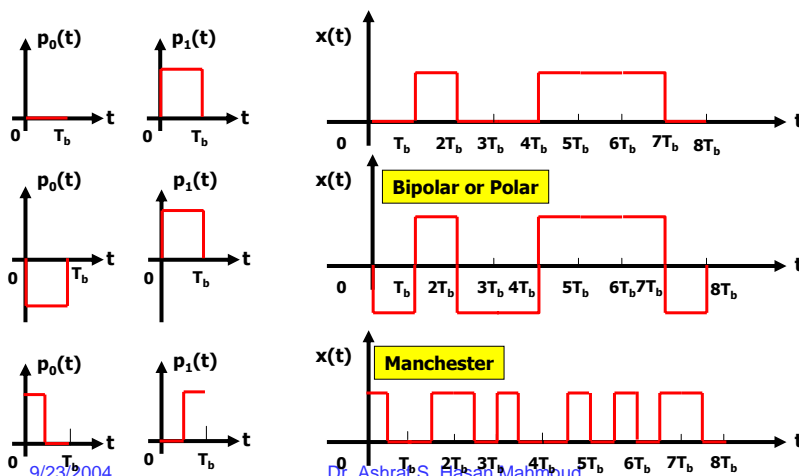
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Signal Elements or Pulses

Definitions of Pulses Encoded Signal: 0 1 0 0 1 1 1 0



Examples of Digital Signaling

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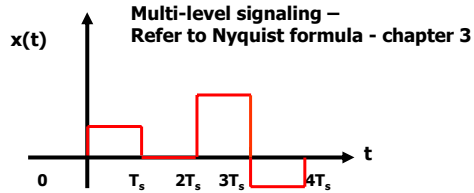
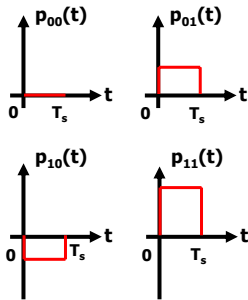
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Signal Elements or Pulses

Pulses Definitions

Encoded Signal: 0 1 0 0 1 1 1 0



Example of Digital Signaling

- Note that each symbol or pulse carries 2 bits
- Symbol duration is $T_s = 2T_b$
- Bit rate R equal to $1/T_b$
- Symbol rate or *baud rate* R_s equal to $1/T_s \rightarrow R = 2R_s$
- In general to encode n bits per pulse, you need 2^n pulses

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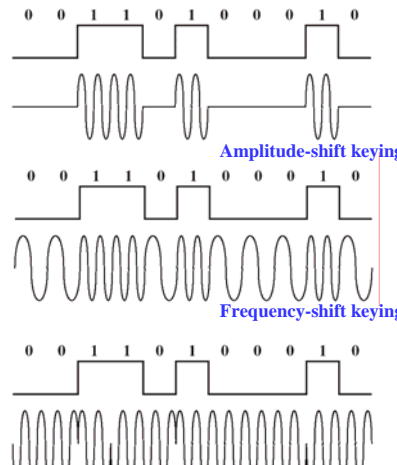
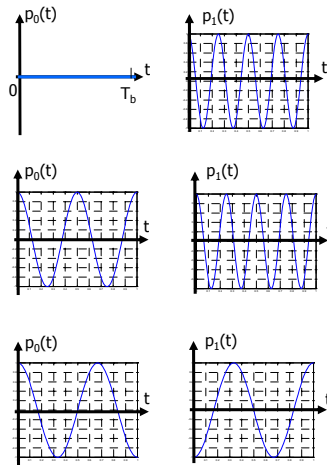
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Signal Elements or Pulses

Definitions of Pulses

Encoded Signal:



Example of Analog Signaling

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Phase-shift keying

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Digital Signal Encoding Formats

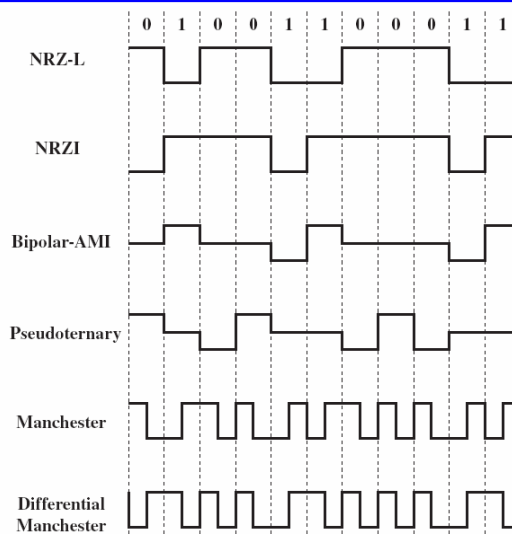
- **Nonreturn to Zero-Level (NRZ-L)**
 - 0 = high level
 - 1 = low level
- **Nonreturn to Zero Inverted (NRZI)**
 - 0 = no transition at beginning of interval
 - 1 = transition at beginning of interval
- **Bipolar-AMI**
 - 0 = no line signal
 - 1 = +ve or -ve level; alternating successive ones
- **Pseudoternary**
 - 0 = +ve or -ve level; alternating for successive ones
 - 1 = no line signal
- **Doubinary**
 - 0 = no line signal
 - 1 = +ve or -ve level; depending on number of separating 0s (even – same polarity, odd – opposite polarity)
- **Manchester**
 - 0 = transition from high to low in middle of interval
 - 1 = transition from low to high in middle of interval
- **Differential Manchester: Always transition in middle of interval**
 - 0 = transition at beginning of interval
 - 1 = no transition at beginning of interval

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Digital Signal Encoding Formats



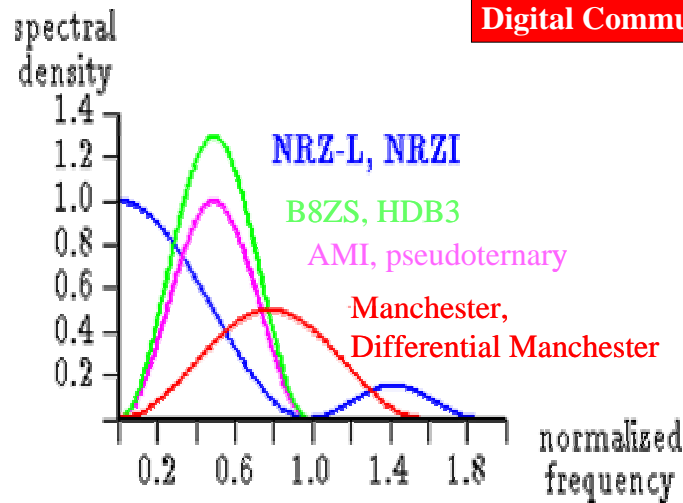
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Spectrum Characteristics of Digital Encoding Schemes

Digital Communications



Asynchronous Data Transmission

- **Digital Info:**
 - Bits
 - Characters
 - Packets
 - Messages or files
- **Serial vs. Parallel character**

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Asynchronous Transmission

- Exploits: Rx-er can remain for short period in synch with Tx-er
- Used for short stream of bits – data transmitted one character (5 ~ 8 bits) at a time
- Synchronization is needed to be maintained for the length of short transmission
- Character is delimited (start & end) by known signal elements: start bit – stop element
- Rx-er re-synchs with the arrival of new character

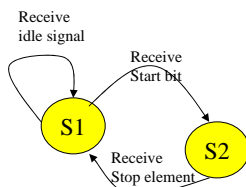
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Asynchronous Transmission

- Simple / Cheap
- Efficiency: transmit 1 start bit + 8 bit of data + 2 stop bits → Efficiency = $8/11 = 72\%$ (or overhead = $3/11 = 28\%$)
- Good for data with large gaps (e.g. keyboard, etc)

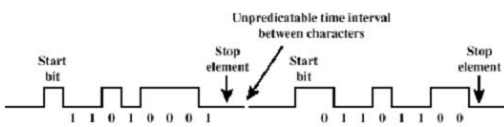


S1: receiver in idle state

S2: receiver is receiving character



(a) Character format



(b) 8-bit asynchronous character stream

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Synchronous Data Transmission

- **Two ends remain in sync for significant period of time**
- **Use of SYNC or PREAMBLE characters**
- **Noise + Data = may create another SYNC character → frame split**
 - **Solution – use two SYNC characters**
- **Rx-er must buffer incoming frames and search for SYNC character(s)**

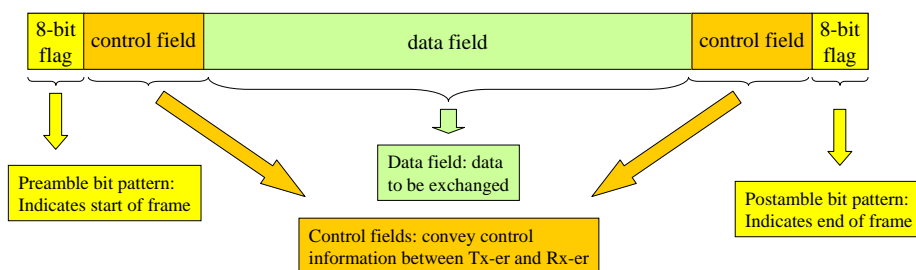
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Synchronous Frame Format

- **Typical Frame Structure**



- For large data blocks, synchronous transmission is far more efficient than asynchronous:
 - E.g. HDLC frame (to be discussed in Chapter 7): 48 bits are used for control, preamble, and postamble – if 1000 bits are used for data → efficiency = 99.4% (or overhead = 0.6%)

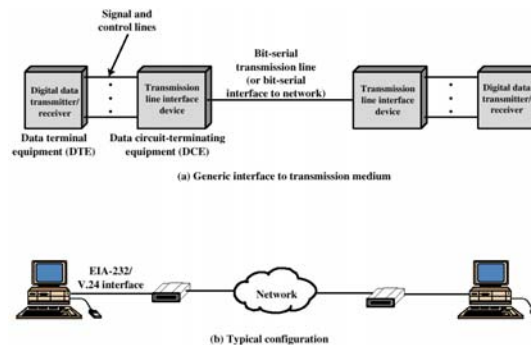
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Interfacing

- Data Terminal Equipment (DTE): terminals or computers
- Data Circuit Equipment (DCE): modem
- Two DCEs exchanging data on behalf of DTEs must use exact same protocol



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DTE-DCE Interface Definition

- Mechanical: physical specification of connection – type, dimensions, location of pins, etc
- Electrical: voltage levels and timing signals used
- Functional: specify functions that are performed for circuits – rx circuit, tx circuit, etc.
- Procedural: specification of sequence of event for transmitting data based on functional specification
- Two examples:
 - V.24/EIA-232-F, and
 - ISDN physical interface

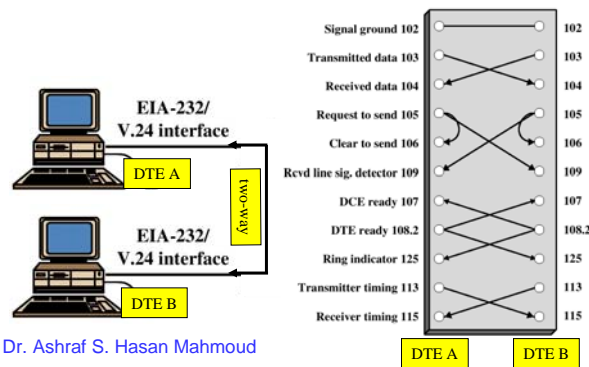
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V.24/EIA-232-F - Procedural Specification – Examples

- Example: Two terminals connected back-to-back through the V.24 interface BUT with no DCEs
- This is referred to as the NULL modem connection
- For short distance connections



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Error Control

- Error Detection
 - Parity Checks
 - Cyclic Redundancy Check (CRC)
- For a channel of bit error rate (or BER) of P , the probability of m bits in error in a block of n bits ($m \leq n$) is given by

$$\binom{n}{m} p^m (1-p)^{n-m}$$

or

$$\frac{n!}{(n-m)!m!} p^m (1-p)^{n-m}$$

The above simple formula assumes iid error probability across all bits in block – How realistic is that?

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Error Control (2)

- The probability the frame or block is correct is given by

$$(1-p)^n$$

Therefore the probability, the frame is in error (one or more bits in error) is given by

$$1-(1-p)^n$$

The above quantity is referred to as FER

Error Control - Example

- Consider a channel with BER = 10^{-3} , for a block (packet of $n = 100$ bits), the probability of having one bit in error is equal to

$$100 \times P \times (1-P)^{99} = 9 \times 10^{-2}$$

While the probability of having 4 bits in error is equal to

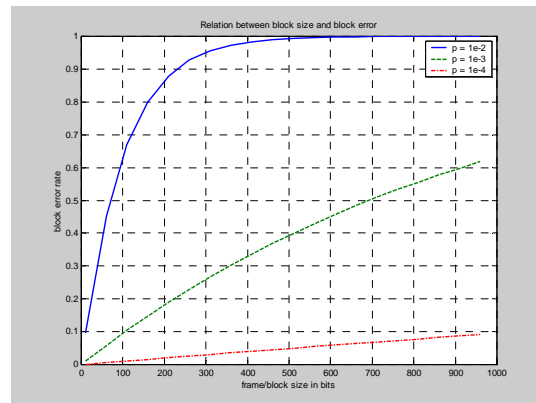
$$(100 \text{ choose } 4) \times P^4 \times (1-P)^{96} = 3.6 \times 10^{-6}$$

The probability that the frame is correct is equal to

$$(1-P)^{100} = 0.905 \rightarrow \text{i.e. } \sim 10\% \text{ of the time the frame is in error!!}$$

Error Control – Example (2)

- Relation between block size (n) and frame error rate (FER)



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Simple Parity Check

- Add one extra bit for each character such that:
 - Even Parity: no of 1s even
 - Odd Parity: no of 1s odd
- Simple
- Can not detect even no of errors in character
- Adding one extra bit to a group of n bits →
Excess redundancy = $1/(n+1)$

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VRC/LRC Parity Check

- Extension of simple parity: Vertical Redundancy Check (VRC) and Longitudinal Redundancy Check (LRC)

	Original data to send								Parity check
Char 1	1	0	0	1	1	0	0	0	1
Char 2	0	1	1	1	0	1	0	1	1
Char 3	1	1	0	0	1	1	0	0	0
Char 4	1	0	0	0	1	0	0	0	0
Char 5	0	1	0	0	1	1	1	0	0
Checking char	1	1	1	0	0	1	1	1	0

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VRC/LRC Parity Check (2)

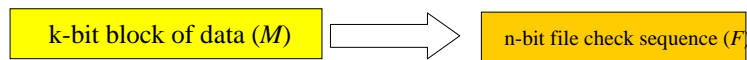
- Can detect all odd errors – same as the simple parity check
- Can detect any combination of even error in characters that DO NOT result in even number of errors in a column
- Excess Redundancy: $14/(40+14) = 0.26$
- There could be undetected errors – How?

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Cyclic Redundancy Check (CRC)



Processing: compute FCS (for some given an $n+1$ bit polynomial P)



$k+n$ bit frame to be transmitted = T

- Modulo 2 arithmetic is used to generate the FCS:
 - $0 \pm 0 = 0; 1 \pm 0 = 1; 0 \pm 1 = 1; 1 \pm 1 = 0$
 - $1 \times 0 = 0; 0 \times 1 = 0; 1 \times 1 = 1$

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CRC – Mapping Binary Bits into Polynomials

- Consider the following k -bit word or frame and its polynomial equivalent:

$$b_{k-1} b_{k-2} \dots b_2 b_1 b_0 \rightarrow b_{k-1}x^{k-1} + b_{k-2}x^{k-2} + \dots + b_1x^1 + b_0$$

where b_i ($k-1 \leq i \leq 0$) is either 1 or 0

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CRC – Mapping Binary Bits into Polynomials - Examples

- Example1: an 8 bit word $M = 11011001$ is represented as $M(x) = x^7+x^6+x^4+x^3+1$

- Example2: What is $x^4M(x)$ equal to?

$x^4M(x) = x^4(x^7+x^6+x^4+x^3+1) = x^{11}+x^{10}+x^8+x^7+x^4$, the equivalent bit pattern is 110110010000 (i.e. four zeros added to the right of the original M pattern)

- Example3: What is $x^4M(x) + (x^3+x+1)$?

$x^4M(x) + (x^3+x+1) = x^{11}+x^{10}+x^8+x^7+x^4 + x^3+x+1$, the equivalent bit pattern is 110110011011 (i.e. pattern 1011 = x^3+x+1 added to the right of the original M pattern)

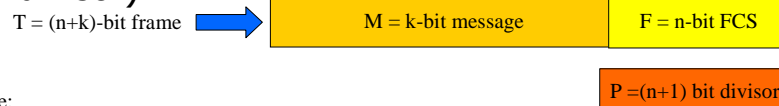
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CRC Calculation

- $T = (k+n)$ -bit frame to be tx-ed, $n < k$
- $M = k$ -bit message, the first k bits of frame T
- $F = n$ -bit FCS, the last n bits of frame T
- $P =$ pattern of $n+1$ bits (a predetermined divisor)



Note:

- $T(x)$ is the polynomial (of $k+n-1^{\text{st}}$ degree or less) representation of frame T
- $M(x)$ is the polynomial (of $k-1^{\text{st}}$ degree or less) representation of message M
- $F(x)$ is the polynomial (of $n-1^{\text{st}}$ degree or less) representation of FCS
- $P(x)$ is the polynomial (of n^{th} degree or less) representation of the divisor P
- $T(x) = X^n M(x) + F(x)$ – refer to example 3 on previous slide

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CRC Calculation (2)

- Design: frame T such that it divides the pattern P with no remainder?
- Solution: Since the first component of T, M, is the data part, it is required to find F (or the FCS) such that T divides P with no remainder

Using the polynomial equivalent:

$$T(x) = X^n M(x) + F(x)$$

One can show that $F(x) = \text{remainder of } x^n M(x) / P(x)$

i.e if $x^n M(x) / P(x)$ is equal to $Q(x) + R(x)/P(x)$, then $F(x)$ is set to be equal to $R(x)$.

Note that:

Polynomial of degree k+n

----- = polynomial of degree k + remainder polynomial of degree n or less

Polynomial of degree n

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CRC Calculation - Procedure

- 1. Shift pattern M n bits to the left**
- 2. Divide the new pattern $2^n M$ by the pattern P**
- 3. The remainder of the division R (n bits) is set to be the FCS**
- 4. The desired frame T is $2^n M$ plus the FCS bits**

Note:

$2^n M$ is the pattern resulting from shifting the pattern M n bits to the left. In other words, the polynomial equivalent of the pattern $2^n M$ is $x^n M(x)$

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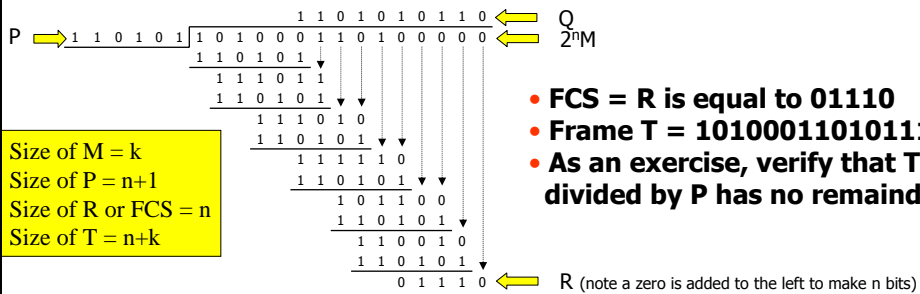
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CRC Calculation – Example

- Message M = 1010001101 (10 bits) → k = 10
- Pattern P = 110101 (6 bits – note 0th and nth bits are 1s) → n + 1 = 6 → n = 5
- Find the frame T to be transmitted?

- Solution:



- FCS = R is equal to 01110
- Frame T = 101000110101110
- As an exercise, verify that T divided by P has no remainder

Size of M = k
 Size of P = n+1
 Size of R or FCS = n
 Size of T = n+k

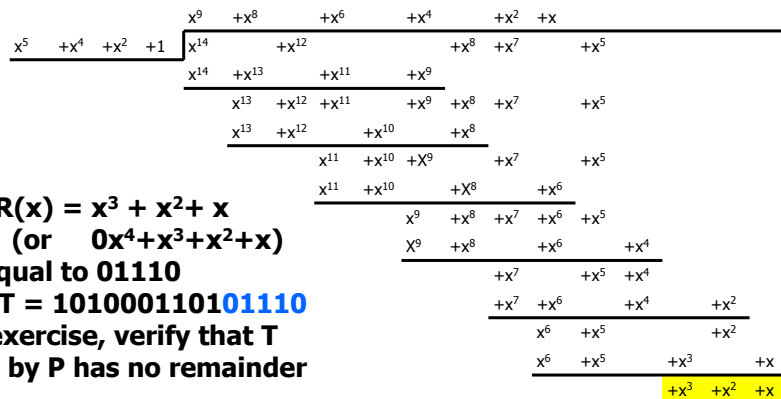
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CRC Calculation – The previous example BUT using Polynomials

- Message M = 1010001101 (10 bits) → k = 10
- $M(x) = x^9 + x^7 + x^3 + x^2 + 1 \rightarrow x^5 M(x) = x^{14} + x^{12} + x^8 + x^7 + x^5$
- Pattern P = 110101 (6 bits – note 0th and nth bits are 1s) → n + 1 = 6 → n = 5
- $P(x) = x^5 + x^4 + x^2 + 1$
- Find the frame T to be transmitted?
- Solution:



- FCS = R(x) = x³ + x² + x (or 0x⁴+x³+x²+x)
- → R is equal to 01110
- Frame T = 101000110101110
- As an exercise, verify that T divided by P has no remainder

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CRC Calculation – The previous example BUT using Polynomials – cont'd

- **Message M = 1010001101 (10 bits)**
- $M(x) = x^9 + x^7 + x^3 + x^2 + 1$
- $x^5M(x) = x^{14} + x^{12} + x^8 + x^7 + x^5$
- **Pattern P = 110101**
- $P(x) = x^5 + x^4 + x^2 + 1$
- $R(x) = x^3 + x^2 + x$
- $Q(x) = x^9 + x^8 + x^6 + x^4 + x^2 + x$
- $T(x) = x^5M(x) + R(x)$
 $= x^{14} + x^{12} + x^8 + x^7 + x^5 + x^3 + x^2 + x$, or
T = 101000110101110
- **Exercise:** Verify that $Q(x)P(x) + R(x) = x^5M(x)$

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CRC – Receiver Procedure

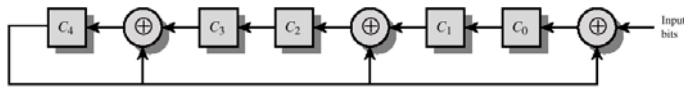
- **Tx-er transmits frame T**
- **Channel introduces error pattern E**
- **Rx-er receives frame $T_r = T \oplus E$** (note that if E = 000..000, then T_r is equal to T, i.e. error free transmission)
- **T_r is divided by P, Remainder of division is R**
- **if R is ZERO, Rx-er assumes no errors in frame; else Rx-er assumes erroneous frame**
- **If an error occurs and T_r is still divisible by P → UNDETECTABLE error** (this means the E is also divisible by P)

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CRC – Transmitter Circuit



□ = 1-bit shift register ⊕ = Exclusive-OR circuit

Shift register circuit for dividing by $P = X^5 + X^4 + X^2 + 1$

(a) Shift-register implementation

	C_4	C_3	C_2	C_1	C_0	$C_4 \oplus C_3$	$C_4 \oplus C_1$	$C_4 \oplus \text{input}$	input
Initial	0	0	0	0	0	0	0	1	1
Step1	0	0	0	0	1	0	0	0	0
Step2	0	0	0	1	0	0	1	1	1
Step3	0	0	1	0	1	0	0	0	0
Step4	0	1	0	1	0	1	1	0	0
Step5	1	0	1	0	0	1	1	1	0
Step6	1	1	1	0	1	0	1	0	1
Step7	0	1	1	1	0	1	1	1	1
Step8	1	1	1	0	1	0	1	1	0
Step9	0	1	1	1	1	1	1	1	1
Step10	1	1	1	1	1	0	0	1	0
Step11	0	1	0	1	1	1	1	0	0
Step12	1	0	1	1	0	1	0	1	0
Step13	1	1	0	0	1	0	1	1	0
Step14	0	0	1	1	1	0	1	0	0
Step15	0	1	1	1	0	1	1	0	—

(b) Example with input of 1010001101

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CRC – Receiver Circuit

- Tx-er transmits frame T

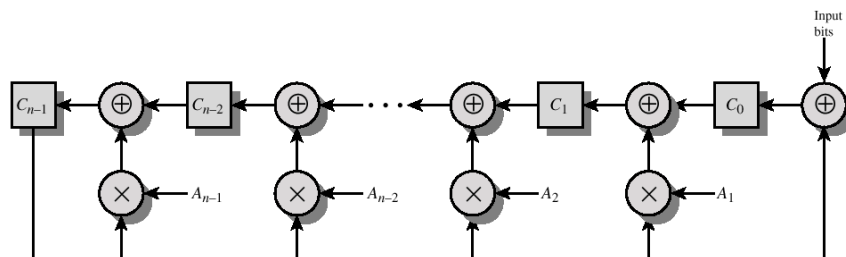


Figure 7.7 General CRC Architecture to Implement Divisor $1 + A_1X + A_2X^2 + \dots + A_{n-1}X^{n-1} + X^n$

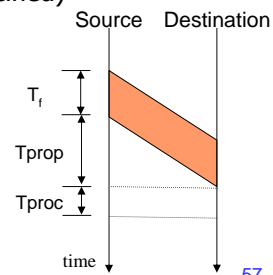
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Flow Control

- A scheme to ensure that transmitter does not overwhelm receiver with data
- Transmission of one frame:
 - T_f : time to transmit frame
 - T_{prop} : time for signal to propagate
 - T_{proc} : time for destination to process received frame – small delay (usually ignored if not specified)
- T_{proc} may be ignored if not specified

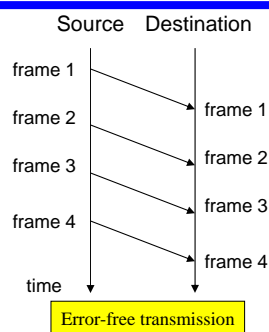


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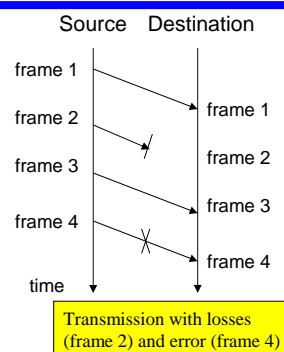
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Flow Control (2)



Models of Frame Transmission



- The destination has a limited buffer space. How will the source know that destination is ready to receive the next frame?
- In case of errors or lost frame, the source need to retransmit frames – i.e. a copy of transmitted frames must be kept. How will the source know when to discard copies of old frames?
- Etc.

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Stop-and-Wait Protocol

- Protocol:
 - Source transmits a frame
 - After the destination receives frame, it sends ACK
 - Source, upon the receipt of ACK, can now send the next frame
- Destination can stop source by withholding the ACK
- Simple
- Animation for [Stop-and-Wait](#)
- NOTE: ONLY one frame can be in transit at any time

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Stop-and-Wait Protocol: Efficiency

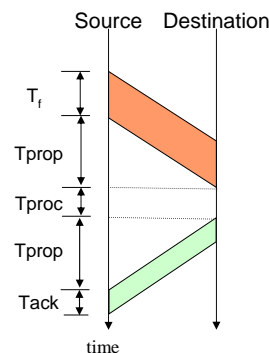
- After every frame, source must wait till acknowledgment → Hence link propagation time is significant
- Total time to for one frame:

$$T_{total} = T_f + 2T_{prop} + T_{proc} + T_{ack}$$
 if we ignore T_{proc} and T_{ack} (usually very small)

$$T_{total} = T_f + 2T_{prop}$$
- Link utilization, U is equal to

$$U = T_f / (T_{total}), \text{ or}$$

$$= 1 / (1 + 2(T_{prop}/T_f)) = 1 / (1 + 2a)$$
 where $a = T_{prop}/T_f = \text{length of link in bits}$
- If $a < 1$ (i.e. $T_f > T_{prop}$ – when 1st transmitted bit reaches destination, source will still be transmitting → U is close 100%)
- If $a > 1$ (i.e. $T_f < T_{prop}$ – frame transmission is completed before 1st bit reaches destination → U is low
- See figure 7.2



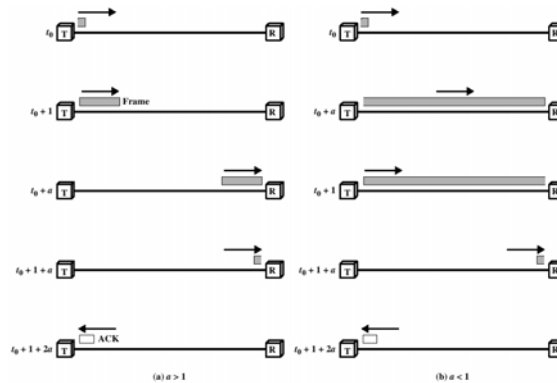
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Stop-and-Wait Protocol: Efficiency (2)

- Remember: $a = T_{prop}/T_f =$ length of link in bits
- If $a < 1$ (i.e. $T_f > T_{prop}$ – when 1st transmitted bit reaches destination, source will still be transmitting \rightarrow U is close 100%)
- If $a > 1$ (i.e. $T_f < T_{prop}$ – frame transmission is completed before 1st bit reaches destination \rightarrow U is low)
- Stop-and-Wait is efficient for links where $a \ll 1$ (long frames compared to propagation time)



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Sliding Window Protocol

- Stop-and-Wait can be very inefficient when $a > 1$
- Protocol:
 - Assumes full duplex line
 - Source A and Destination B have buffers each of size W frames
 - For k -bit sequence numbers:
 - Frames are numbered: $0, 1, 2, \dots, 2^k-1, 0, 1, \dots$ (modulo 2^k)
 - ACKs (RRs) are numbered: $0, 1, 2, \dots, 2^k-1, 0, 1, \dots$ (modulo 2^k)
 - A is allowed to transmit up to W frames without waiting for an ACK
 - B can receive up to W consecutive frames
 - ACK J (or RR J), where $0 \leq J < 2^k-1$, sent by B means B has received frames up to frame $J-1$ and is ready to receive frame J
 - B can also send RNR J : B has received all frames up to $J-1$ and is not ready to receive any more
- Window size, W can be less or equal to 2^k-1

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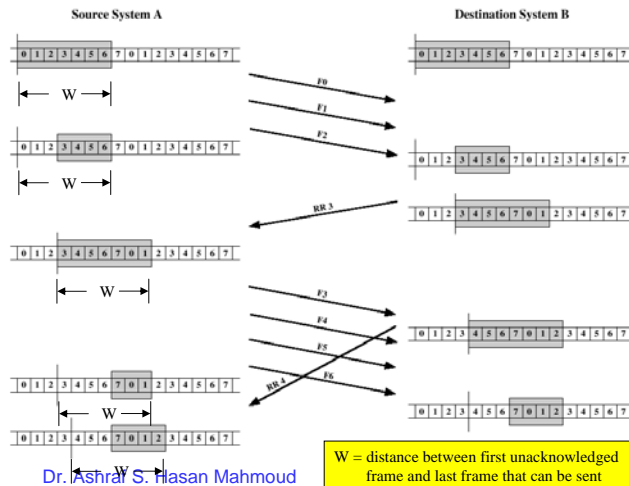
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Sliding Window Protocol (2)

- Example of Sliding-Window-Protocol: $k = 3$ bits, $W = 7$

Observations:

- A may tx $W = 7$ frames (F0, F1, ..., F6)
- After F0, F1, & F2 are tx-ed, window is shrunk (i.e. can not transmit except F3, F4, ..., F6)
- When B sends RR3, A knows F0, F1 & F2 have been received and B is ready to receive F3
- Window is advanced to cover 7 frames (starting with F3 up to F1)
- A sends F3, F4, F5, & F6
- B responds with RR4 when F3 is received – A advances the window by one position to include F2



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W = distance between first unacknowledged frame and last frame that can be sent

Sliding Window Protocol - Piggybacking

- When using sliding window protocol in full duplex connections:
 - Node A maintains its own transmit window
 - Node B maintains its own transmit window
 - A frame contains: data field + ACK field
 - There is a sequence number for the data field, and a sequence number for the ACK field

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Sliding Window Protocol - Efficiency

- Again we can distinguish two cases:
- Case 1: $W \geq 2a + 1$
- Case 2: $W < 2a + 1$

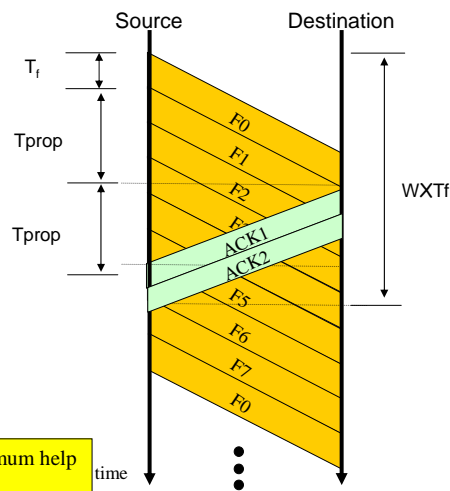
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Sliding Window Protocol - Efficiency - Case 1

- Assume $k=3, W = 7$
(ignoring Tack)
- Source can continuously keep transmitting!!
 - Because the ACK can arrive to source before the window is completed
- Utilization = 100%



Sending ACK0 as soon as F0 is received is the maximum help the destination can do to increase utilization

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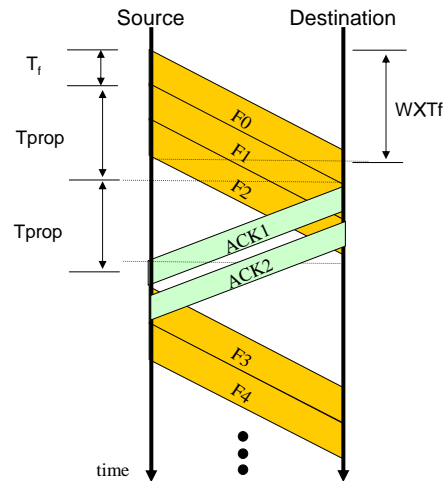
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Sliding Window Protocol - Efficiency - Case 2

- Assume $k = 3$, $W = 3$ (ignoring T_{ack})
- Source can NOT continuously keep transmitting!!
 - Because the ACK can NOT arrive to source before the window is completed

• Utilization =
$$\frac{W \times T_f}{T_f + 2 \times T_{prop}}$$

$$= \frac{W}{1 + 2a}$$



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Sliding Window Protocol - Efficiency

- Refer to Appendix A
- When window size is W (for error free), link utilization, U , is given by

$$U = \begin{cases} 1 & W \geq (2a + 1) \\ \frac{W}{2a + 1} & W < (2a + 1) \end{cases}$$

where $a = T_{prop}/T_f$ or length of link in bits

- Sliding window protocol can achieve 100% utilization if $W \geq (2a + 1)$

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Sliding Window Protocol

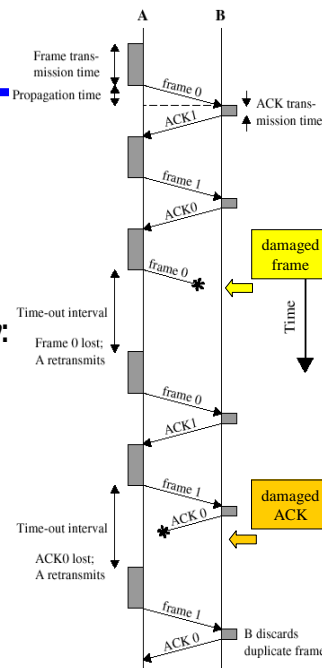
- Animation for Sliding Window protocol
- Sliding Window Protocol Simulation
(<http://www.cs.stir.ac.uk/~kjt/software/comms/jasper/SWP3.html>)

Automatic Repeat Request - ARQ

- Types of Errors:
 - Lost frame
 - Damaged frame
- Error control Techniques:
 - Error detection – discussed previously
 - +ve ACK
 - Retransmission after timeout
 - -ve ACK and retransmission
- ARQ Procedures: convert an unreliable data link into a reliable one.
 - Stop-and-wait
 - Go-back-N
 - Selective-reject

Stop-and-Wait ARO

- Based on the stop-and-wait control flow procedure
- Two types of errors:
 1. Frame lost or damaged – *Solution*: timeout timer
 2. Damaged or lost ACK – The timeout timer solves this problem



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Sliding Window Protocol

- Stop-and-Wait can be very inefficient when $a > 1$
- Protocol:
 - Assumes full duplex line
 - Source A and Destination B have buffers each of size W frames
 - For k -bit sequence numbers:
 - Frames are numbered: $0, 1, 2, \dots, 2^k-1, 0, 1, \dots$ (modulo 2^k)
 - ACKs (RRs) are numbered: $0, 1, 2, \dots, 2^k-1, 0, 1, \dots$ (modulo 2^k)
 - A is allowed to transmit up to W frames without waiting for an ACK
 - B can receive up to W consecutive frames
 - ACK J (or RR J), where $0 \leq J < 2^k-1$, sent by B means B has received frames up to frame $J-1$ and is ready to receive frame J
 - B can also send RNR J : B has received all frames up to $J-1$ and is not ready to receive any more
- Window size, W can be less or equal to 2^k-1

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Go-Back-N ARQ

- Based on the sliding-window flow control procedure
- If the i^{th} frame is lost or deemed lost (i.e. $i+1^{\text{st}}$ is received before the i^{th} frame), the i^{th} frame and all subsequent frames are retransmitted

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Selective-Reject ARQ

- In contrast to Go-Back-N, the only frames retransmitted are those that receive -ve ACK (called SREJ) or those that time out
- More efficient:
 - Rx-er must have large enough buffer to save *post*-SREJ frames
 - Buffer manipulation – re-insertion of out-of-order frames

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Window Size for Selective-Reject ARQ – Why?

- Window size: should be less or equal to half range of sequence numbers
 - For n-bit sequence numbers, Window size is $\leq 2^{n-1}$ (remember sequence numbers range from 0,1, ..., 2^n-1)
- Why? See next example

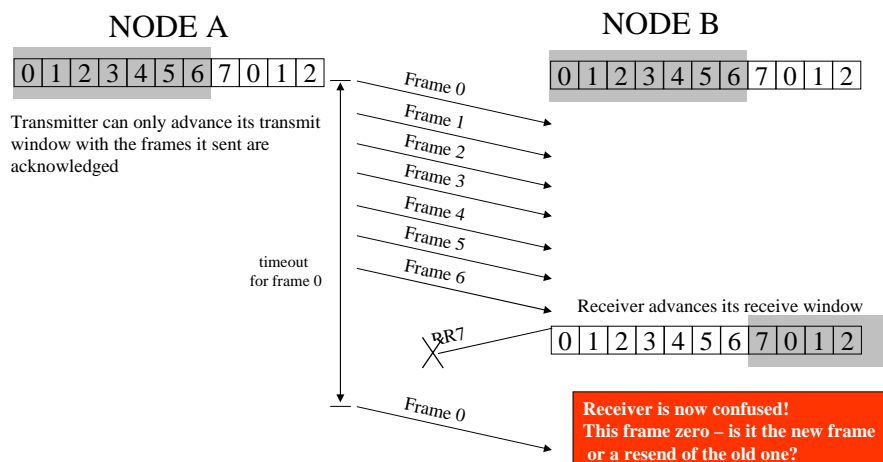
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Window Size for Selective-Reject ARQ – Why? (2)

- Example: Consider 3-bit sequence number and window size of 7



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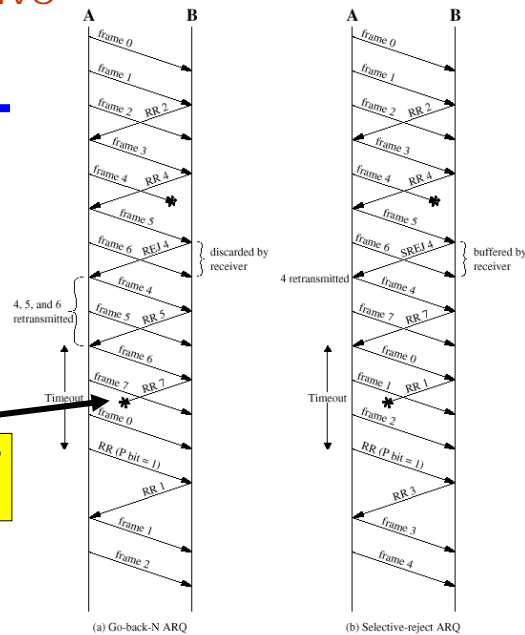
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Go-Back-N/Selective-Reject ARQ Examples

- With Go-back-N frames 4,5 and 6 are retransmitted
- With Selective-Reject only frame 4 is retransmitted

Did this lost RR7 affect flow?
How did the link recover?



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Switching

- Circuit Switching
 - Call Setup
 - Data Exchange
 - Call Termination
- Store-and-forward (Packet Switching)
 - Virtual Circuit
 - Datagram

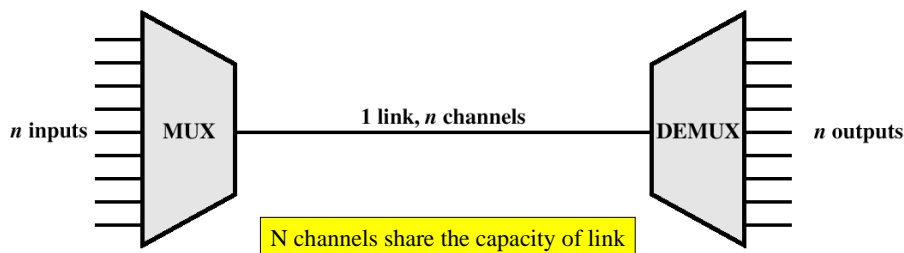
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What is MULTIPLEXING?

- A generic term used where more than one application or connection share the capacity of one link
- Why?
 - To achieve better utilization of resources



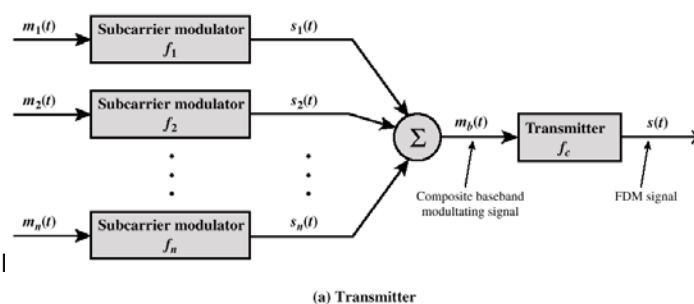
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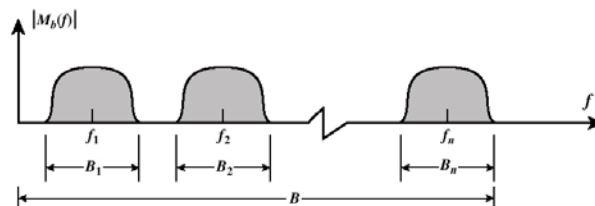
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Frequency-Division Multiplexing - Transmitter

- $m_i(t)$: analog or digital information
- Modulated with subcarrier $f_i \rightarrow s_i(t)$
- $m_b(t)$ composite baseband modulating signal
- $m_b(t)$ modulated by $f_c \rightarrow$ The overall FDM signal $s(t)$



(a) Transmitter



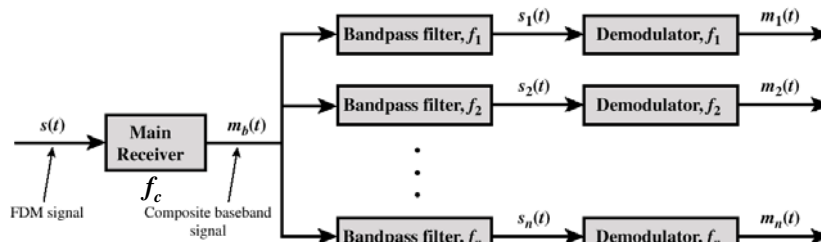
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Spectrum function of composite baseband modulating signal $m_b(t)$

Frequency-Division Multiplexing - Receiver

- $m_b(t)$ is retrieved by demodulating the FDM signal $s(t)$ using carrier f_c
- $m_b(t)$ is passed through a parallel bank of bandpass filters – centered around f_i
- The output of the i^{th} filter is the i^{th} signal $s_i(t)$
- $m_i(t)$ is retrieved by demodulating $s_i(t)$ using subcarrier f_i



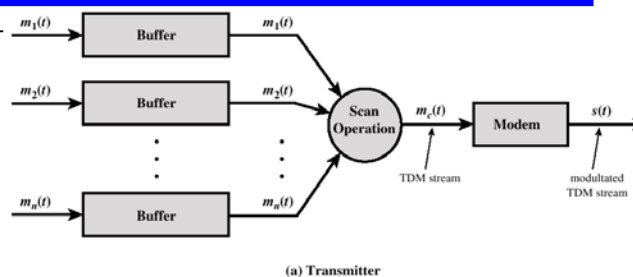
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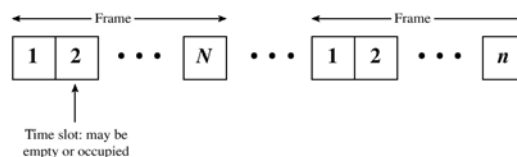
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Synchronous Time-Division Multiplexing - Transmitter

- Digital sources $m_i(t)$ – usually buffered
- A scanner samples sources in a cyclic manner to form a frame
- $m_c(t)$ is the TDM stream or frame → frame structure is fixed
- Frame $m_c(t)$ is then transmitted using a modem → resulting analog signal is $s(t)$



(a) Transmitter



(b) TDM Frames

Fixed assignments – slot left empty if no data
No address requirement
Bit stuffing

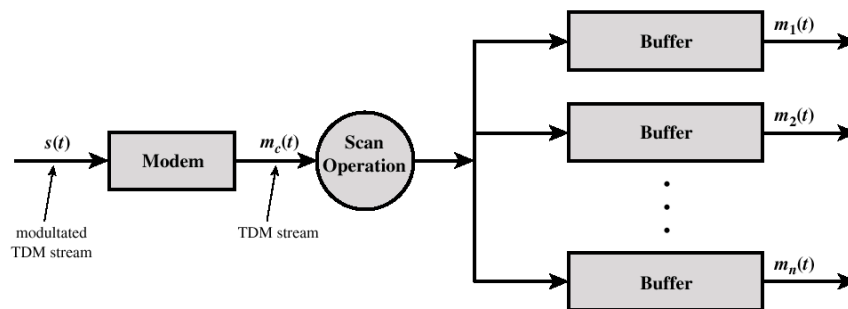
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Synchronous Time-Division Multiplexing - Receiver

- TDM signal $s(t)$ is demodulated \rightarrow result is TDM digital frame $m_c(t)$
- $m_c(t)$ is then scanned into n parallel buffers;
- The i^{th} buffer correspond to the original $m_i(t)$ digital information



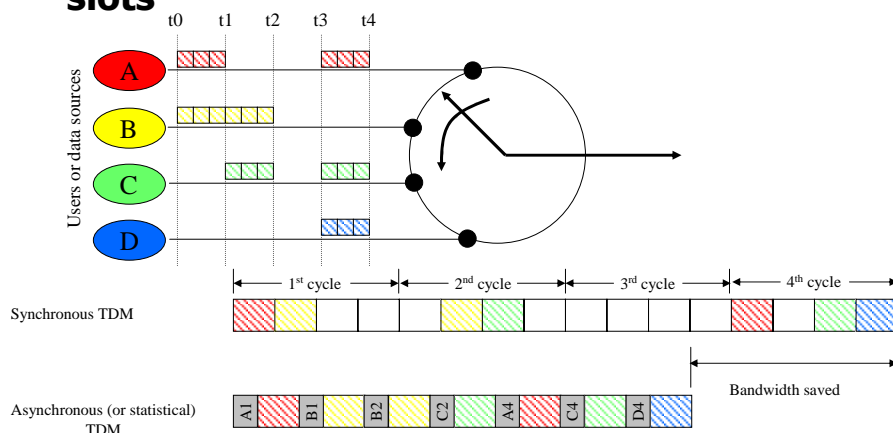
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Statistical Time-Division Multiplexing

- **Dynamic and on-demand allocation of time slots**



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Statistical Time-Division Multiplexing Frame Format

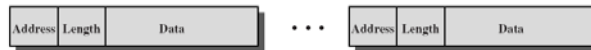
- Clearly, the aim of statistical TDM is increase efficiency by not sending empty slots
- But it requires overhead info to work:
 - Address field
 - Length field



(a) Overall frame



(b) Subframe with one source per frame



(c) Subframe with multiple sources per frame

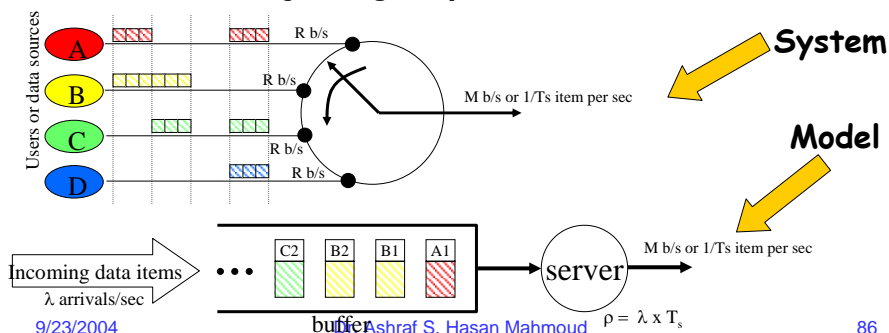
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Statistical Time-Division Multiplexing – Modeling

- Data items (bits, bytes, etc) are generated at any time – source may be intermittent (bursty) not constant
- R b/s is the peak rate for single source
 - αR b/s is the average rate for single source ($0 \leq \alpha \leq 1$)
- The *effective* multiplexing line rate is M b/s
- Each data item requires T_s sec to be served or tx-ed
- Data items may accumulate in buffer before server is able to transmit them → Queuing delay



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$\rho = \lambda \times T_s$

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