COE 202, Term 162
Digital Logic Design
HW# 3 Solution

Q.1. For the Boolean function E and F, as given in the following truth table:

X	Y	Z	E	F
0	0	0	1	0
0	0	1	1	0
0	1	0	1	1
0	1	1	0	0
1	0	0	1	0
1	0	1	0	0
1	1	0	0	1
1	1	1	0	1

(i) List the minterms and the maxterms of each function.

(ii) List the minterms of E' and F'.

(iii) List the minterms of E + F and E . F.

(iv) Express E and F in sum-of-minterms algebraic form.

(v) Simplify E and F to expressions with a minimum number of literals.

Q.2. Simplify the following Boolean functions **F** together with the don\`t care conditions **d**. Find all prime implicants and essential prime implicants, and apply the selection rule.

(i) F(A, B, C)=Σ m(3, 5, 6), d(A,B,C)=Σ m(0, 7)

(ii) F(A, B, C, D)=Σ m(4, 6, 7, 8, 12, 15), d(A, B, C, D)=Σ m(2, 3, 5, 10, 11, 14)

(iii) F(A, B, C, D) =)=Π M(1, 3, 5, 6, 7, 9, 10, 11, 14)

Q.3. Simplify the following Boolean functions **F** together with the don\`t care conditions **d** in (1) sum-of-products and (2) product-of-sums form:

(i) F(W, X, Y, Z)=Σ m(0, 1, 2, 3, 7, 8, 10), d(W, X, Y, Z)=Σ m(5, 6, 11, 15)

(ii) F(A, B, C, D)=Σ m(3, 4, 13, 15), d(A, B, C, D)=Σ m(1, 2, 5, 6, 8, 10, 12, 14)

(iii) F(A, B, C, D, E, F)=Σ m(6, 9, 13, 18, 19, 25, 27, 29, 41, 45, 57, 61)

Q.4. The following Boolean expression: BE + B\`DE\` is a simplified version of the expression: A\`BE + BCDE + BC\`D\`E + A\`B\`DE\` + B\`C\`DE\`. Are there any don\`t care conditions? If so, what are they?

Q.5. Simplify each of the following expressions, and implement them with (1) NAND gates, (2) NOR gates. Assume that both true and complement versions of the input variables are available.

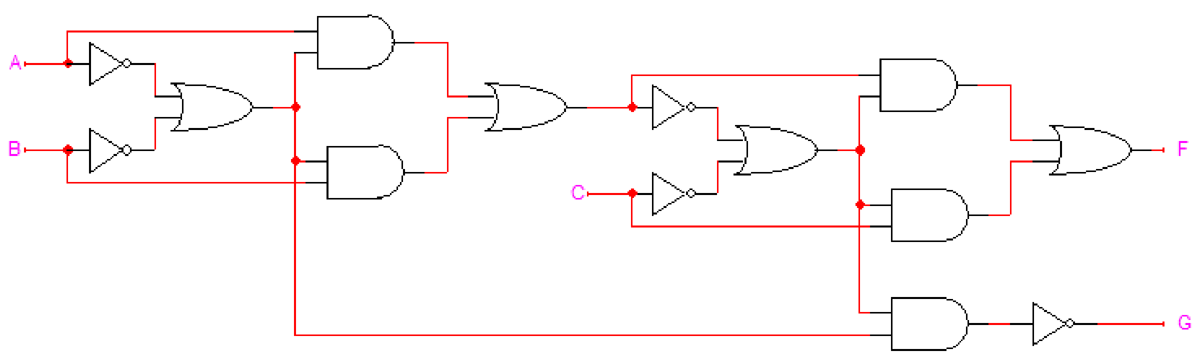
(i) WX` + WXZ + W`Y`Z` + W`XY` + WXZ`

(ii) XZ + XYZ` + WX`Y`

Q.6. Implement the following Boolean function with XOR and AND gates:

AB`C`D + A`BC`D + AB`CD` + A`BCD`

Q.7. Convert the AND/OR/NOT logic diagram shown below to (a) a NAND logic diagram, and (b) a NOR logic diagram.



Q.8. Derive the exclusive-OR/exclusive–NOR circuits for three-bit parity generator and a four-bit parity checker, using an even parity bit.

Q.9. A NAND gate with seven inputs is required. For each of the following cases, minimize the number of gates used in the multiple-level result:

(i) Design the 7-input NAND gate using 2-input NAND gates and NOT gates.

(ii) Design the 7-input NAND gate using 2-input NAND gates, 2-input NOR gates, and NOT gates.

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HW#3

Q1 (a) $E = \Sigma(m_0, m_1, m_2, m_4)$

$= \Pi(M_3, M_5, M_6, M_7)$

$F = \Sigma(m_2, m_6, m_7)$

$= \Pi(M_0, M_1, M_3, M_4, M_5)$

(b) $\bar{E} = \Sigma(m_3, m_5, m_6, m_7)$

$\bar{F} = \Sigma(m_0, m_1, m_3, m_4, m_5)$

(c) The minterms of E+F is the set of minterms in E union the set of minterms in F

$E+F = \Sigma(m_0, m_1, m_2, m_4, m_6, m_7)$

The minterms of EF is the set of minterms in E intersected with the set of minterms in E, i.e. the minterms common to both E and F

$EF = \Sigma(m_2)$

(d) $E = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z}$

$F = \bar{x}y\bar{z} + xy\bar{z} + xyz$

(e)

x \ yz	00	01	11	10
0	1	1	0	1
1	1	0	0	0

$E = \bar{x}\bar{y} + \bar{y}\bar{z} + \bar{x}\bar{z}$

$= \bar{y}(\bar{x} + \bar{z}) + \bar{x}\bar{z}$

5 literals

x \ yz	00	01	11	10
0	0	0	0	1
1	0	0	1	1

$F = xy + y\bar{z}$

$= y(x + \bar{z})$

3 literals

Q2 (i) $F(A,B,C) = \Sigma m(3,5,6)$, $d(A,B,C) = \Sigma m(0,7)$

A \ BC	00	01	11	10
0	X	0	1*	0
1	0	1*	X	1*

<u>Prime Implicants</u> : BC, AC, AB

All of them are essential

$F = BC + AC + AB = C(B+A) + AB$

(ii) $F(A,B,C,D) = \Sigma m(4,6,7,8,12,15)$

$d(A,B,C,D) = \Sigma m(2,3,5,10,11,14)$

AB \ CD	00	01	11	10
00	0	0	X	X
01	1	X	1	1
11	1	0	1*	X
10	*1	0	X	X

<u>Prime Implicants</u> : C, $\bar{A}B$, $B\bar{D}$, $A\bar{D}$

<u>Essential prime implicants</u>: C, $A\bar{D}$

After selecting the essential prime implicants, only minterm m4 remains uncovered. This can be covered by selecting the prime implicant $\bar{A}B$ or $B\bar{D}$. Thus,

$F = C + A\bar{D} + \bar{A}B$ 5 literals

or $F = C + A\bar{D} + B\bar{D}$ 5 literals

$= C + \bar{D}(A+B)$ 4 literals

Note that the 2nd expression is better since it can be factored. This results in a multilevel circuit.

(iii) $F(A,B,C,D) = \prod M(1,3,5,6,7,9,10,11,14)$
$= \sum m(0,2,4,8,12,13,15)$

AB \ CD	00	01	11	10
00	1	0	0	1*
01	*1	0	0	0
11	1	1	*1	0
10	1*	0	0	0

Prime implicants: $\bar{C}\bar{D}$, $\bar{A}\bar{B}\bar{D}$, $AB\bar{C}$, ABD

Essential prime implicants: $\bar{C}\bar{D}$, $\bar{A}\bar{B}\bar{D}$, ABD

After selecting the essential prime implicants, all the minterms are covered.

$$F = \bar{C}\bar{D} + \bar{A}\bar{B}\bar{D} + ABD$$
$$= \bar{D}(\bar{C} + \bar{A}\bar{B}) + ABD$$

Q3

(i) $F(w,x,y,z) = \sum m(0,1,2,3,7,8,10)$
$d(w,x,y,z) = \sum m(5,6,11,15)$

Sum of products:

wx \ yz	00	01	11	10
00	1	1	1	1
01	0	X	1	X
11	0	0	X	0
10	*1	0	X	1

Prime implicants: $\bar{w}\bar{x}$, $\bar{x}\bar{z}$, $\bar{w}z$, $\bar{w}y$, yz, $\bar{x}y$

Essential prime implicants: $\bar{x}\bar{z}$

$$F = \bar{x}\bar{z} + \bar{w}z$$

Product of sums:

$$F(w,x,y,z) = \sum m(4,9,12,13,14)$$
$$d(w,x,y,z) = \sum m(5,6,11,15)$$

wx \ yz	00	01	11	10
00	0	0	0	0
01	1	X	0	X
11	1	1	X	1
10	0	1*	X	0

Prime implicants: $x\bar{y}$, $x\bar{z}$, wx, wz

Essential prime implicants: wz

$$\bar{F} = wz + x\bar{z}$$
$$F = \bar{\bar{F}} = (\bar{w}+\bar{z})(\bar{x}+z)$$

(ii) $F(A,B,C,D) = \sum m(3,4,13,15)$
$d(A,B,C,D) = \sum m(1,2,5,6,8,10,12,14)$

Sum of products:

AB \ CD	00	01	11	10
00	0	X	1	X
01	1	X	0	X
11	X	1	1*	X
10	X	0	0	X

Prime implicants: $\bar{A}\bar{B}D$, $\bar{A}\bar{B}C$, $B\bar{C}$, $B\bar{D}$, AB

Essential prime implicants: AB

$F = AB + B\bar{C} + \bar{A}\bar{B}D$
or $F = AB + B\bar{C} + \bar{A}\bar{B}C$
or $F = AB + B\bar{D} + \bar{A}\bar{B}D$
or $F = AB + B\bar{D} + \bar{A}\bar{B}C$

Product of Sums:

$F(A,B,C,D) = \Sigma m(0,7,9,11)$

$d(A,B,C,D) = \Sigma m(1,2,5,6,8,10,12,14)$

AB \ CD	00	01	11	10
00	1	X	0	X
01	0	X	1	X
11	X	0	0	X
10	X	1	1*	X

Prime implicants: $\overline{B}\,\overline{C}$, $\overline{B}\,\overline{D}$, $A\overline{B}$, $\overline{A}BD$, $\overline{A}BC$

Essential prime implicants: $A\overline{B}$

$\overline{F} = A\overline{B} + \overline{B}\,\overline{C} + \overline{A}BD \Rightarrow F = (\overline{A}+B)(B+C)(A+\overline{B}+\overline{D})$

or $\overline{F} = A\overline{B} + \overline{B}\,\overline{C} + \overline{A}BC$ or $F = (\overline{A}+B)(B+C)(A+\overline{B}+\overline{C})$

or $\overline{F} = A\overline{B} + \overline{B}\,\overline{D} + \overline{A}BD$ or $F = (\overline{A}+B)(B+D)(A+\overline{B}+\overline{D})$

or $\overline{F} = A\overline{B} + \overline{B}\,\overline{D} + \overline{A}BC$ or $F = (\overline{A}+B)(B+D)(A+\overline{B}+\overline{C})$

(iii) $F(A,B,C,D,E,F) = \Sigma m(6,9,13,18,19,25,27,29,41,45,57,61)$

Sum of products:

AB=00

CD \ EF	00	01	11	10
00	0	0	0	0
01	0	0	0	1*
11	0	1*	0	0
10	0	1*	0	0

AB=01

CD \ EF	00	01	11	10
00	0	0	1	1*
01	0	0	0	0
11	0	1*	0	0
10	0	1	1	0

AB=10

CD \ EF	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	1*	0	0
10	0	1*	0	0

AB=11

CD \ EF	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	1*	0	0
10	0	1*	0	0

Prime Implicants: $C\bar{E}F$, $\bar{A}\bar{B}\bar{C}DE\bar{F}$, $\bar{A}B\bar{C}\bar{D}E$, $\bar{A}B\bar{D}EF$, $\bar{A}BC\bar{D}F$

Essential prime implicants: $C\bar{E}F$, $\bar{A}\bar{B}\bar{C}DE\bar{F}$, $\bar{A}B\bar{C}\bar{D}E$

$$= C\bar{E}F + \bar{A}\bar{B}\bar{C}DE\bar{F} + \bar{A}B\bar{C}\bar{D}E + \bar{A}BC\bar{D}F$$

or $$= C\bar{E}F + \bar{A}\bar{B}\bar{C}DE\bar{F} + \bar{A}B\bar{C}\bar{D}E + \bar{A}B\bar{D}EF$$

Product of sums:

$\bar{F}(A,B,C,D,E,F) = \sum m(0, 1, 2, 3, 4, 5, 7, 8, 10, 11, 12, 14, 15, 16, 17, 20, 21, 22, 23, 24, 26, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 62, 63)$

AB=00

CD \ EF	00	01	11	10
00	1	1	1	1
01	1	1	1	0
11	1	0	1	1
10	1	0	1	1

AB=01

CD \ EF	00	01	11	10
00	1	*1	0	0
01	1	1	1	1
11	1	0	1	1
10	1	0	0	1

AB=10

CD \ EF	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	0	1	1
10	1	0	1	1

AB=11

CD \ EF	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	0	1	1
10	1	0	1*	1

Prime implicants: $\bar{C}\bar{E}$, $\bar{E}\bar{F}$, $\bar{B}\bar{C}\bar{D}$, $\bar{B}EF$, $\bar{B}\bar{C}F$, $\bar{B}CE$, $C\bar{F}$, CDE, $A\bar{C}$, AE, $A\bar{C}D$, $B\bar{C}D$, $A\bar{C}F$, BDE, DEF, $\bar{B}\bar{D}E$

Essential prime implicants: $\bar{C}\bar{E}$, AE

$$\bar{F} = \bar{C}\bar{E} + AE + C\bar{F} + \bar{B}EF + BDE + \bar{B}\bar{D}E$$

This is one possible minimal expression. There are also other possibilities.

$$F = \bar{\bar{F}} = (C+E)(\bar{A}+\bar{E})(\bar{C}+F)(B+\bar{E}+\bar{F})(\bar{B}+\bar{D}+\bar{E})(B+D+\bar{E})$$

Q4

E=0

AB \ CD	00	01	11	10
00		1	1	
01				
11				
10		1	X	

E=1

AB \ CD	00	01	11	10
00				
01	1	1	1	1
11	1	X	1	X
10				

The don't care conditions are:

$$d(A,B,C,D,E) = \sum m(22, 27, 29)$$

Q5

(i) $w\bar{x} + wxz + \bar{w}\bar{y}\bar{z} + \bar{w}x\bar{y} + wx\bar{z}$

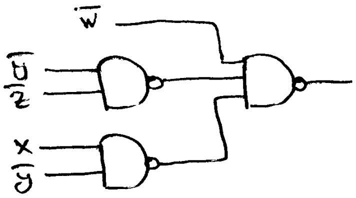
wx \ yz	00	01	11	10
00	1	0	0	0
01	1	1	0	0
11	1	1	1	1
10	1	1	1	1

$$F = w + \bar{y}\bar{z} + x\bar{y}$$

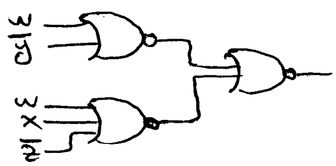
$$\bar{F} = \bar{w}y + \bar{w}\bar{x}z$$

$$F = \bar{\bar{F}} = (w+\bar{y})(w+x+\bar{z})$$

Nand implementation



Nor implementation



(ii) $xz + xy\bar{z} + w\bar{x}\bar{y}$

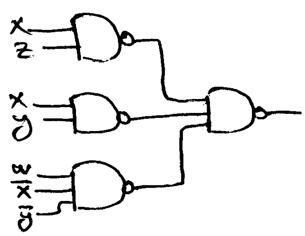
wx \ yz	00	01	11	10
00	0	0	0	0
01	0	1	1	1
11	0	1	1	1
10	1	1	0	0

$$F = xz + xy + w\bar{x}\bar{y}$$

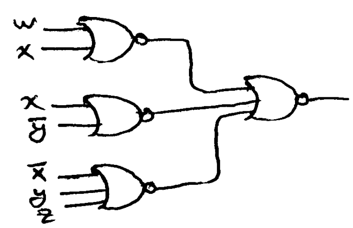
$$\bar{F} = \bar{w}\bar{x} + \bar{x}y + x\bar{y}\bar{z}$$

$$F = \bar{\bar{F}} = (w+x)(x+\bar{y})(\bar{x}+y+z)$$

Nand implementation



Nor implementation



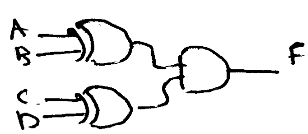
Q6

$$F = A\bar{B}\bar{C}D + \bar{A}B\bar{C}D + A\bar{B}C\bar{D} + \bar{A}BC\bar{D}$$

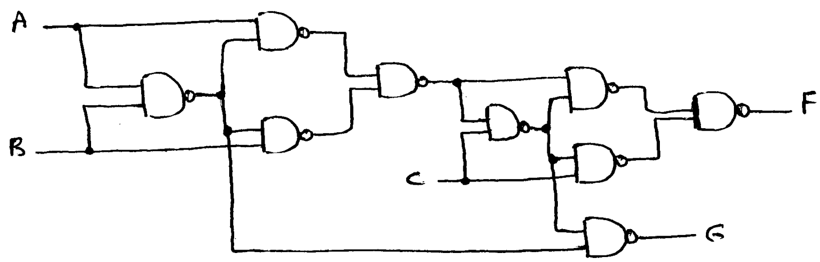
$$= A\bar{B}(\bar{C}D + C\bar{D}) + \bar{A}B(\bar{C}D + C\bar{D})$$

$$= (\bar{C}D + C\bar{D})(A\bar{B} + \bar{A}B)$$

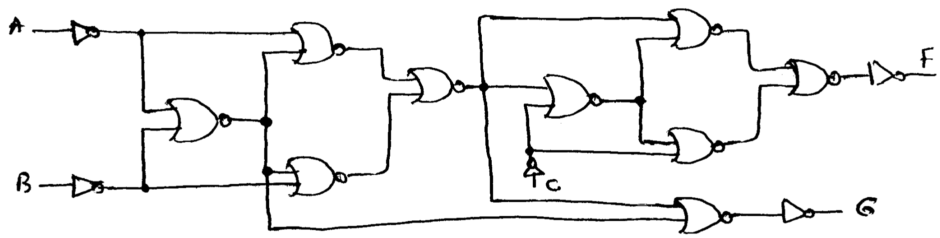
$$= (C \oplus D)(A \oplus B)$$



Q7 a. Nand implementation



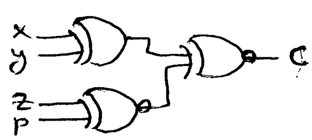
b. NOR implementation



Q8

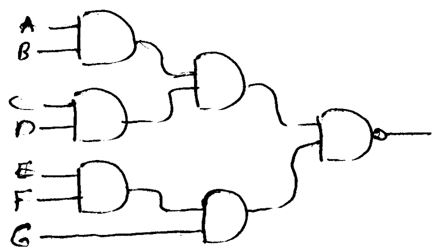
3-bit parity generator with even parity:

4-bit parity checker with even parity:

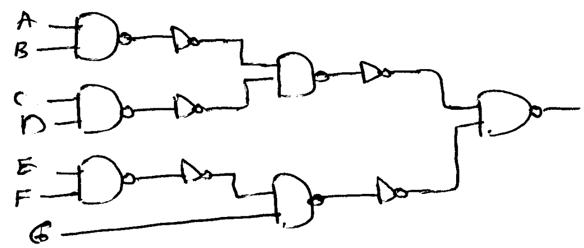


Note that an error occurs if $c = 1$.

Q9 A 7-input NAND gate can be implemented as follows:



a. Using 2-input NAND gates and NOT gates



b. Using 2-input NAND gates, 2-input NOR gates and NOT gates