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CHAPTER 2: INTRODUCTION TO ENGINEERING CALCULATIONS

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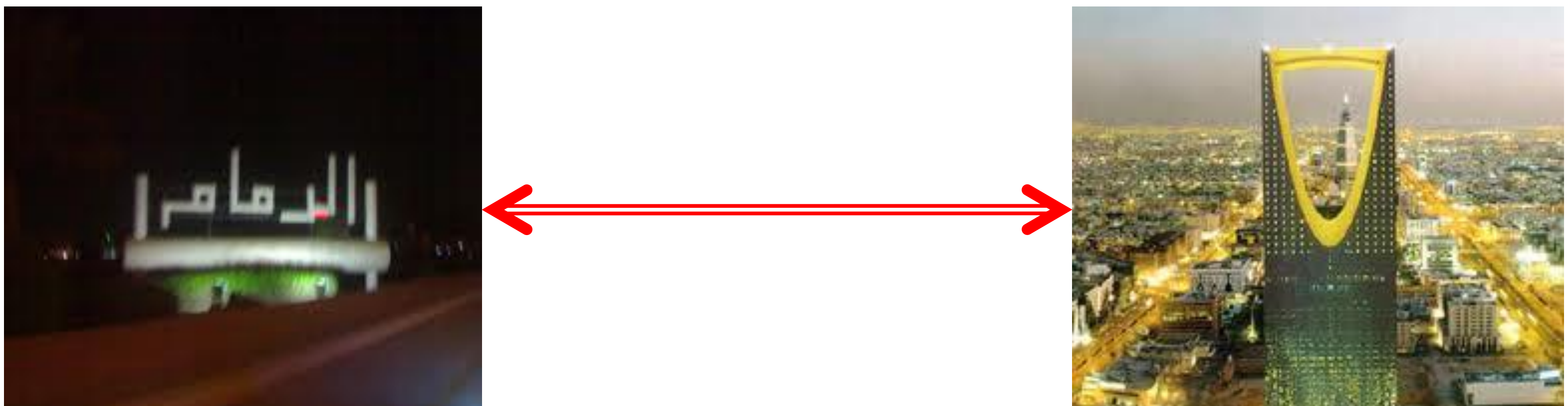
2.1 Units and Dimensions:

Consider the following Case:



400 km

Numerical Value

Unit

2.1 Units and Dimensions:

✓ Dimension is a property that can measured such as length, time, temperature, … etc or calculated by multiplying or dividing other dimensions such as volume (length^3), area (length^2), velocity (length/time), … etc.

✓ Units can be divided into:

1. Measurable units (specific values of dimensions defined by convention, custom, laws)
2. Countable units (Don't have specific values of dimensions).

Examples:

grams	mass
seconds	times
centimeters or feet	length

Relation to mathematical operations:

✓ Addition and Subtraction:

Quantities of the same units can be added or subtracted.

Quantity 1 + Quantity 2 = Quantity 3

Numerical values are added (subtracted) while units are copied as are.

✓ Multiplication and Division:

Quantities of the same or different units are multiplied or divided

Quantity 1 × Quantity 2 = Quantity 3

Numerical values as well as units are multiplied or divided.

Example 2.1-1:

Perform the following mathematical operations if possible:

1. $3.0 \text{ cm} + 4.0 \text{ cm}$
2. $5.0 \text{ g} + 10.0 \text{ kg}$
3. $10.0 \text{ N} \times 5.0 \text{ m}^2$
4. $12.0 \text{ kg/hr} \times 4.0 \text{ hr}$
5. $10.0 \text{ km} / 2.0 \text{ hr}$
6. $5.0 \text{ kg/s} \div 0.2 \text{ kg/m}^3$

2.2 Conversion of Units:

Given: $a = b$

can be written as $\frac{a}{b}$ leading to:

Equivalence between two expressions of the same quantities may be defined as a ratio.

$$1.0 \text{ m} = 100 \text{ cm}$$

Conversion Factors

$$\frac{1.0\,\text{m}}{100\,\text{cm}}$$ 1.0 m per 100 centimeter

$$\frac{100\,\text{cm}}{1.0\,\text{m}}$$ 100 cm per 1.0 meter

2.2 Conversion of Units:

✓ Conversion factor is defined as a ratio of new unit to old unit used to convert between quantities,

$$\text{Given Unit} \times \frac{\text{Desired Unit}}{\text{Given Unit}}$$

Example 2.2-1Convert the following:

1. 36 mg to grams
2. 5 km to cm
3. 1.0 cm/s^2 to km/yr^2

2.3 Systems of Units

Units can be broken up into:

1. Basic units, i.e. units of length, mass, time, temperature, …. etc
2. Multiple units, i.e. multiples or fractions of basic units such as hour, minute, millisecond, centimeter, … etc
3. Derived units are those obtained by:
   a. Multiplying or dividing basic or multiple units such as m^3, m^2, m/s, m/s^2.
   b. Equivalents of compound units such as $1\ N = 1\ kg \times m/s^2$, $1\ dyne = 1\ g \times cm/s^2$, $1\ lb_f = 32.174\ lb_m \times ft/s^2$.

2.3 Systems of Units

Quantity	Unit
Length	m
Mass	kg
Time	s
Moles	Gram-mole
Temperature	Kelvin
Electric Current	Ampere
Light Intensity	candela

2.3 Systems of Units

Quantity	SI	CGS	American Engineering
Length	m	cm	ft
Mass	kg	g	lb_m
Time	s	s	s
Force	N	Dyne	lb_f

2.3 Systems of Units

Prefixes are used with the SI units indicating powers of tens as follows:

Prefix	Indication
Mega (M)	10^6
Kilo (k)	10^3
Centi (c)	10^{-2}
Mili (m)	10^{-3}
Micro (μ)	10^{-6}
Nano (n)	10^{-9}

2.3 Systems of Units

FACTORS FOR UNIT CONVERSIONS

Quantity	Equivalent Values
Mass	1 kg = 1000 g = 0.001 metric ton = 2.20462 lb_m = 35.27392 oz 1 lb_m = 16 oz = 5×10^{-4} ton = 453.593 g = 0.453593 kg
Length	1 m = 100 cm = 1000 mm = 10^6 microns (μm) = 10^{10} angstroms (Å) = 39.37 in. = 3.2808 ft = 1.0936 yd = 0.0006214 mile 1 ft = 12 in. = 1/3 yd = 0.3048 m = 30.48 cm
Volume	1 m^3 = 1000 L = 10^6 cm^3 = 10^6 mL = 35.3145 ft^3 = 220.83 imperial gallons = 264.17 gal = 1056.68 qt 1 ft^3 = 1728 $in.^3$ = 7.4805 gal = 0.028317 m^3 = 28.317 L = 28,317 cm^3
Force	1 N = 1 kg·m/s^2 = 10^5 dynes = 10^5 g·cm/s^2 = 0.22481 lb_f 1 lb_f = 32.174 lb_m·ft/s^2 = 4.4482 N = 4.4482×10^5 dynes
Pressure	1 atm = 1.01325×10^5 N/m^2 (Pa) = 101.325 kPa = 1.01325 bar = 1.01325×10^6 dynes/cm^2 = 760 mm Hg at 0°C (torr) = 10.333 m H_2O at 4°C = 14.696 lb_f/$in.^2$ (psi) = 33.9 ft H_2O at 4°C = 29.921 in. Hg at 0°C
Energy	1 J = 1 N·m = 10^7 ergs = 10^7 dyne·cm = 2.778×10^{-7} kW·h = 0.23901 cal = 0.7376 ft-lb_f = 9.486×10^{-4} Btu
Power	1 W = 1 J/s = 0.23901 cal/s = 0.7376 ft·lb_f/s = 9.486×10^{-4} Btu/s = 1.341×10^{-3} hp

Example: The factor to convert grams to lb_m is $\left(\frac{2.20462\ lb_m}{1000\ g}\right)$.

Example 2.3-1: Conversion Between Systems of Units

Convert 23 $lb_m \times ft/min^2$ to its equivalent in $kg \times cm/s^2$?

2.4 Force and Weight

✓According to Newton second law, force is defied as the product of the mass and acceleration.

$$F = m \times a$$

Units are:

SI $\rightarrow$ $kg \times m/s^2$ ($1\ N \equiv 1\ kg \times m/s^2$)

CGS $\rightarrow$ $g \times cm/s^2$ ($1\ dyne \equiv 1\ g \times cm/s^2$)

Ameri. $\rightarrow$ $lb_m \times ft/s^2$ ($1\ lb_f \equiv 32.174\ lb_m \times ft/s^2$)

✓Pound-force is defined as the product of 1 lb_m and acceleration of gravity at sea level and latitude of 45°, i.e. a = 32.174 ft/s^2.

2.4 Force and Weight

Example 2.4-1: Calculate:

1. Force in Newton required to accelerate a mass of 4.0 kg at a rate of 9.00 m/s^2?
2. Force in dyne required to accelerate a mass of 4.0 g at a rate of 9.00 cm/s^2?
3. Force in lb_f required to accelerate a mass of 4.00 lb_m at a rate of 9.00 ft/s^2?

2.4 Force and Weight

Define $g_c \equiv$ conversion factor as:

SI: $$g_c = \frac{1.0\,\text{kg} \times \text{m/s}^2}{1.0\,\text{N}}$$

CGS: $$g_c = \frac{1.0\,\text{g} \times \text{cm/s}^2}{1.0\,\text{dyne}}$$

Ameri: $$g_c = \frac{32.173\,\text{lb}_\text{m} \times \text{ft/s}^2}{1.0\,\text{lb}_\text{f}}$$

✓Weight of an object is the force exerted by gravitational attraction. It is calculated according to:

$$W = m\frac{g}{g_c}$$

2.4 Force and Weight

- ✓ Gravitational acceleration is given at sea level and 45° latitude as:

$$g = 9.8066\ m/s^2$$

$$= 980.66\ cm/s^2$$

$$= 32.174\ ft/s^2$$

2.6 Dimensional Homogeneity and Dimensionless Quantities

Consider:

$$u\left(\frac{m}{s}\right)=u_o\left(\frac{m}{s}\right)+g\left(\frac{m}{s^2}\right)t\left(s\right)$$

Dimensions: $$\frac{\text{length}}{\text{time}}=\frac{\text{length}}{\text{time}}+\frac{\text{length}}{\text{time}^2}\times\text{time}$$

- An equation is **dimensionally homogeneous** if and only if all additive terms on both sides of the equation have the same dimensions. The equation is also **consistent** in terms of units, i.e. length is in meters and time is in second.

2.6 Dimensional Homogeneity and Dimensionless Quantities

Consider:

$$u\left(\frac{m}{s}\right)=u_o\left(\frac{m}{s}\right)+g\left(\frac{m}{s^2}\right)t\left(\min\right)$$

Dimensions: $$\frac{\text{length}}{\text{time}}=\frac{\text{length}}{\text{time}}+\frac{\text{length}}{\text{time}^2}\times\text{time}$$

- An equation is **dimensionally homogeneous** if and only if all additive terms on both sides of . The equation is **inconsistent** in terms of units, i.e. length in meter but time in seconds and minutes. A proper conversion factor must be introduced.

$$u\left(\frac{m}{s}\right)=u_o\left(\frac{m}{s}\right)+g\left(\frac{m}{s^2}\right)t\left(\min\right)\times\frac{60\,s}{1\min}$$

2.6 Dimensional Homogeneity and Dimensionless Quantities

Consider:

$$u\left(\frac{m}{s}\right)=u_o\left(\frac{m}{s}\right)+g\left(\frac{m}{s^2}\right)$$

Dimensions: $$\frac{\text{length}}{\text{time}}=\frac{\text{length}}{\text{time}}+\frac{\text{length}}{\text{time}^2}$$

- An equation is **not valid** if it is dimensionally non-homogeneous.

Summary of Cases:

	Dimensionally Homogeneous	Consistent	Example	Actions
Case 1	✓	✓	$u\left(\frac{m}{s}\right)=u_o\left(\frac{m}{s}\right)+g\left(\frac{m}{s^2}\right)t(s)$	Valid It can be used
Case 2	✓	✗	$u\left(\frac{m}{s}\right)=u_o\left(\frac{m}{s}\right)+g\left(\frac{m}{s^2}\right)t(\min)$	A conversion factor is needed
Case 3	✗	----	$u\left(\frac{m}{s}\right)=u_o\left(\frac{m}{s}\right)+g\left(\frac{m}{s^2}\right)$	Invalid It can't be used

Example 2.6-1:

Consider the equation:

$$D(ft) = 3t(s) + 4$$

1. If the equation is valid, what are the dimensions of constants 3 and 4?
2. If the equation is consistent in its units, what are the units of 3 and 4?
3. Derive an equation for the distance in meters in terms of time in minutes?

Procedure to drive equivalent equations in different units:

1. Define new variables by affixing primes to the old variable names with the new units.
2. Write the old variables in terms of the new ones.
3. Substitute in the old expression and simplify the equation.

Dimensionless Quantities:

1. Pure Numbers such as 1, 2 , 3/2, 1/4, and so on.
2. Multiplicative combination of variables may result in dimensionless quantities such as:

$$\text{Rynold's no.} = \frac{D(\text{cm}) \times u\left(\frac{\text{cm}}{\text{s}}\right) \times \varrho\left(\frac{\text{g}}{\text{cm}^3}\right)}{\mu\left(\frac{\text{g}}{\text{cm} \times \text{s}}\right)}$$

$$\text{Mass Ratio} = \frac{M(\text{g})}{M_o(\text{g})}$$

3. Exponents such as "a" in X^a .
4. Arguments of transcendental functions such as:

X	in	sin(X)
Y	in	exp(Y)
Z	in	log(Z)

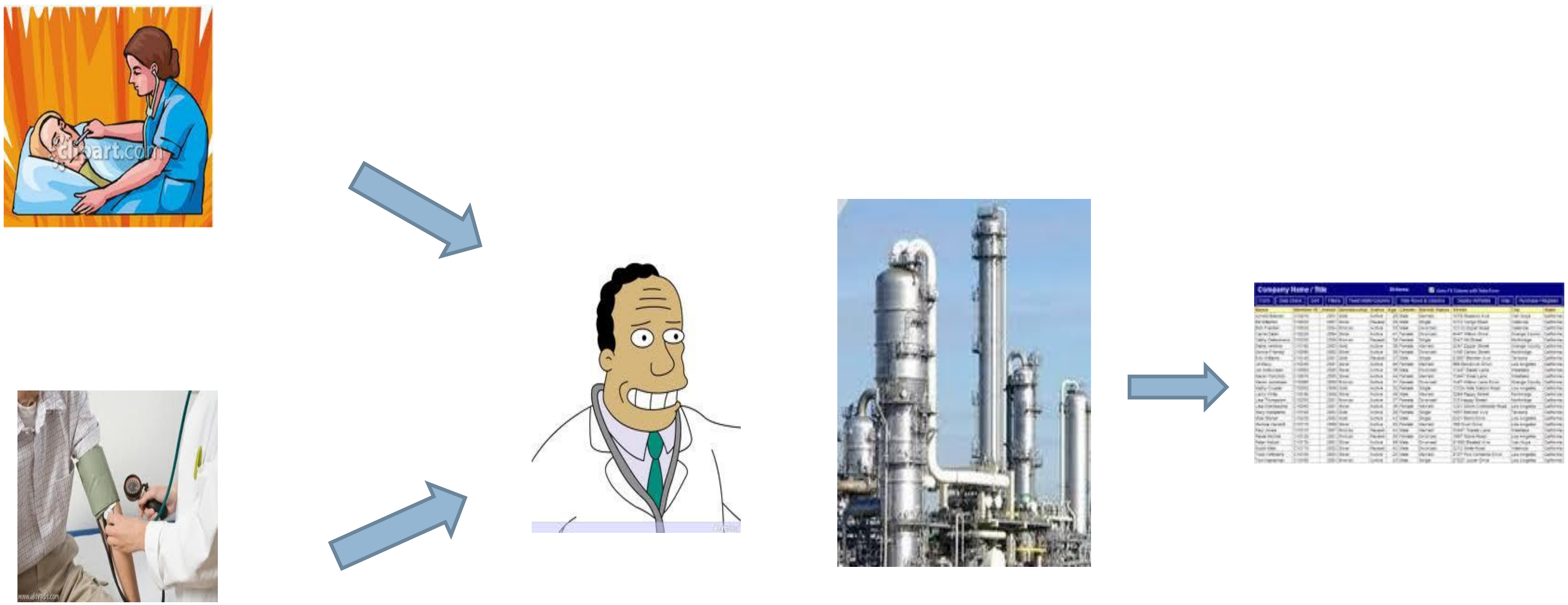
Example 2.6-2: Dimensional homogeneity and dimensionless groups

A quantity k depends on temperature T in the following manner:

$$k\left(\frac{mol}{cm^3 \times s}\right) = 1.2\times10^5 \exp\left(-\frac{20{,}000}{1.987T}\right)$$

The units of the quantity 20,000 are cal/mol, and T in K (Kelvin). What are the units of 1.2×10^5 and 1.987?

2.7 Process Data Representation and Analysis



Operating chemical plants is based on measuring data, such as: temperature, flowrates, pressure, concentrations….etc

2.7 Process Data Representation and Analysis

Consider the following:

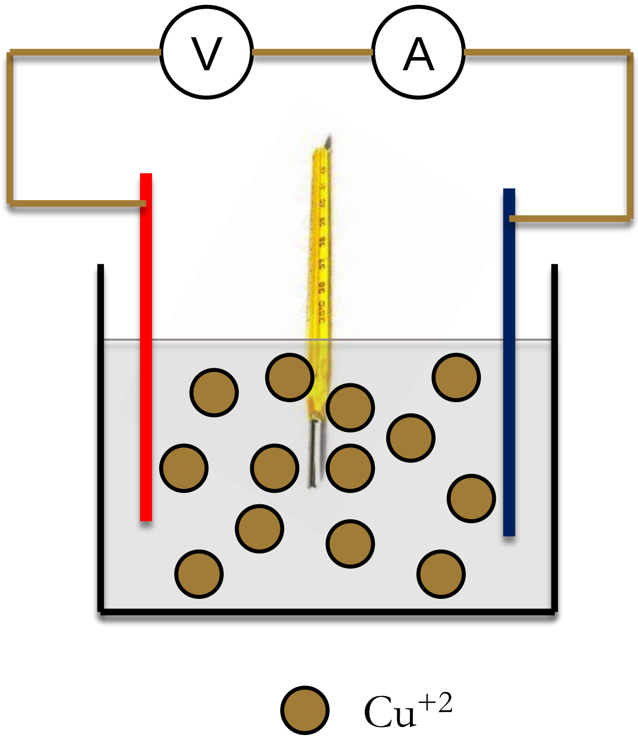
✓Temperature:

Dip a thermometer
(Direct measurement)

✓Concentration:

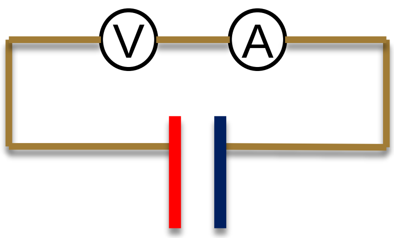
Higher the concentration of ions in the solution, higher the current passes in the circuit

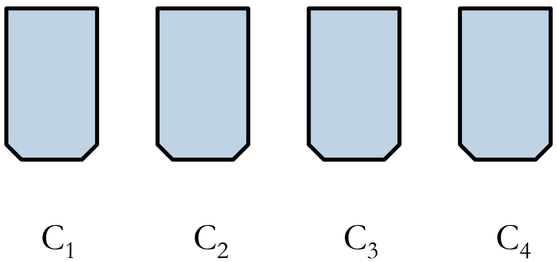
Indirect measurement (Measuring the variable through a known relation with another one).



2.7 Process Data Representation and Analysis

✓In the previous experiment, only **current can be measured!**





<u>Calibration Experiment:</u>

Experiment in which solution of known concentration are prepared and X is measured for each solution.

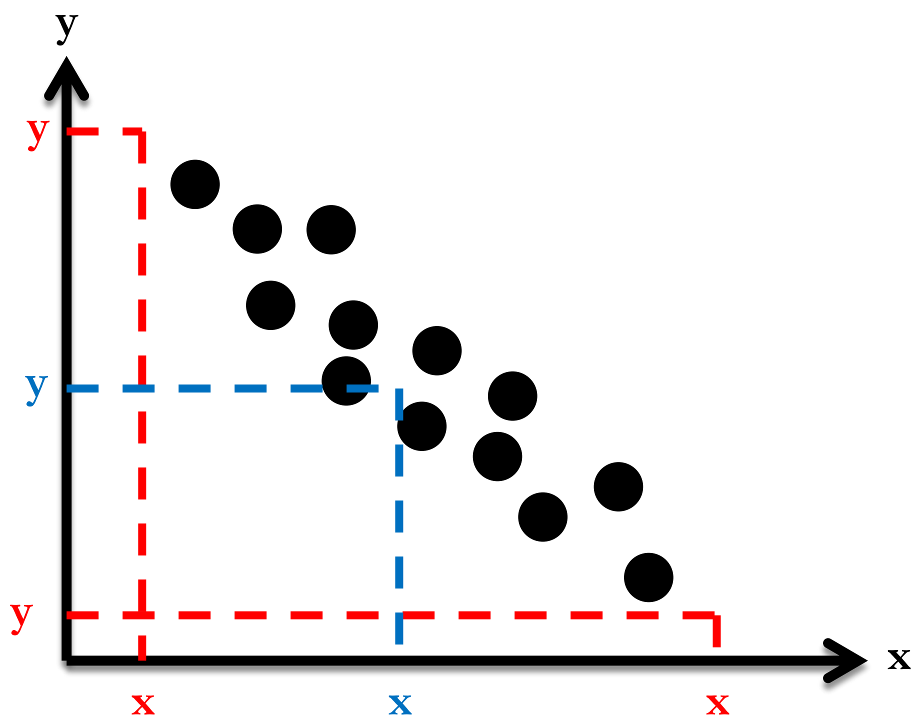
Concentration	Current
C_1	I_1
C_2	I_2
C_3	I_3
C_4	I_4

2.7 Process Data Representation and Analysis

What would I do if the current I got was not one of the listed values in the table?

2.7 Process Data Representation and Analysis

For any set of data x and y:



Interpolation: estimation of "y" for a given "x" located within the date range.

Extrapolation: estimation of "y" for a given "x" located outside the data range.

2.7 Process Data Representation and Analysis

Techniques to obtain an estimate for a given "x":

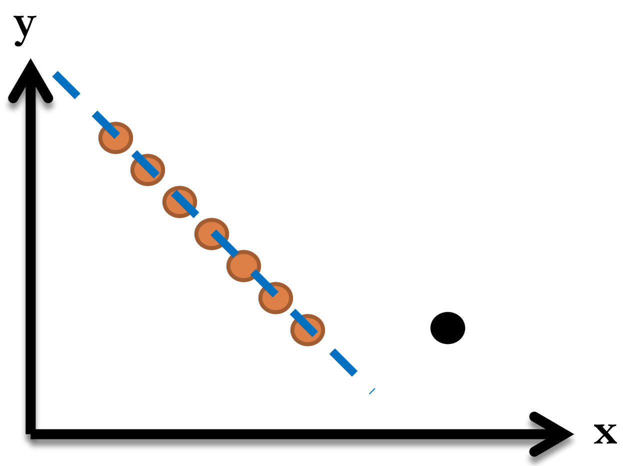
1. Two-point linear interpolation.
2. Graphical method.
3. Least squared method

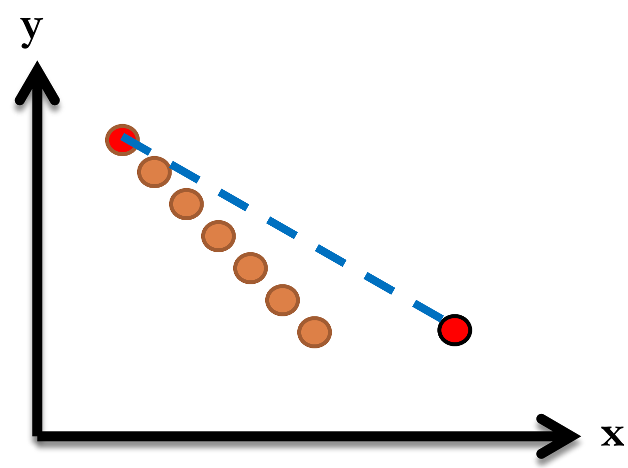
2.7 Process Data Representation and Analysis

✓For any two points (x_1, y_1) and (x_2, y_2), a straight line equation connecting them is:

$$y = y_1 + \frac{x - x_1}{x_2 - x_1}\left(y_2 - y_1\right)$$

- Suitable if the points in the table are close to each other.
- Not recommended if the data points are widely spread.





2.7 Process Data Representation and Analysis

Example: Values of a variable (f) are measured at several times (t):

f	1	4	8
t	1	2	3

Using the two-point linear interpolation, calculate:

a. f(t=1.3)

b. t(f=5.0)

2.7 Process Data Representation and Analysis

- A straight line equation passing through two points (x_1, y_1) and (x_2, y_2) is given by:

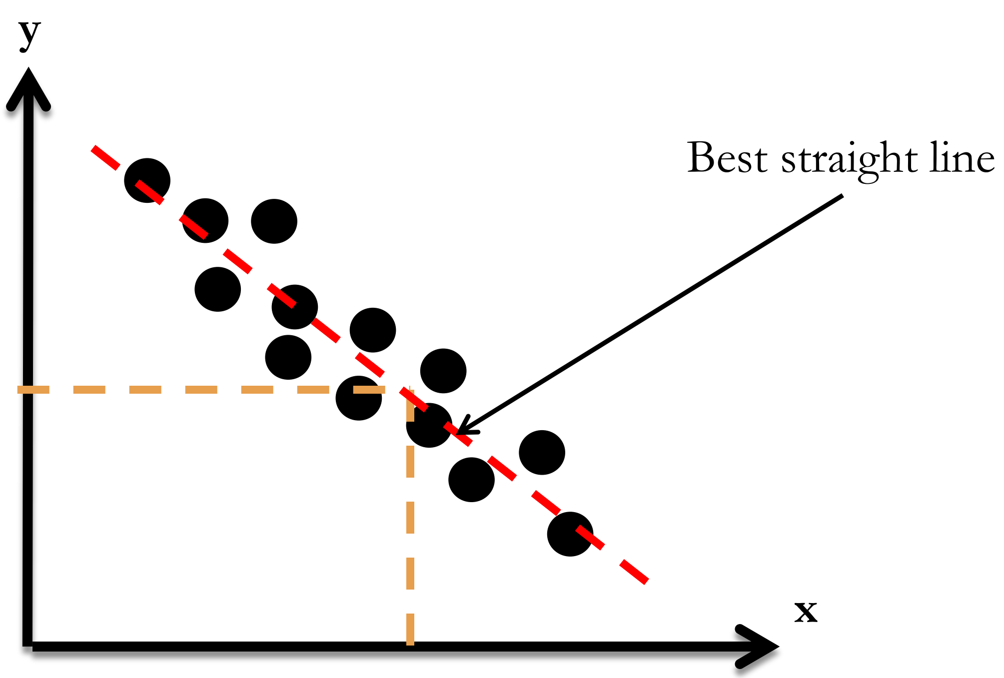
$$\boxed{y = ax + b}$$

$$a \equiv \text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$b \equiv \text{intercept} = \begin{cases} y_1 - ax_1 \\ y_2 - ax_2 \end{cases}$$

2.7 Process Data Representation and Analysis

1. Plot the data points.
2. Look for the best curve passing through the data points.
3. Read your values from the resulting curve OR
4. Obtain an equation for the straight line considering any two points.



2.7 Process Data Representation and Analysis

✓If a set of data and a nonlinear model is available, the following procedure is applied:

1. Try to rewrite the model in a linear form:

$$f(x,y) = ag(x,y) + b$$

2. Calculate $f(x,y)$ and $g(x,y)$
3. Plot $f(x,y)$ vs. $g(x,y)$.
4. Obtain the values of the model constants, a & b.

2.7 Process Data Representation and Analysis

Example: two variables P and t, are related by the equation:

$$P = \frac{1}{mt^{1/2} + r}$$

The following data are taken:

P	0.279	0.194	0.168	0.120	0.083
t	1.0	2.0	3.0	5.0	10.0

Calculate m and r?

2.7 Process Data Representation and Analysis

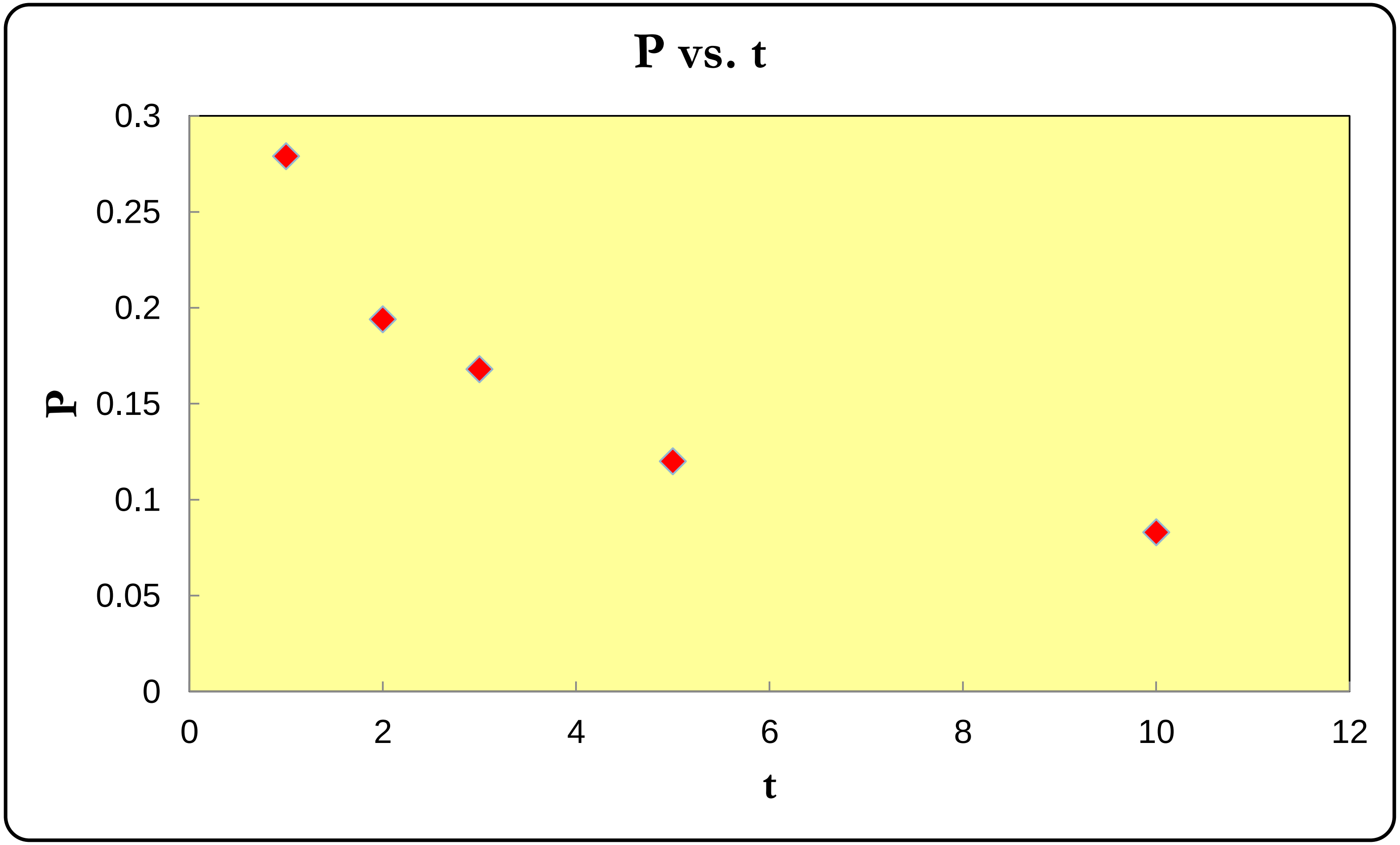
1. Rewrite the nonlinear model in a linear form, i.e.

$$f(x,y) = ag(x,y) + b \qquad \frac{1}{P} = mt^{1/2} + r$$

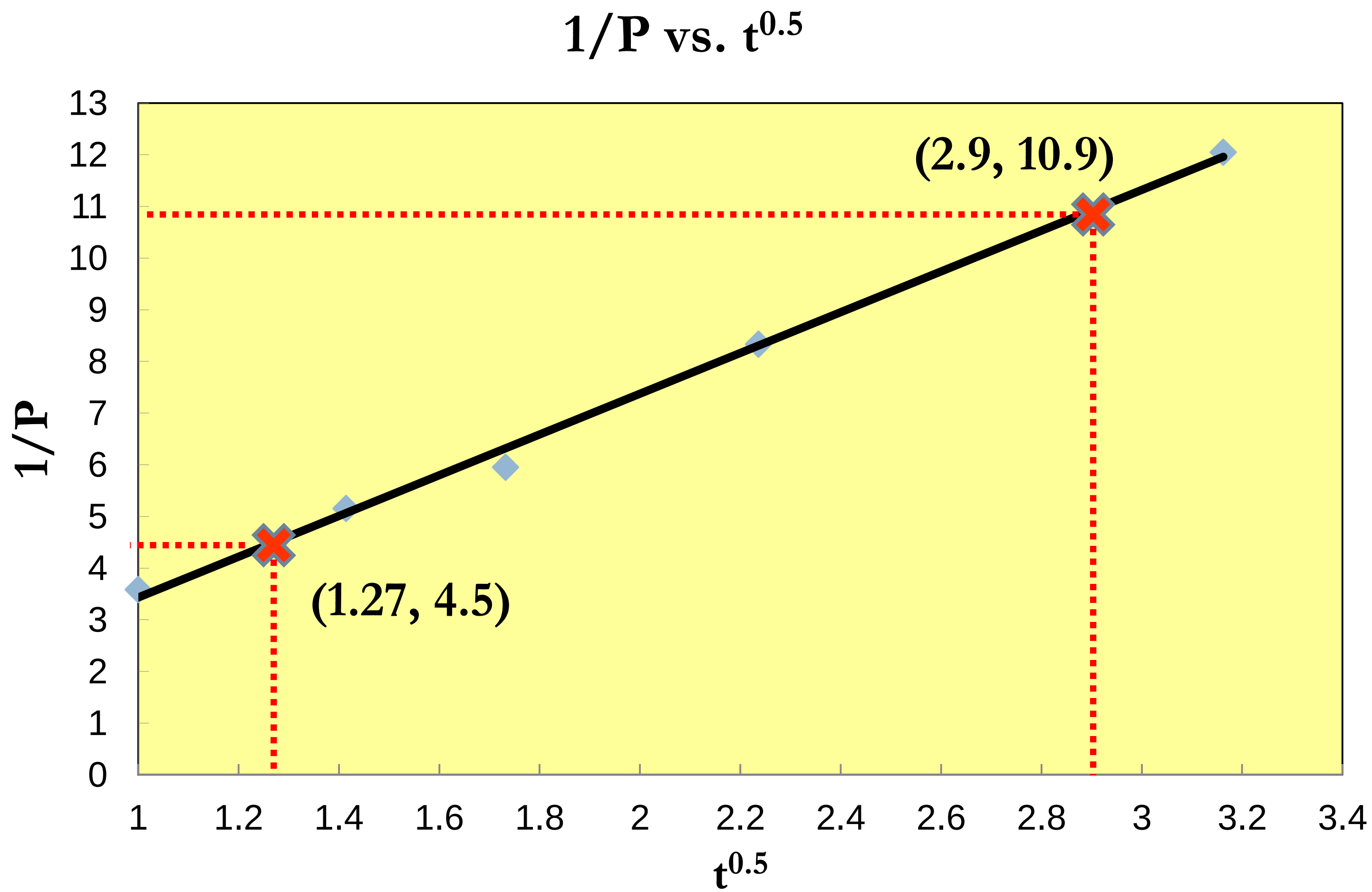
2. Calculate and using the data given:

P	0.279	0.194	0.168	0.120	0.083
t	1.0	2.0	3.0	5.0	10.0
1/P	3.58	5.15	5.95	8.33	12.05
$t^{1/2}$	1.0	1.41	1.73	2.24	3.16

2.7 Process Data Representation and Analysis



2.7 Process Data Representation and Analysis



2.7 Process Data Representation and Analysis

1. Power law functions:

$$y = ax^b$$

$$\ln(y) = b\ln(x) + \ln(a)$$

For a given set x and y, ln(y) vs. ln(x) on a rectangular coordinate gives a straight line with "slope = b" and "intercept = ln(a)".

2.7 Process Data Representation and Analysis

2. Exponential functions:

$$y = b\exp(ax)$$

$$\ln(y) = ax + \ln(b)$$

For a given set x and y, ln(y) vs. x on a rectangular coordinate gives a straight line with "slope = a" and "intercept = ln(b)".

2.7 Process Data Representation and Analysis

1. Rectangular Coordinate Graph Paper.
2. Semi-log Graph Paper.
3. Log-log Graph Paper

2.7 Process Data Representation and Analysis

A component A depleted in a solution with time due to an unknown chemical reaction. The data of concentration of A collected vs. time are as follows:

t (min)	0	1	2	3	4
C_A (mole/L)	10	6.07	3.68	2.23	1.35

It is requested to fit the above data into the following model:

$$C_A(t) = C_{A0} \exp(-kt)$$

What are the values of "k" and "C_{A0}"?

2.7 Process Data Representation and Analysis

✓The given model can be rewritten as:

$$\ln(C_A) = \ln(C_{A0}) - kt$$

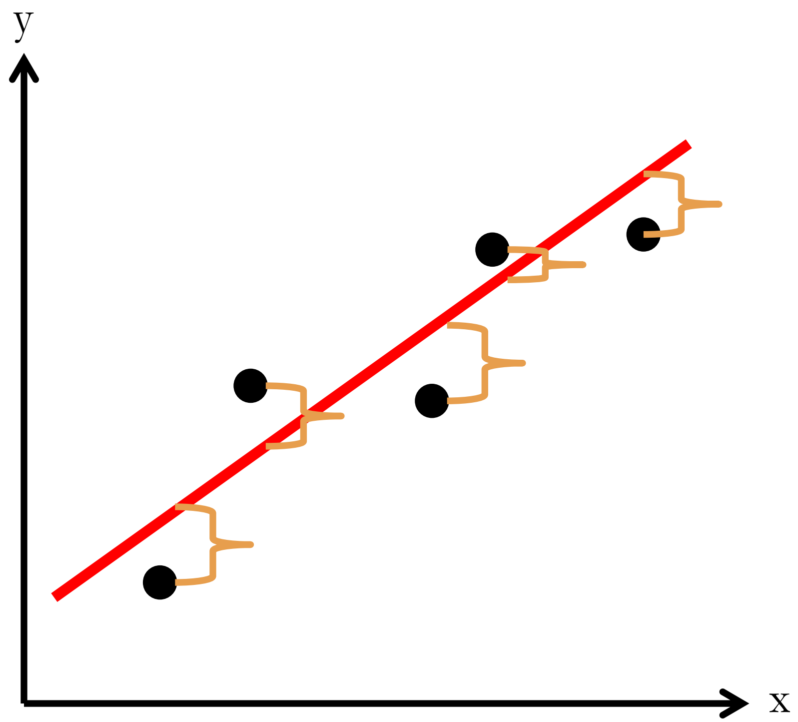
✓Two options for plotting:

1. Plot $\ln(C_A)$ vs. t on rectangular coordinate to obtain a straight line of slope=-k, and intercept=$\ln(C_{A0})$.

t (min)	0	1	2	3	4
C_A (mole/L)	10	6.07	3.68	2.23	1.35
$\ln(C_A)$	2.30	1.80	1.30	0.80	0.30

2. Plot C_A vs. t on a semi-log graph paper.

2.7e Linear Regression (Least squared method)



This technique is based on minimizing the error between the data and the model predictions.

$$OF(a,b)=\sum_{i=1}^{n}\left[y_i-(ax_i+b)\right]^2$$

2.7e Linear Regression (Least squared method)

Best Line: $$y = ax + b$$

$$s_x = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad s_{xx} = \frac{1}{n}\sum_{i=1}^{n} x_i^2$$

$$s_y = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad s_{xy} = \frac{1}{n}\sum_{i=1}^{n} x_i y_i$$

Slope: $$a = \frac{s_{xy} - s_x s_y}{s_{xx} - \left(s_x\right)^2}$$

Intercept: $$b = \frac{s_{xx} s_y - s_{xy} s_x}{s_{xx} - \left(s_x\right)^2}$$

2.7e Linear Regression (Least squared method)

Best Line: $$y = ax$$

$$s_{xx} = \frac{1}{n}\sum_{i=1}^{n} x_i^2$$

$$s_{xy} = \frac{1}{n}\sum_{i=1}^{n} x_i y_i$$

Slope: $$a = \frac{s_{xy}}{s_{xx}}$$

2.7e Linear Regression (Least squired method)

t (min)	0	1	2	3	4
C_A (mol/L)	10	6.07	3.68	2.23	1.35
$\ln(C_A)$	2.30	1.80	1.30	0.80	0.30

2.7e Linear Regression (Least squired method)

t (min)	C_A (mol/L)	$\ln(C_A)$	$t^2 (min^2)$	t*ln(CA)
0	10	2.30	0	0
1	6.07	1.80	1	1.80
2	3.68	1.30	4	2.60
3	2.23	0.80	9	2.4
4	1.35	0.30	16	1.2
10	----	6.50	30	8.0

Thank You