

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

King Fahd University of Petroleum & Minerals

Department of Chemical Engineering

**CHAPTER 2: INTRODUCTION TO ENGINEERING
CALCULATIONS**

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2.1 Units and Dimensions:

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Consider the following Case:



400 km

Numerical Value

Unit

2.1 Units and Dimensions:

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✓ Dimension is a property that can measured such as length, time, temperature, ... etc or calculated by multiplying or dividing other dimensions such as volume (length^3), area (length^2), velocity ($\text{length}/\text{time}$), ... etc.

✓ Units can be divided into:

1. Measurable units (specific values of dimensions defined by convention, custom, laws)
2. Countable units (Don't have specific values of dimensions).

Examples:

grams

mass

seconds

times

centimeters or feet

length

Relation to mathematical operations:

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✓ Addition and Subtraction:

Quantities of the same units can be added or subtracted.

$$\text{Quantity 1} + \text{Quantity 2} = \text{Quantity 3}$$

Numerical values are added (subtracted) while units are copied as are.

✓ Multiplication and Division:

Quantities of the same or different units are multiplied or divided

$$\text{Quantity 1} \times \text{Quantity 2} = \text{Quantity 3}$$

Numerical values as well as units are multiplied or divided.

Example 2.1-1:

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Perform the following mathematical operations if possible:

1. $3.0 \text{ cm} + 4.0 \text{ cm}$
2. $5.0 \text{ g} + 10.0 \text{ kg}$
3. $10.0 \text{ N} \times 5.0 \text{ m}^2$
4. $12.0 \text{ kg/hr} \times 4.0 \text{ hr}$
5. $10.0 \text{ km} / 2.0 \text{ hr}$
6. $5.0 \text{ kg/s} \div 0.2 \text{ kg/m}^3$

2.2 Conversion of Units:

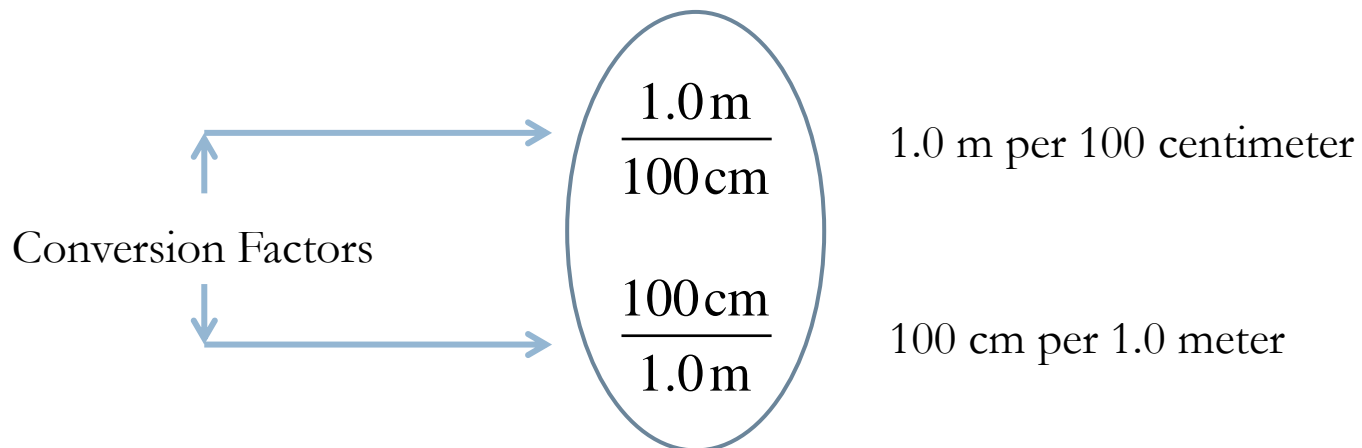
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Given: $a = b$

can be written as $\frac{a}{b}$ leading to:

Equivalence between two expressions of the same quantities may be defined as a ratio.

$$1.0 \text{ m} = 100 \text{ cm}$$



2.2 Conversion of Units:

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- ✓ Conversion factor is defined as a ratio of new unit to old unit used to convert between quantities,

$$\text{Given Unit} \times \frac{\text{Desired Unit}}{\text{Given Unit}}$$

Example 2.2-1 Convert the following:

1. 36 mg to grams
2. 5 km to cm
3. 1.0 cm/s^2 to km/yr^2

2.3 Systems of Units

Units can be broken up into:

1. Basic units, i.e. units of length, mass, time, temperature, etc
2. Multiple units, i.e. multiples or fractions of basic units such as hour, minute, millisecond, centimeter, ... etc
3. Derived units are those obtained by:
 - a. Multiplying or dividing basic or multiple units such as m^3 , m^2 , m/s , m/s^2 .
 - b. Equivalents of compound units such as $1 \text{ N} = 1 \text{ kg} \times m/s^2$, $1 \text{ dyne} = 1 \text{ g} \times \text{cm}/s^2$, $1 \text{ lb}_f = 32.174 \text{ lb}_m \times \text{ft}/s^2$.

2.3 Systems of Units

Quantity	Unit
Length	m
Mass	kg
Time	s
Moles	Gram-mole
Temperature	Kelvin
Electric Current	Ampere
Light Intensity	candela

2.3 Systems of Units

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Quantity	SI	CGS	American Engineering
Length	m	cm	ft
Mass	kg	g	lb _m
Time	s	s	s
Force	N	Dyne	lb _f

2.3 Systems of Units

Prefixes are used with the SI units indicating powers of tens as follows:

Prefix	Indication
Mega (M)	10^6
Kilo (k)	10^3
Centi (c)	10^{-2}
Mili (m)	10^{-3}
Micro (μ)	10^{-6}
Nano (n)	10^{-9}

2.3 Systems of Units

FACTORS FOR UNIT CONVERSIONS

Quantity	Equivalent Values
Mass	1 kg = 1000 g = 0.001 metric ton = 2.20462 lb _m = 35.27392 oz 1 lb _m = 16 oz = 5 × 10 ⁻⁴ ton = 453.593 g = 0.453593 kg
Length	1 m = 100 cm = 1000 mm = 10 ⁶ microns (μm) = 10 ¹⁰ angstroms (Å) = 39.37 in. = 3.2808 ft = 1.0936 yd = 0.0006214 mile 1 ft = 12 in. = 1/3 yd = 0.3048 m = 30.48 cm
Volume	1 m ³ = 1000 L = 10 ⁶ cm ³ = 10 ⁶ mL = 35.3145 ft ³ = 220.83 imperial gallons = 264.17 gal = 1056.68 qt 1 ft ³ = 1728 in. ³ = 7.4805 gal = 0.028317 m ³ = 28.317 L = 28,317 cm ³
Force	1 N = 1 kg·m/s ² = 10 ⁵ dynes = 10 ⁵ g·cm/s ² = 0.22481 lb _f 1 lb _f = 32.174 lb _m ·ft/s ² = 4.4482 N = 4.4482 × 10 ⁵ dynes
Pressure	1 atm = 1.01325 × 10 ⁵ N/m ² (Pa) = 101.325 kPa = 1.01325 bar = 1.01325 × 10 ⁶ dynes/cm ² = 760 mm Hg at 0°C (torr) = 10.333 m H ₂ O at 4°C = 14.696 lb _f /in. ² (psi) = 33.9 ft H ₂ O at 4°C = 29.921 in. Hg at 0°C
Energy	1 J = 1 N·m = 10 ⁷ ergs = 10 ⁷ dyne·cm = 2.778 × 10 ⁻⁷ kW·h = 0.23901 cal = 0.7376 ft·lb _f = 9.486 × 10 ⁻⁴ Btu
Power	1 W = 1 J/s = 0.23901 cal/s = 0.7376 ft·lb _f /s = 9.486 × 10 ⁻⁴ Btu/s = 1.341 × 10 ⁻³ hp

Example: The factor to convert grams to lb_m is $\left(\frac{2.20462 \text{ lb}_m}{1000 \text{ g}}\right)$.

Example 2.3-1: Conversion Between Systems of Units

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Convert $23 \text{ lb}_m \times \text{ft}/\text{min}^2$ to its equivalent in $\text{kg} \times \text{cm}/\text{s}^2$?

2.4 Force and Weight

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- ✓ According to Newton second law, force is defined as the product of the mass and acceleration.

$$F = m \times a$$

Units are:

SI \rightarrow $\text{kg} \times \text{m}/\text{s}^2$

CGS \rightarrow $\text{g} \times \text{cm}/\text{s}^2$

Ameri. \rightarrow $\text{lb}_m \times \text{ft}/\text{s}^2$

$$(1 \text{ N} \equiv 1 \text{ kg} \times \text{m}/\text{s}^2)$$

$$(1 \text{ dyne} \equiv 1 \text{ g} \times \text{cm}/\text{s}^2)$$

$$(1 \text{ lb}_f \equiv 32.174 \text{ lb}_m \times \text{ft}/\text{s}^2)$$

- ✓ Pound-force is defined as the product of 1 lb_m and acceleration of gravity at sea level and latitude of 45° , i.e. $a = 32.174 \text{ ft}/\text{s}^2$.

2.4 Force and Weight

Example 2.4-1: Calculate:

1. Force in Newton required to accelerate a mass of 4.0 kg at a rate of 9.00 m/s^2 ?
2. Force in dyne required to accelerate a mass of 4.0 g at a rate of 9.00 cm/s^2 ?
3. Force in lb_f required to accelerate a mass of 4.00 lb_m at a rate of 9.00 ft/s^2 ?

2.4 Force and Weight

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Define $g_c \equiv$ conversion factor as:

$$\text{SI:} \quad g_c = \frac{1.0 \text{ kg} \times \text{m/s}^2}{1.0 \text{ N}}$$

$$\text{CGS:} \quad g_c = \frac{1.0 \text{ g} \times \text{cm/s}^2}{1.0 \text{ dyne}}$$

$$\text{Ameri:} \quad g_c = \frac{32.173 \text{ lb}_m \times \text{ft/s}^2}{1.0 \text{ lb}_f}$$

✓ Weight of an object is the force exerted by gravitational attraction. It is calculated according to:

$$W = m \frac{g}{g_c}$$

2.4 Force and Weight

- ✓ Gravitational acceleration is given at sea level and 45° latitude as:

$$\begin{aligned}g &= 9.8066 \text{ m/s}^2 \\ &= 980.66 \text{ cm/s}^2 \\ &= 32.174 \text{ ft/s}^2\end{aligned}$$

2.6 Dimensional Homogeneity and Dimensionless Quantities

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Consider:

$$u \left(\frac{m}{s} \right) = u_o \left(\frac{m}{s} \right) + g \left(\frac{m}{s^2} \right) t (s)$$

Dimensions:
$$\frac{\text{length}}{\text{time}} = \frac{\text{length}}{\text{time}} + \frac{\text{length}}{\text{time}^2} \times \text{time}$$

- An equation is **dimensionally homogeneous** if and only if all additive terms on both sides of the equation have the same dimensions. The equation is also **consistent** in terms of units, i.e. length is in meters and time is in second.

2.6 Dimensional Homogeneity and Dimensionless Quantities

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Consider:

$$u\left(\frac{m}{s}\right) = u_o\left(\frac{m}{s}\right) + g\left(\frac{m}{s^2}\right)t(\text{min})$$

Dimensions:
$$\frac{\text{length}}{\text{time}} = \frac{\text{length}}{\text{time}} + \frac{\text{length}}{\text{time}^2} \times \text{time}$$

- An equation is **dimensionally homogeneous** if and only if all additive terms on both sides of . The equation is **inconsistent** in terms of units, i.e. length in meter but time in seconds and minutes. A proper conversion factor must be introduced.

$$u\left(\frac{m}{s}\right) = u_o\left(\frac{m}{s}\right) + g\left(\frac{m}{s^2}\right)t(\text{min}) \times \frac{60s}{1\text{min}}$$

2.6 Dimensional Homogeneity and Dimensionless Quantities

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Consider:

$$u \left(\frac{m}{s} \right) = u_o \left(\frac{m}{s} \right) + g \left(\frac{m}{s^2} \right)$$

Dimensions:

$$\frac{\text{length}}{\text{time}} = \frac{\text{length}}{\text{time}} + \frac{\text{length}}{\text{time}^2}$$

- An equation is **not valid** if it is dimensionally non-homogeneous.

Summary of Cases:

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	Dimensionally Homogeneous	Consistent	Example	Actions
Case 1	✓	✓	$u\left(\frac{m}{s}\right) = u_o\left(\frac{m}{s}\right) + g\left(\frac{m}{s^2}\right)t(s)$	Valid It can be used
Case 2	✓	✗	$u\left(\frac{m}{s}\right) = u_o\left(\frac{m}{s}\right) + g\left(\frac{m}{s^2}\right)t(\text{min})$	A conversion factor is needed
Case 3	✗	---	$u\left(\frac{m}{s}\right) = u_o\left(\frac{m}{s}\right) + g\left(\frac{m}{s^2}\right)$	Invalid It can't be used

Example 2.6-1:

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Consider the equation:

$$D(ft) = 3t(s) + 4$$

1. If the equation is valid, what are the dimensions of constants 3 and 4?
2. If the equation is consistent in its units, what are the units of 3 and 4?
3. Derive an equation for the distance in meters in terms of time in minutes?

Procedure to drive equivalent equations in different units:

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1. Define new variables by affixing primes to the old variable names with the new units.
2. Write the old variables in terms of the new ones.
3. Substitute in the old expression and simplify the equation.

Dimensionless Quantities:

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1. Pure Numbers such as 1, 2, 3/2, 1/4, and so on.
2. Multiplicative combination of variables may result in dimensionless quantities such as:

$$\text{Rynold's no.} = \frac{D(\text{cm}) \times u \left(\frac{\text{cm}}{\text{s}} \right) \times \rho \left(\frac{\text{g}}{\text{cm}^3} \right)}{\mu \left(\frac{\text{g}}{\text{cm} \times \text{s}} \right)}$$

$$\text{Mass Ratio} = \frac{M(\text{g})}{M_o(\text{g})}$$

3. Exponents such as “a” in X^a .
4. Arguments of transcendental functions such as:

X in sin(X)

Y in exp(Y)

Z in log(Z)

Example 2.6-2: Dimensional homogeneity and dimensionless groups

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A quantity k depends on temperature T in the following manner:

$$k\left(\frac{\text{mol}}{\text{cm}^3 \times \text{s}}\right) = 1.2 \times 10^5 \exp\left(-\frac{20,000}{1.987T}\right)$$

The units of the quantity 20,000 are cal/mol, and T in K (Kelvin). What are the units of 1.2×10^5 and 1.987?

2.7 Process Data Representation and Analysis

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Company Name / Size		Address		Phone		Fax		E-mail		Website	
Company Name / Size	Address	Phone	Fax	E-mail	Website	Company Name / Size	Address	Phone	Fax	E-mail	Website
ABC Company	123 Main St	555-123-4567	555-123-4568	info@abc.com	www.abc.com	DEF Corp	456 Elm St	555-987-6543	555-987-6544	info@def.com	www.def.com
GHI Inc	789 Oak St	555-234-5678	555-234-5679	info@ghi.com	www.ghi.com	JKL LLC	101 Pine St	555-345-6789	555-345-6790	info@jkl.com	www.jkl.com
MNO Corp	202 Cedar St	555-456-7890	555-456-7891	info@mno.com	www.mno.com	PQR Ltd	303 Birch St	555-567-8901	555-567-8902	info@pqr.com	www.pqr.com
STU Group	404 Spruce St	555-678-9012	555-678-9013	info@stu.com	www.stu.com	VWX Partners	505 Willow St	555-789-0123	555-789-0124	info@vwx.com	www.vwx.com
YZA Systems	606 Ash St	555-890-1234	555-890-1235	info@yza.com	www.yza.com	BCD Solutions	707 Hickory St	555-901-2345	555-901-2346	info@bcd.com	www.bcd.com
EFG Networks	808 Sycamore St	555-012-3456	555-012-3457	info@efg.com	www.efg.com	HJK Analytics	909 Magnolia St	555-123-4567	555-123-4568	info@hjk.com	www.hjk.com
LMN Data	1010 Dogwood St	555-234-5678	555-234-5679	info@lmn.com	www.lmn.com	OPQ Cloud	1111 Redwood St	555-345-6789	555-345-6790	info@opq.com	www.opq.com
RST Security	1212 Juniper St	555-456-7890	555-456-7891	info@rst.com	www.rst.com	UVW Energy	1313 Cypress St	555-567-8901	555-567-8902	info@uvw.com	www.uvw.com
XYZ Logistics	1414 Fir St	555-678-9012	555-678-9013	info@xyz.com	www.xyz.com	ABC Manufacturing	1515 Palm St	555-789-0123	555-789-0124	info@abc.com	www.abc.com
DEF Retail	1616 Cedar St	555-890-1234	555-890-1235	info@def.com	www.def.com	GHI Services	1717 Birch St	555-901-2345	555-901-2346	info@ghi.com	www.ghi.com
JKL Wholesale	1818 Spruce St	555-012-3456	555-012-3457	info@jkl.com	www.jkl.com	MNO Distribution	1919 Willow St	555-123-4567	555-123-4568	info@mno.com	www.mno.com
PQR Import	2020 Ash St	555-234-5678	555-234-5679	info@pqr.com	www.pqr.com	STU Export	2121 Hickory St	555-345-6789	555-345-6790	info@stu.com	www.stu.com
VWX Logistics	2222 Sycamore St	555-456-7890	555-456-7891	info@vwx.com	www.vwx.com	YZA Freight	2323 Magnolia St	555-567-8901	555-567-8902	info@yza.com	www.yza.com
BCD Shipping	2424 Dogwood St	555-678-9012	555-678-9013	info@bcd.com	www.bcd.com	EFG Warehousing	2525 Redwood St	555-789-0123	555-789-0124	info@efg.com	www.efg.com
HIJ Freight	2626 Juniper St	555-890-1234	555-890-1235	info@hij.com	www.hij.com	KLM Logistics	2727 Cypress St	555-901-2345	555-901-2346	info@klm.com	www.klm.com
MNO Supply	2828 Fir St	555-012-3456	555-012-3457	info@mno.com	www.mno.com	PQR Demand	2929 Palm St	555-123-4567	555-123-4568	info@pqr.com	www.pqr.com
RST Inventory	3030 Cedar St	555-234-5678	555-234-5679	info@rst.com	www.rst.com	UVW Procurement	3131 Birch St	555-345-6789	555-345-6790	info@uvw.com	www.uvw.com
VWX Procurement	3232 Spruce St	555-456-7890	555-456-7891	info@vwx.com	www.vwx.com	YZA Sourcing	3333 Willow St	555-567-8901	555-567-8902	info@yza.com	www.yza.com
ABC Sourcing	3434 Ash St	555-678-9012	555-678-9013	info@abc.com	www.abc.com	DEF Logistics	3535 Hickory St	555-789-0123	555-789-0124	info@def.com	www.def.com
GHI Freight	3636 Sycamore St	555-890-1234	555-890-1235	info@ghi.com	www.ghi.com	JKL Shipping	3737 Magnolia St	555-901-2345	555-901-2346	info@jkl.com	www.jkl.com
MNO Logistics	3838 Dogwood St	555-012-3456	555-012-3457	info@mno.com	www.mno.com	PQR Warehousing	3939 Redwood St	555-123-4567	555-123-4568	info@pqr.com	www.pqr.com
RST Freight	4040 Juniper St	555-234-5678	555-234-5679	info@rst.com	www.rst.com	UVW Inventory	4141 Cypress St	555-345-6789	555-345-6790	info@uvw.com	www.uvw.com
VWX Supply	4242 Fir St	555-456-7890	555-456-7891	info@vwx.com	www.vwx.com	YZA Procurement	4343 Palm St	555-567-8901	555-567-8902	info@yza.com	www.yza.com
ABC Sourcing	4444 Cedar St	555-678-9012	555-678-9013	info@abc.com	www.abc.com	DEF Logistics	4545 Birch St	555-789-0123	555-789-0124	info@def.com	www.def.com
GHI Freight	4646 Spruce St	555-890-1234	555-890-1235	info@ghi.com	www.ghi.com	JKL Shipping	4747 Willow St	555-901-2345	555-901-2346	info@jkl.com	www.jkl.com
MNO Logistics	4848 Ash St	555-012-3456	555-012-3457	info@mno.com	www.mno.com	PQR Warehousing	4949 Hickory St	555-123-4567	555-123-4568	info@pqr.com	www.pqr.com
RST Freight	5050 Sycamore St	555-234-5678	555-234-5679	info@rst.com	www.rst.com	UVW Inventory	5151 Magnolia St	555-345-6789	555-345-6790	info@uvw.com	www.uvw.com
VWX Supply	5252 Dogwood St	555-456-7890	555-456-7891	info@vwx.com	www.vwx.com	YZA Procurement	5353 Redwood St	555-567-8901	555-567-8902	info@yza.com	www.yza.com
ABC Sourcing	5454 Juniper St	555-678-9012	555-678-9013	info@abc.com	www.abc.com	DEF Logistics	5555 Cypress St	555-789-0123	555-789-0124	info@def.com	www.def.com
GHI Freight	5656 Fir St	555-890-1234	555-890-1235	info@ghi.com	www.ghi.com	JKL Shipping	5757 Palm St	555-901-2345	555-901-2346	info@jkl.com	www.jkl.com
MNO Logistics	5858 Cedar St	555-012-3456	555-012-3457	info@mno.com	www.mno.com	PQR Warehousing	5959 Birch St	555-123-4567	555-123-4568	info@pqr.com	www.pqr.com
RST Freight	6060 Spruce St	555-234-5678	555-234-5679	info@rst.com	www.rst.com	UVW Inventory	6161 Willow St	555-345-6789	555-345-6790	info@uvw.com	www.uvw.com
VWX Supply	6262 Ash St	555-456-7890	555-456-7891	info@vwx.com	www.vwx.com	YZA Procurement	6363 Hickory St	555-567-8901	555-567-8902	info@yza.com	www.yza.com
ABC Sourcing	6464 Sycamore St	555-678-9012	555-678-9013	info@abc.com	www.abc.com	DEF Logistics	6565 Magnolia St	555-789-0123	555-789-0124	info@def.com	www.def.com
GHI Freight	6666 Dogwood St	555-890-1234	555-890-1235	info@ghi.com	www.ghi.com	JKL Shipping	6767 Redwood St	555-901-2345	555-901-2346	info@jkl.com	www.jkl.com
MNO Logistics	6868 Juniper St	555-012-3456	555-012-3457	info@mno.com	www.mno.com	PQR Warehousing	6969 Cypress St	555-123-4567	555-123-4568	info@pqr.com	www.pqr.com
RST Freight	7070 Fir St	555-234-5678	555-234-5679	info@rst.com	www.rst.com	UVW Inventory	7171 Palm St	555-345-6789	555-345-6790	info@uvw.com	www.uvw.com
VWX Supply	7272 Cedar St	555-456-7890	555-456-7891	info@vwx.com	www.vwx.com	YZA Procurement	7373 Birch St	555-567-8901	555-567-8902	info@yza.com	www.yza.com
ABC Sourcing	7474 Spruce St	555-678-9012	555-678-9013	info@abc.com	www.abc.com	DEF Logistics	7575 Willow St	555-789-0123	555-789-0124	info@def.com	www.def.com
GHI Freight	7676 Ash St	555-890-1234	555-890-1235	info@ghi.com	www.ghi.com	JKL Shipping	7777 Hickory St	555-901-2345	555-901-2346	info@jkl.com	www.jkl.com
MNO Logistics	7878 Sycamore St	555-012-3456	555-012-3457	info@mno.com	www.mno.com	PQR Warehousing	7979 Magnolia St	555-123-4567	555-123-4568	info@pqr.com	www.pqr.com
RST Freight	8080 Dogwood St	555-234-5678	555-234-5679	info@rst.com	www.rst.com	UVW Inventory	8181 Redwood St	555-345-6789	555-345-6790	info@uvw.com	www.uvw.com
VWX Supply	8282 Juniper St	555-456-7890	555-456-7891	info@vwx.com	www.vwx.com	YZA Procurement	8383 Cypress St	555-567-8901	555-567-8902	info@yza.com	www.yza.com
ABC Sourcing	8484 Fir St	555-678-9012	555-678-9013	info@abc.com	www.abc.com	DEF Logistics	8585 Palm St	555-789-0123	555-789-0124	info@def.com	www.def.com
GHI Freight	8686 Cedar St	555-890-1234	555-890-1235	info@ghi.com	www.ghi.com	JKL Shipping	8787 Birch St	555-901-2345	555-901-2346	info@jkl.com	www.jkl.com
MNO Logistics	8888 Spruce St	555-012-3456	555-012-3457	info@mno.com	www.mno.com	PQR Warehousing	8989 Willow St	555-123-4567	555-123-4568	info@pqr.com	www.pqr.com
RST Freight	9090 Ash St	555-234-5678	555-234-5679	info@rst.com	www.rst.com	UVW Inventory	9191 Hickory St	555-345-6789	555-345-6790	info@uvw.com	www.uvw.com
VWX Supply	9292 Sycamore St	555-456-7890	555-456-7891	info@vwx.com	www.vwx.com	YZA Procurement	9393 Magnolia St	555-567-8901	555-567-8902	info@yza.com	www.yza.com
ABC Sourcing	9494 Dogwood St	555-678-9012	555-678-9013	info@abc.com	www.abc.com	DEF Logistics	9595 Redwood St	555-789-0123	555-789-0124	info@def.com	www.def.com
GHI Freight	9696 Juniper St	555-890-1234	555-890-1235	info@ghi.com	www.ghi.com	JKL Shipping	9797 Cypress St	555-901-2345	555-901-2346	info@jkl.com	www.jkl.com
MNO Logistics	9898 Fir St	555-012-3456	555-012-3457	info@mno.com	www.mno.com	PQR Warehousing	9999 Palm St	555-123-4567	555-123-4568	info@pqr.com	www.pqr.com
RST Freight	10000 Cedar St	555-234-5678	555-234-5679	info@rst.com	www.rst.com	UVW Inventory					

Operating chemical plants is based on measuring data, such as: temperature, flowrates, pressure, concentrations....etc

2.7 Process Data Representation and Analysis

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Consider the following:

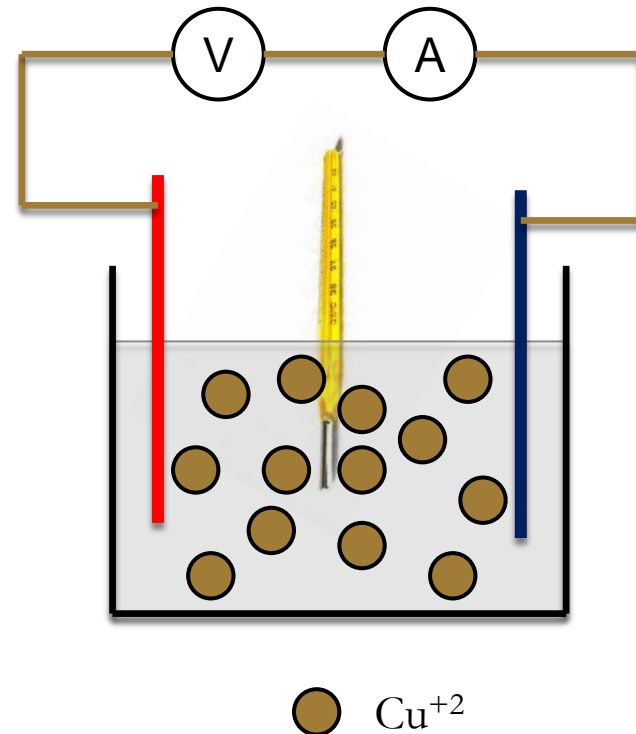
✓ Temperature:

Dip a thermometer
(Direct measurement)

✓ Concentration:

Higher the concentration of ions in the solution, higher the current passes in the circuit

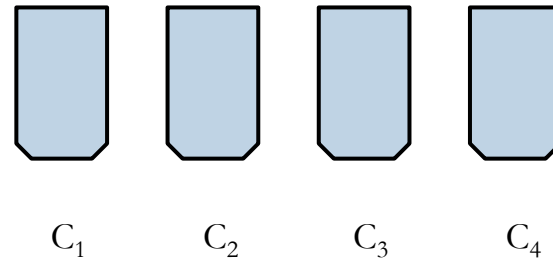
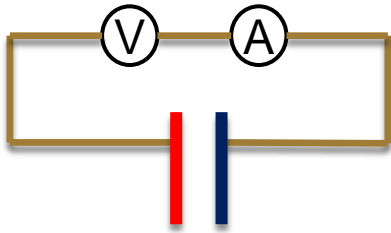
Indirect measurement (Measuring the variable through a known relation with another one).



2.7 Process Data Representation and Analysis

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✓ In the previous experiment, only **current can be measured!**



Calibration Experiment:

Experiment in which solution of known concentration are prepared and X is measured for each solution.

Concentration	Current
C_1	I_1
C_2	I_2
C_3	I_3
C_4	I_4

2.7 Process Data Representation and Analysis

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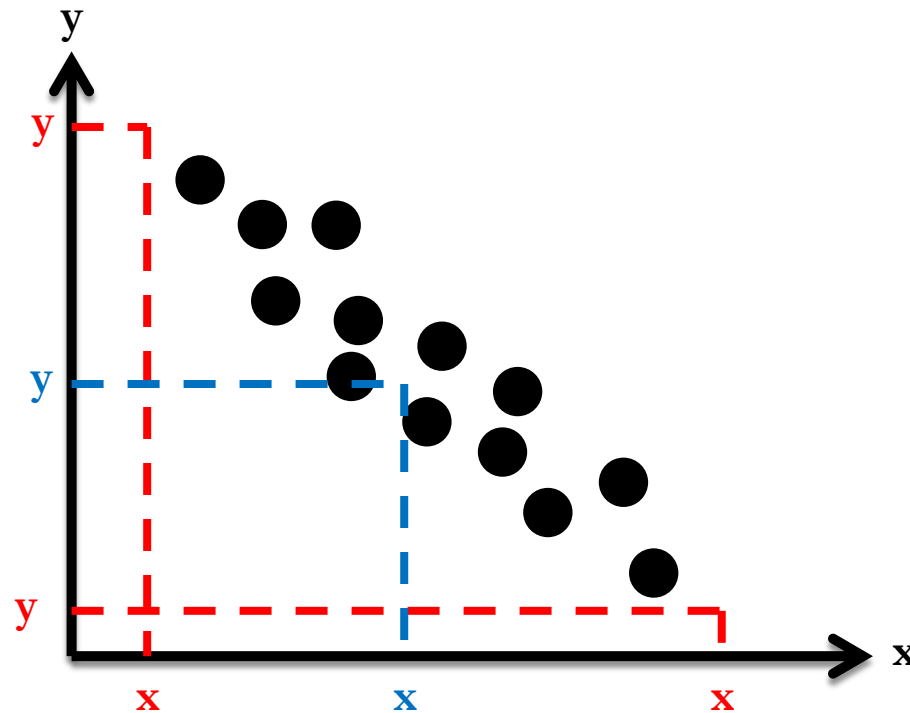


What would I do if the current I got was not one of the listed values in the table?

2.7 Process Data Representation and Analysis

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For any set of data x and y :



Interpolation: estimation of “ y ” for a given “ x ” located within the data range.

Extrapolation: estimation of “ y ” for a given “ x ” located outside the data range.

2.7 Process Data Representation and Analysis

Techniques to obtain an estimate for a given “x”:

1. Two-point linear interpolation.
2. Graphical method.
3. Least squared method

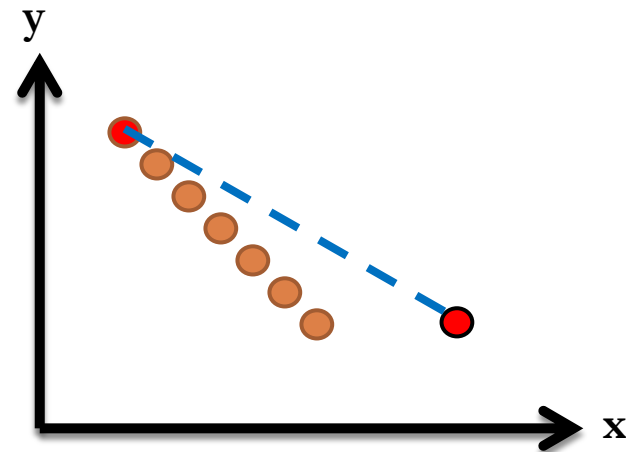
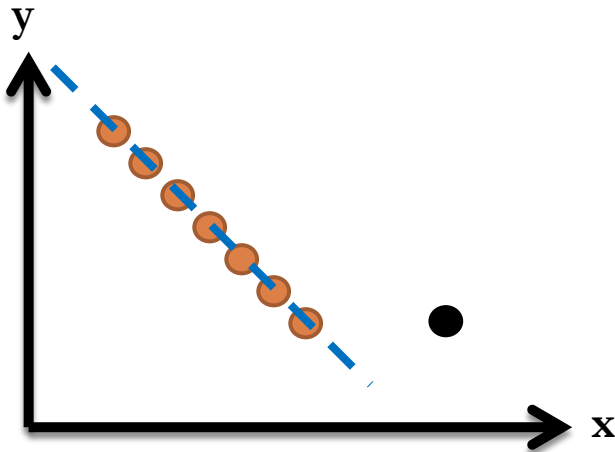
2.7 Process Data Representation and Analysis

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✓ For any two points (x_1, y_1) and (x_2, y_2) , a straight line equation connecting them is:

$$y = y_1 + \frac{x - x_1}{x_2 - x_1} (y_2 - y_1)$$

- Suitable if the points in the table are close to each other.
- Not recommended if the data points are widely spread.



2.7 Process Data Representation and Analysis

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Example: Values of a variable (f) are measured at several times (t):

f	1	4	8
t	1	2	3

Using the two-point linear interpolation, calculate:

a. $f(t=1.3)$

b. $t(f=5.0)$

2.7 Process Data Representation and Analysis

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- A straight line equation passing through two points (x_1, y_1) and (x_2, y_2) is given by:

$$y = ax + b$$

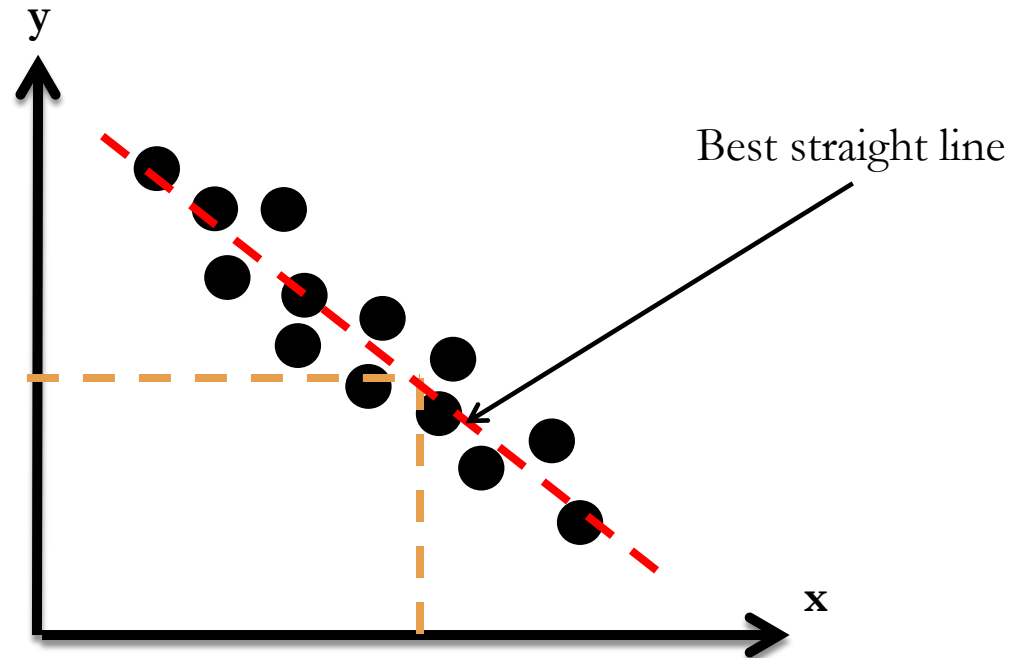
$$a \equiv \text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$b \equiv \text{intercept} = \begin{cases} y_1 - ax_1 \\ y_2 - ax_2 \end{cases}$$

2.7 Process Data Representation and Analysis

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1. Plot the data points.
2. Look for the best curve passing through the data points.
3. Read your values from the resulting curve OR
4. Obtain an equation for the straight line considering any two points.



2.7 Process Data Representation and Analysis

✓ If a set of data and a nonlinear model is available, the following procedure is applied:

1. Try to rewrite the model in a linear form:

$$f(x, y) = ag(x, y) + b$$

2. Calculate $f(x, y)$ and $g(x, y)$
3. Plot $f(x, y)$ vs. $g(x, y)$.
4. Obtain the values of the model constants, a & b.

2.7 Process Data Representation and Analysis

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Example: two variables P and t , are related by the equation:

$$P = \frac{1}{mt^{1/2} + r}$$

The following data are taken:

P	0.279	0.194	0.168	0.120	0.083
t	1.0	2.0	3.0	5.0	10.0

Calculate m and r ?

2.7 Process Data Representation and Analysis

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1. Rewrite the nonlinear model in a linear form, i.e.

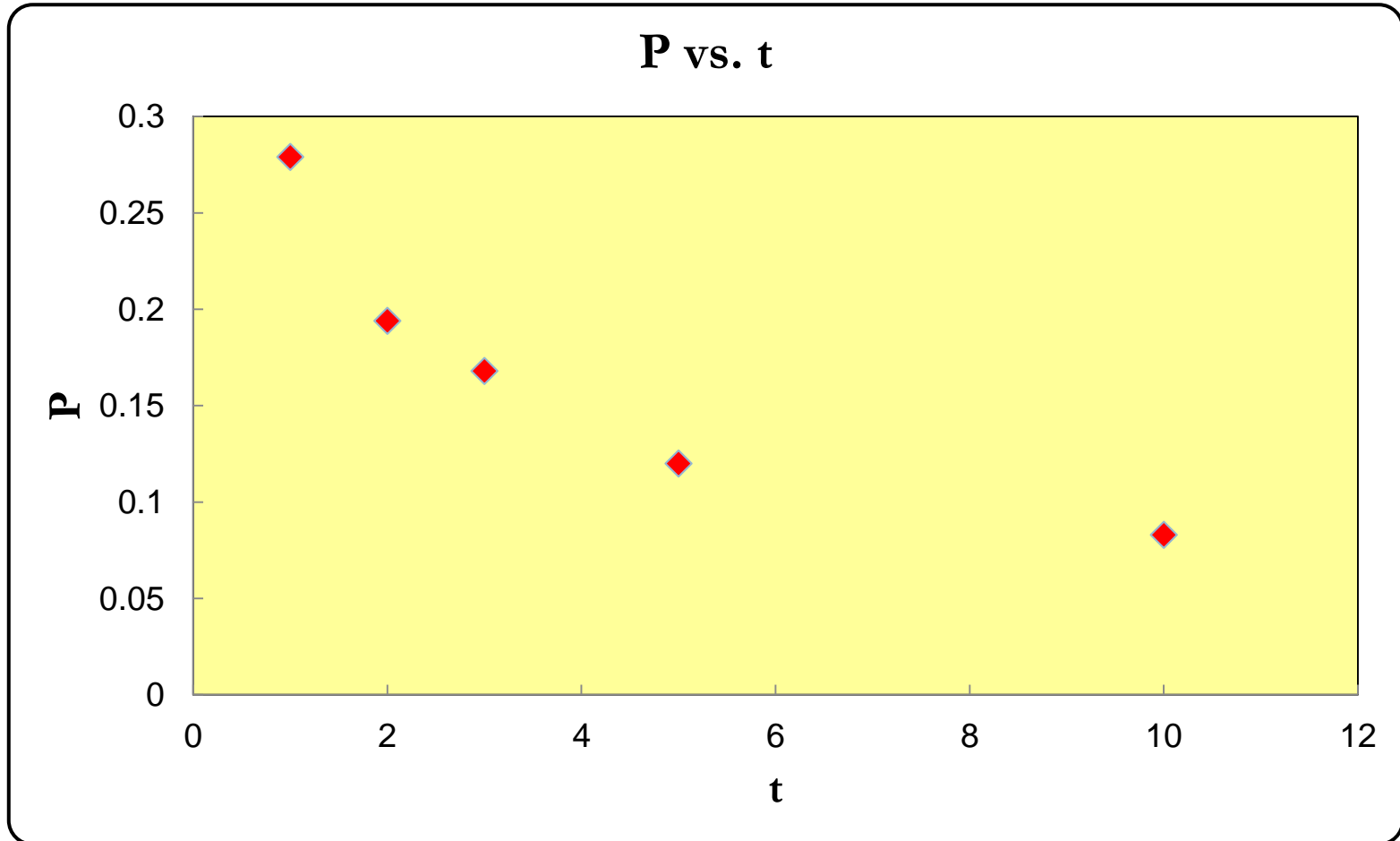
$$f(x, y) = ag(x, y) + b \qquad \frac{1}{P} = mt^{1/2} + r$$

2. Calculate and using the data given:

P	0.279	0.194	0.168	0.120	0.083
t	1.0	2.0	3.0	5.0	10.0
1/P	3.58	5.15	5.95	8.33	12.05
$t^{1/2}$	1.0	1.41	1.73	2.24	3.16

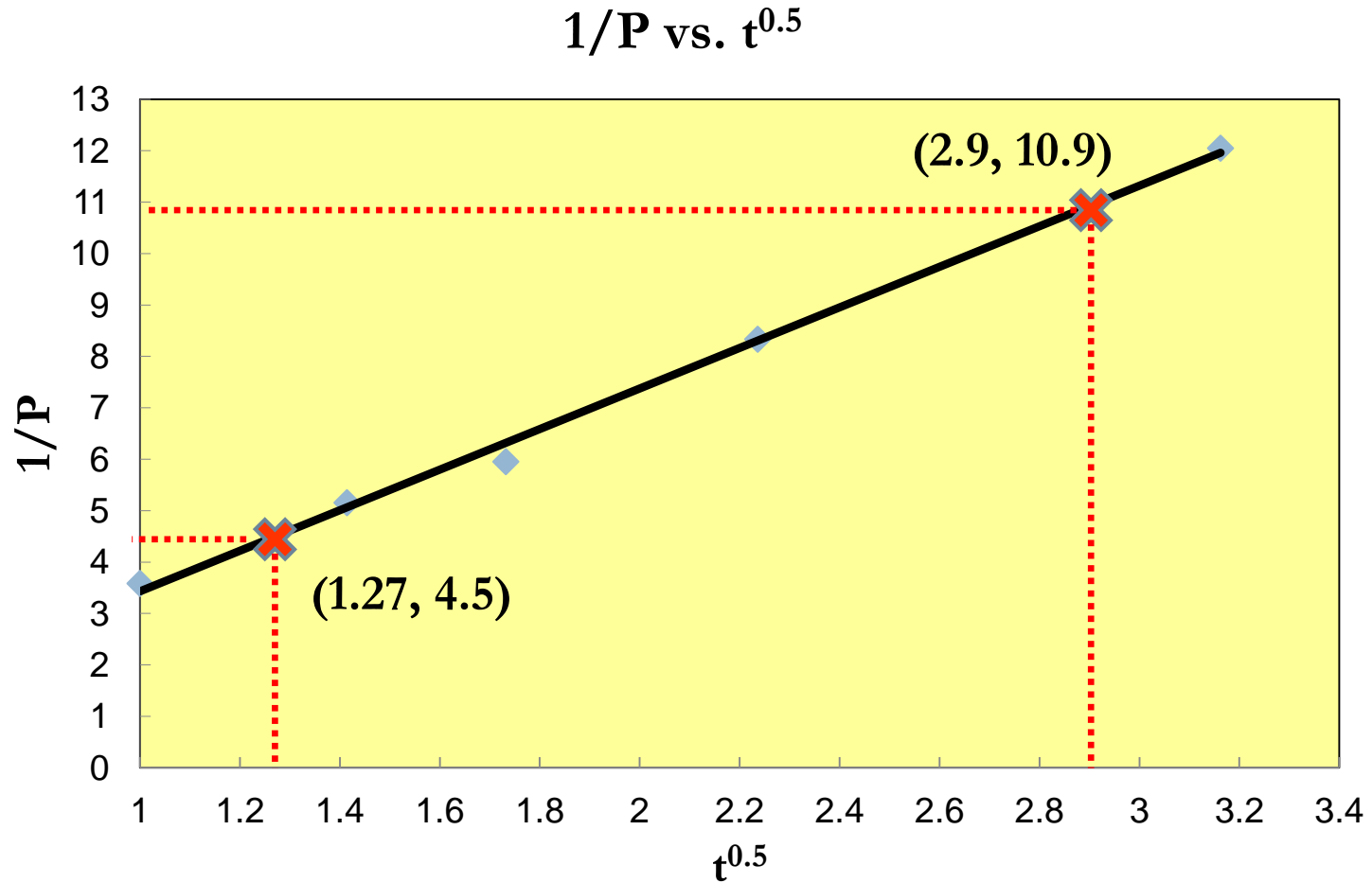
2.7 Process Data Representation and Analysis

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2.7 Process Data Representation and Analysis

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2.7 Process Data Representation and Analysis

1. Power law functions:

$$y = ax^b$$

$$\ln(y) = b \ln(x) + \ln(a)$$

For a given set x and y , $\ln(y)$ vs. $\ln(x)$ on a rectangular coordinate gives a straight line with “slope = b ” and “intercept = $\ln(a)$ ”.

2.7 Process Data Representation and Analysis

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2. Exponential functions:

$$y = b \exp(ax)$$

$$\ln(y) = ax + \ln(b)$$

For a given set x and y , $\ln(y)$ vs. x on a rectangular coordinate gives a straight line with “slope = a ” and “intercept = $\ln(b)$ ”.

2.7 Process Data Representation and Analysis

1. Rectangular Coordinate Graph Paper.
2. Semi-log Graph Paper.
3. Log-log Graph Paper

2.7 Process Data Representation and Analysis

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A component A depleted in a solution with time due to an unknown chemical reaction. The data of concentration of A collected vs. time are as follows:

t (min)	0	1	2	3	4
C_A (mole/L)	10	6.07	3.68	2.23	1.35

It is requested to fit the above data into the following model:

$$C_A(t) = C_{A0} \exp(-kt)$$

What are the values of “k” and “ C_{A0} ”?

2.7 Process Data Representation and Analysis

✓ The given model can be rewritten as:

$$\ln(C_A) = \ln(C_{A0}) - kt$$

✓ Two options for plotting:

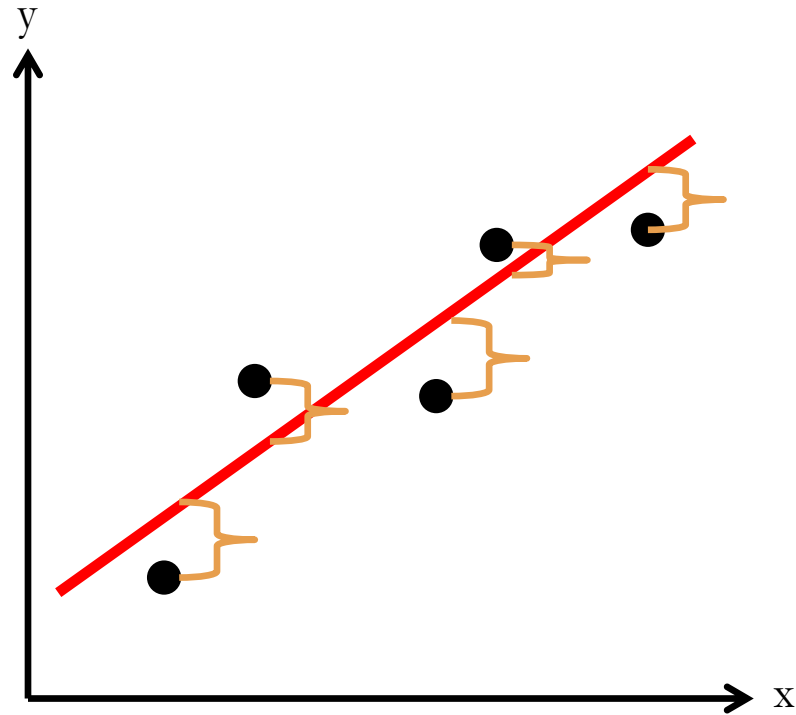
1. Plot $\ln(C_A)$ vs. t on rectangular coordinate to obtain a straight line of slope $= -k$, and intercept $= \ln(C_{A0})$.

t (min)	0	1	2	3	4
C_A (mole/L)	10	6.07	3.68	2.23	1.35
$\ln(C_A)$	2.30	1.80	1.30	0.80	0.30

2. Plot C_A vs. t on a semi-log graph paper.

2.7e Linear Regression (Least squared method)

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This technique is based on minimizing the error between the data and the model predictions.

$$OF(a,b) = \sum_{i=1}^n [y_i - (ax_i + b)]^2$$

2.7e Linear Regression (Least squared method)

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Best Line:

$$y = ax + b$$

$$s_x = \frac{1}{n} \sum_{i=1}^n x_i$$

$$s_{xx} = \frac{1}{n} \sum_{i=1}^n x_i^2$$

$$s_y = \frac{1}{n} \sum_{i=1}^n y_i$$

$$s_{xy} = \frac{1}{n} \sum_{i=1}^n x_i y_i$$

Slope:

$$a = \frac{s_{xy} - s_x s_y}{s_{xx} - (s_x)^2}$$

Intercept:

$$b = \frac{s_{xx} s_y - s_{xy} s_x}{s_{xx} - (s_x)^2}$$

2.7e Linear Regression (Least squared method)

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Best Line:

$$y = ax$$

$$s_{xx} = \frac{1}{n} \sum_{i=1}^n x_i^2$$

$$s_{xy} = \frac{1}{n} \sum_{i=1}^n x_i y_i$$

Slope:

$$a = \frac{s_{xy}}{s_{xx}}$$

2.7e Linear Regression (Least squared method)

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t (min)	0	1	2	3	4
C_A (mol/L)	10	6.07	3.68	2.23	1.35
$\ln(C_A)$	2.30	1.80	1.30	0.80	0.30

2.7e Linear Regression (Least squared method)

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t (min)	C_A (mol/L)	$\ln(C_A)$	$t^2(\text{min}^2)$	$t*\ln(CA)$
0	10	2.30	0	0
1	6.07	1.80	1	1.80
2	3.68	1.30	4	2.60
3	2.23	0.80	9	2.4
4	1.35	0.30	16	1.2
10	----	6.50	30	8.0

Thank You