Decision Analysis

Objective of Decision Analysis

Determine an optimal decision under uncertain future events

Formulation of Decision Problem

- Clear statement of the problem
- Identify:
  - The decision alternatives
  - The future events that could impact the decision. These are referred to as chance events
  - The consequences: the outcomes associated with each decision alternative and chance event

Example: PDC Constructing Luxury Condominium Complex

- Problem: How large should the complex be
  PDC limited the options to:
  d_1 = small complex with 30 condominiums
  d_2 = medium complex with 60 condominiums
  d_3 = large complex with 90 condominiums
- Future chance events were limited to:
  s_1 = strong demand for the condominiums
  s_2 = weak demand for the condominiums

Representation of the Decision Problem

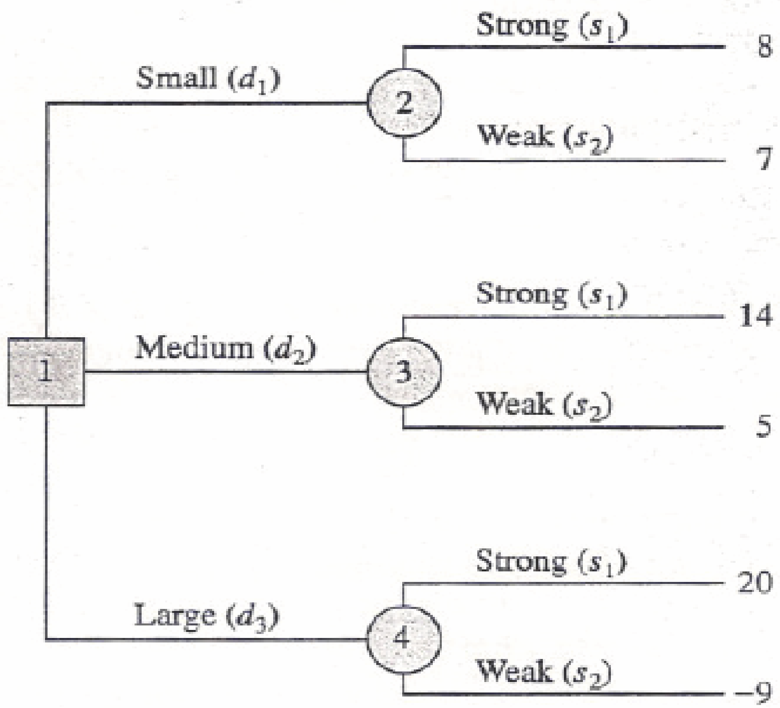
- Payoff Tables
- Decision Tree

Payoff Table

Decision Alternative	State of Nature: Strong Demand s_1	Weak Demand s_2
Small Complex, d_1	8	7
Medium Complex, d_2	14	5
Large Complex, d_3	20	-9

Decision Tree

FIGURE 4.2 DECISION TREE FOR THE PDC CONDOMINIUM PROJECT (PAYOFFS IN $ MILLION)



Decision Making without Probabilities

- Optimistic Approach
- Conservative Approach
- Minimax Regret Approach

Optimistic Approach

- Select the highest payoff (for maximization) or lowest payoff (for minimization)

Decision Alternative	Maximum Payoff
Small Complex, d_1	8
Medium Complex, d_2	14
Large Complex, d_3	20

Maximum of the maximum payoffs

Conservative Approach

Select the maximum of the minimum payoffs

Decision Alternative	Minimum Payoff
Small Complex, d_1	7
Medium Complex, d_2	5
Large Complex, d_3	-9

Maximum of the minimum payoffs

Minimax Regret Approach

- An approach between the optimistic and the conservative
- Regret is the opportunity loss between the payoffs of the best decision given a state of nature and the decision you made

Regret Table

Decision Alternative	State of Nature for Demand	
	Strong, s_1	Weak, s_2
Small Complex, d_1	12	0
Medium Complex, d_2	6	2
Large Complex, d_3	0	16

Minimax Regret Approach

Select the minimum of the maximum regrets

Decision Alternative	Maximum Regrets
Small Complex, d_1	12
Medium Complex, d_2	6
Large Complex, d_3	16

Minimum of the maximum regrets

Decision Making with Probabilities

The expected value for an alternative d_i is defined as:

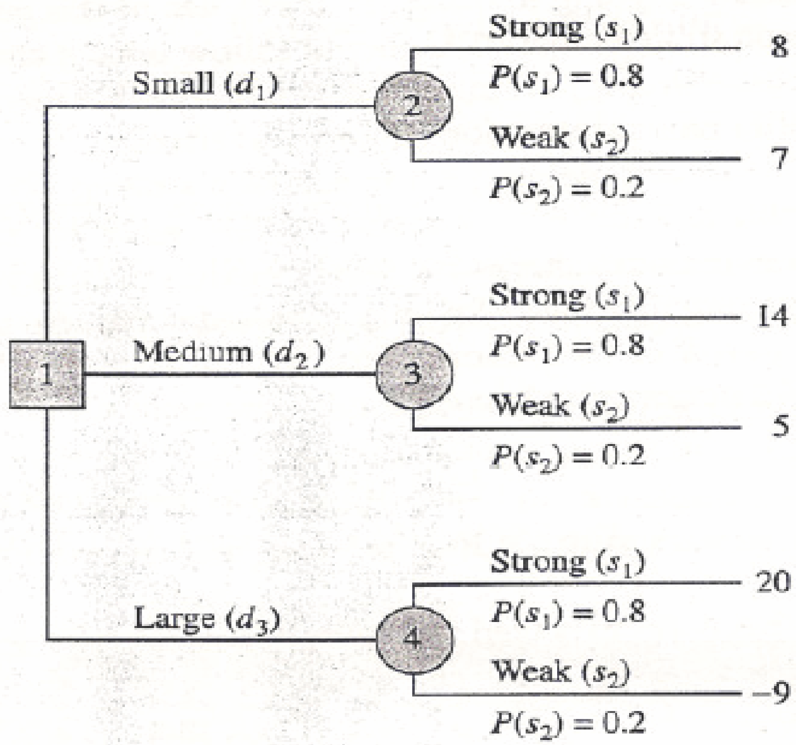
$$EV(d_i) = \sum_{j=1}^{N} P(s_j) V_{ij}$$

N = Number of states of nature

$P(s_j)$ = the probability of the state of nature s_j

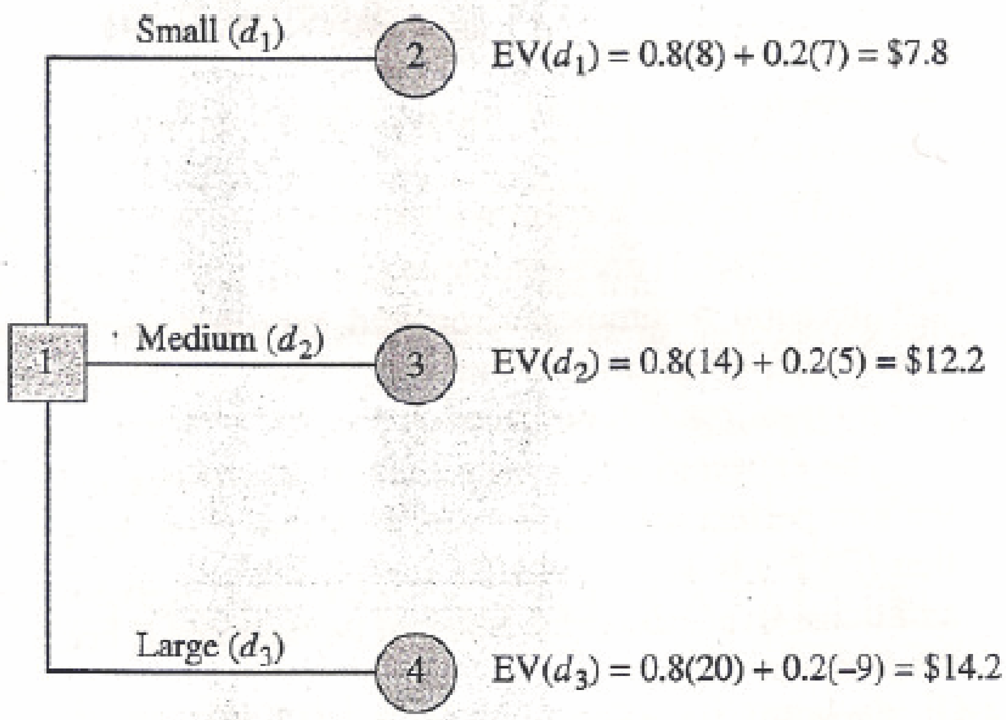
The Expected Value

FIGURE 4.3 PDC DECISION TREE WITH STATE-OF-NATURE BRANCH PROBABILITIES



The Expected Value

FIGURE 4.4 APPLYING THE EXPECTED VALUE APPROACH USING DECISION TREES



The Expected Value of Perfect Information

- If an expert could tell PDC the level of demand (either s_1 or s_2), what would that information be worth?
- Given the payoff table, PDC decision should be

Expert says Demand is	PDC Decision should be	Payoff
Strong, s_1	Large Complex, d_3	20
Weak, s_2	Small Complex, d_1	7

Payoff Table

Decision Alternative	State of Nature: Strong Demand s_1	Weak Demand s_2
Small Complex, d_1	8	7
Medium Complex, d_2	14	5
Large Complex, d_3	20	-9

The Expected Value of Perfect Information

- The EV of the above strategy is referred to as the EV with perfect information and can be determined as:

$$EVwPI = 0.8(20) + 0.2(7) = 17.4$$

The Expected Value of Perfect Information

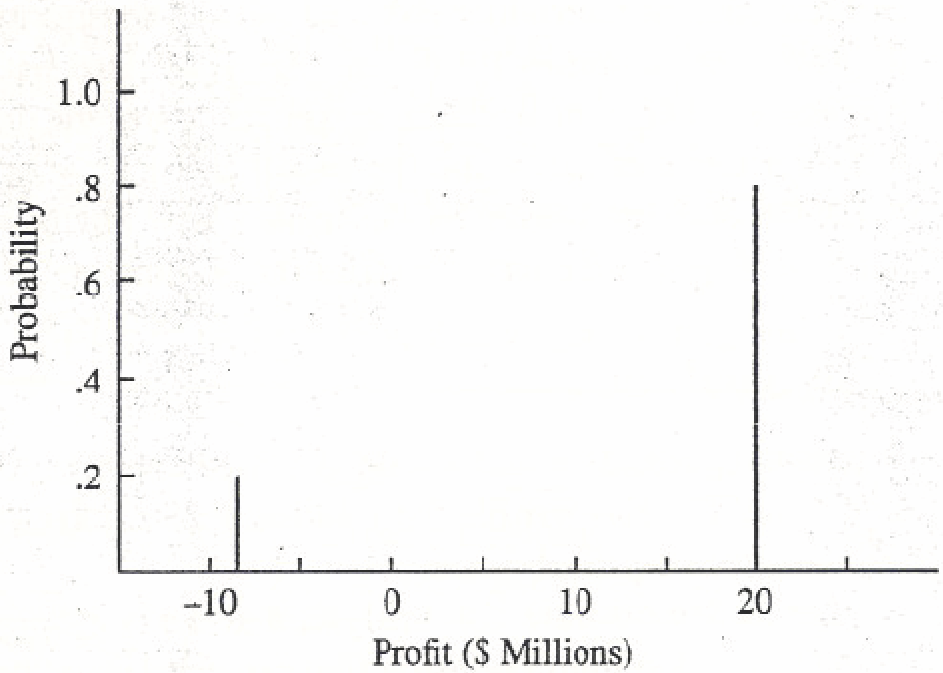
- Without PI, the EV was previously determined as 14.2 and can be referred to as EVwoPI
- Hence, Expected Value of Perfect Information:

$$EVPI = EVwoPI – EVwPI$$
$$= 17.4 – 14.2 = 3.2$$

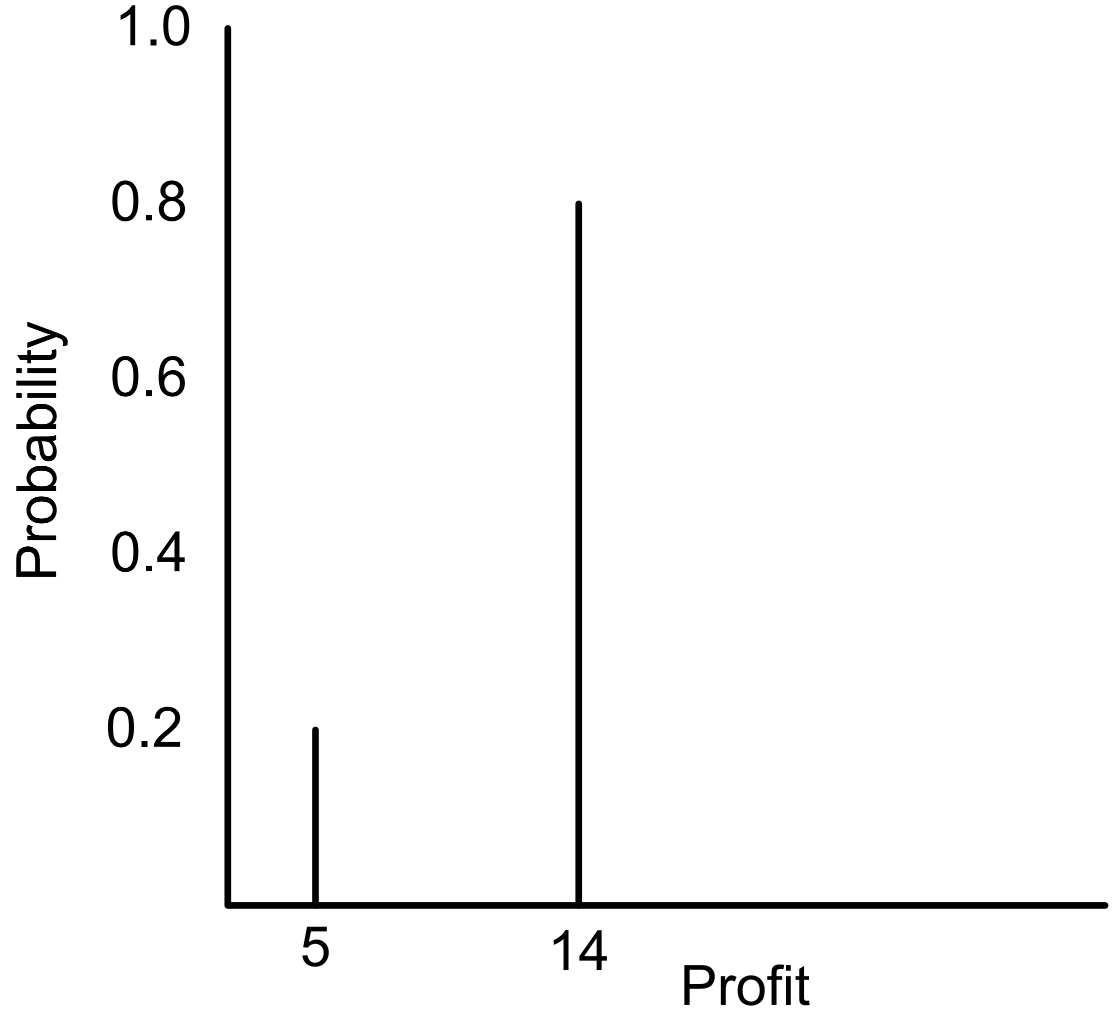
Risk Profile

- A risk profile is a graph showing the probability associated with each of the payoffs for a decision.
- A risk profile gives and indication of the degree of risk of a decision. It helps the decision maker to properly consider the risk and may lead to a decision other than that arrived at by the EV approach.
- Compare d_2 and d_3

FIGURE 4.5 RISK PROFILE FOR THE LARGE COMPLEX DECISION ALTERNATIVE FOR THE PDC CONDOMINIUM PROJECT



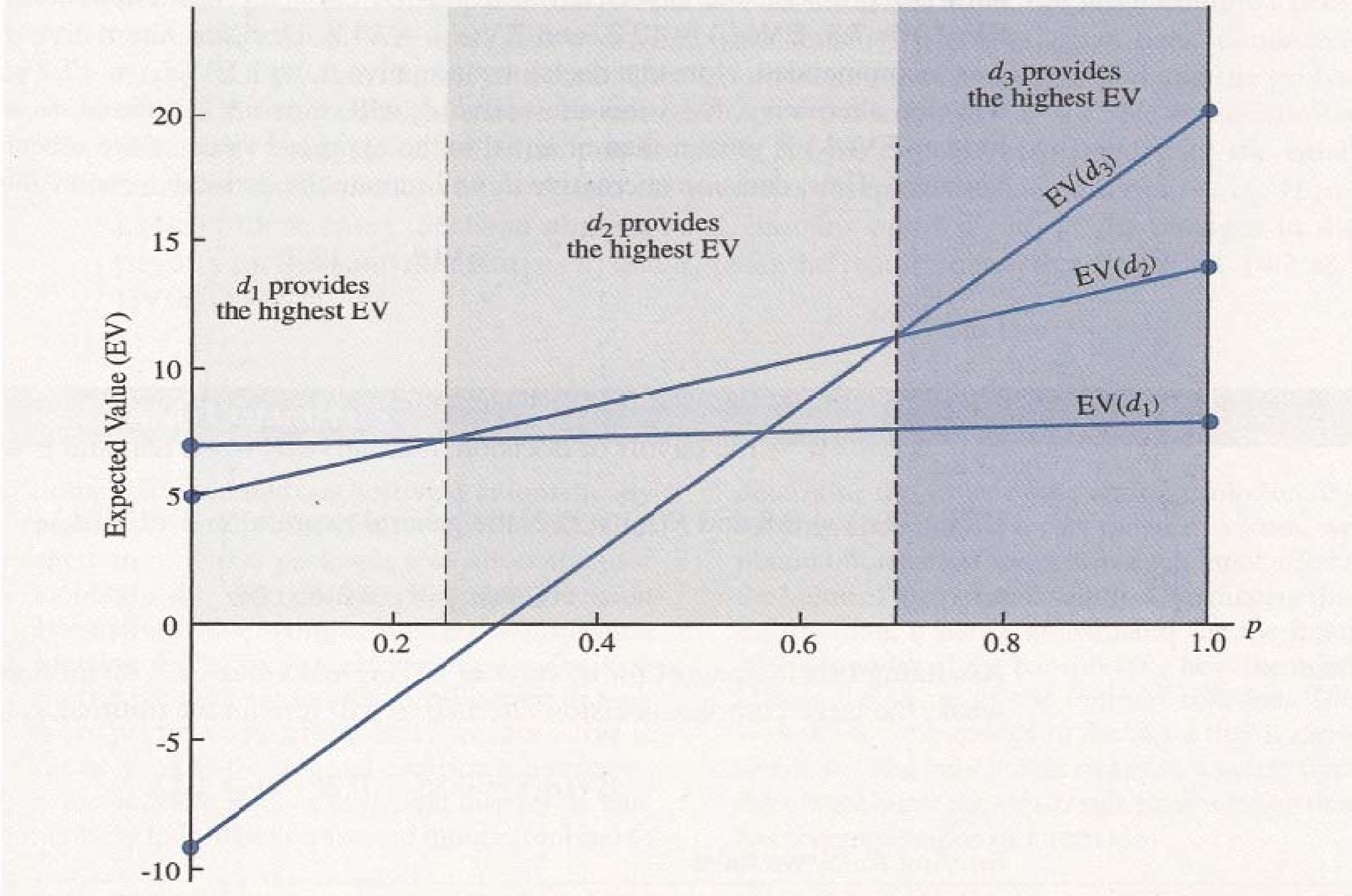
Risk Profile for the Medium Complex Decision Alternative

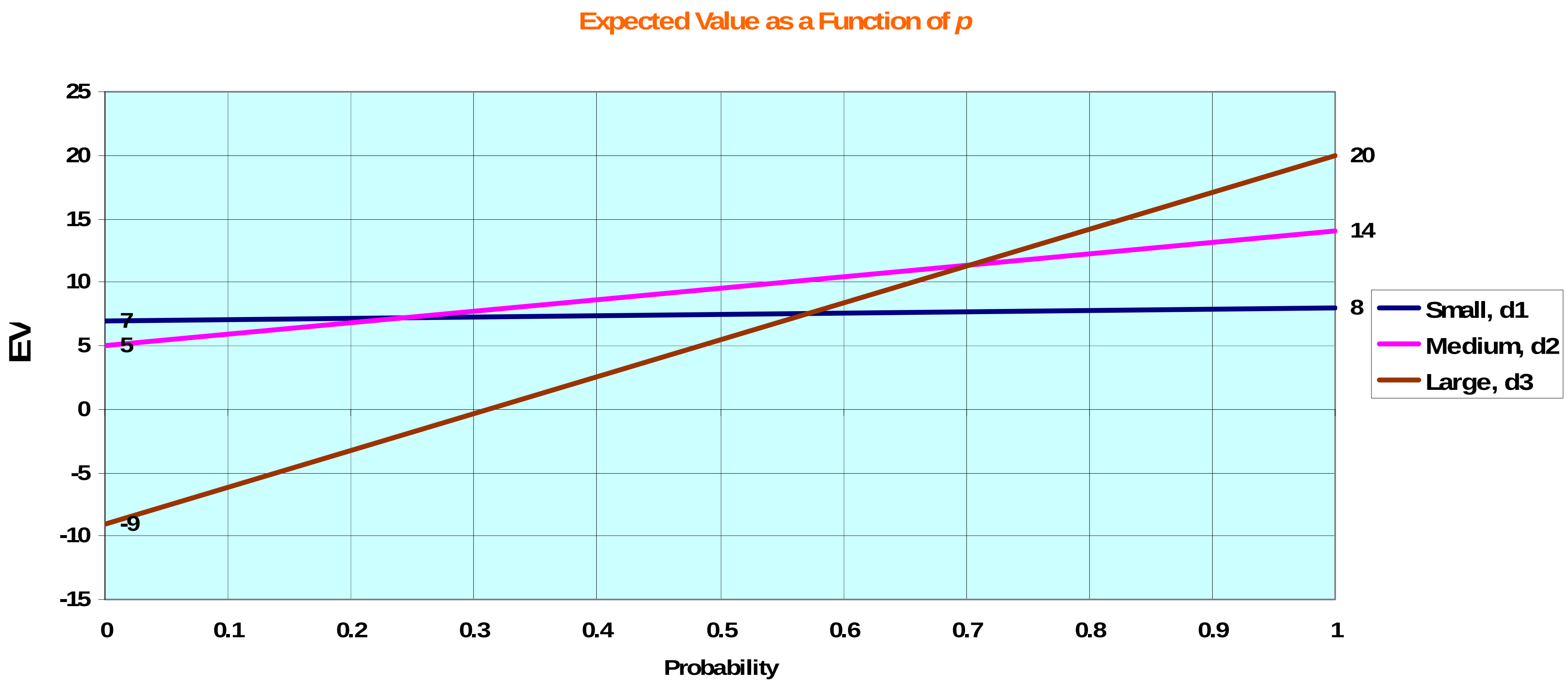


Sensitivity Analysis

- A study of how changes in value of one item, while maintaining the others constant, can effect the solution (decision selected)
- If the decision is changed with minor changes in the value of an item ➔ Decision is sensitive to the item ➔ Decision maker should try to obtain best estimate of value
- If the decision is not changed even with moderate changes in the value of an item ➔ Decision is not sensitive to the item ➔ Decision maker should not worry about obtaining better estimate of item.

FIGURE 4.6 EXPECTED VALUE FOR THE PDC DECISION ALTERNATIVES AS A FUNCTION OF p




Expected Value as a Function of p
EV
25
20
15
10
5
0
-5
-10
-15
7
5
-9
20
14
8
Small, d1
Medium, d2
Large, d3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Probability

Decision Analysis with Sample Information

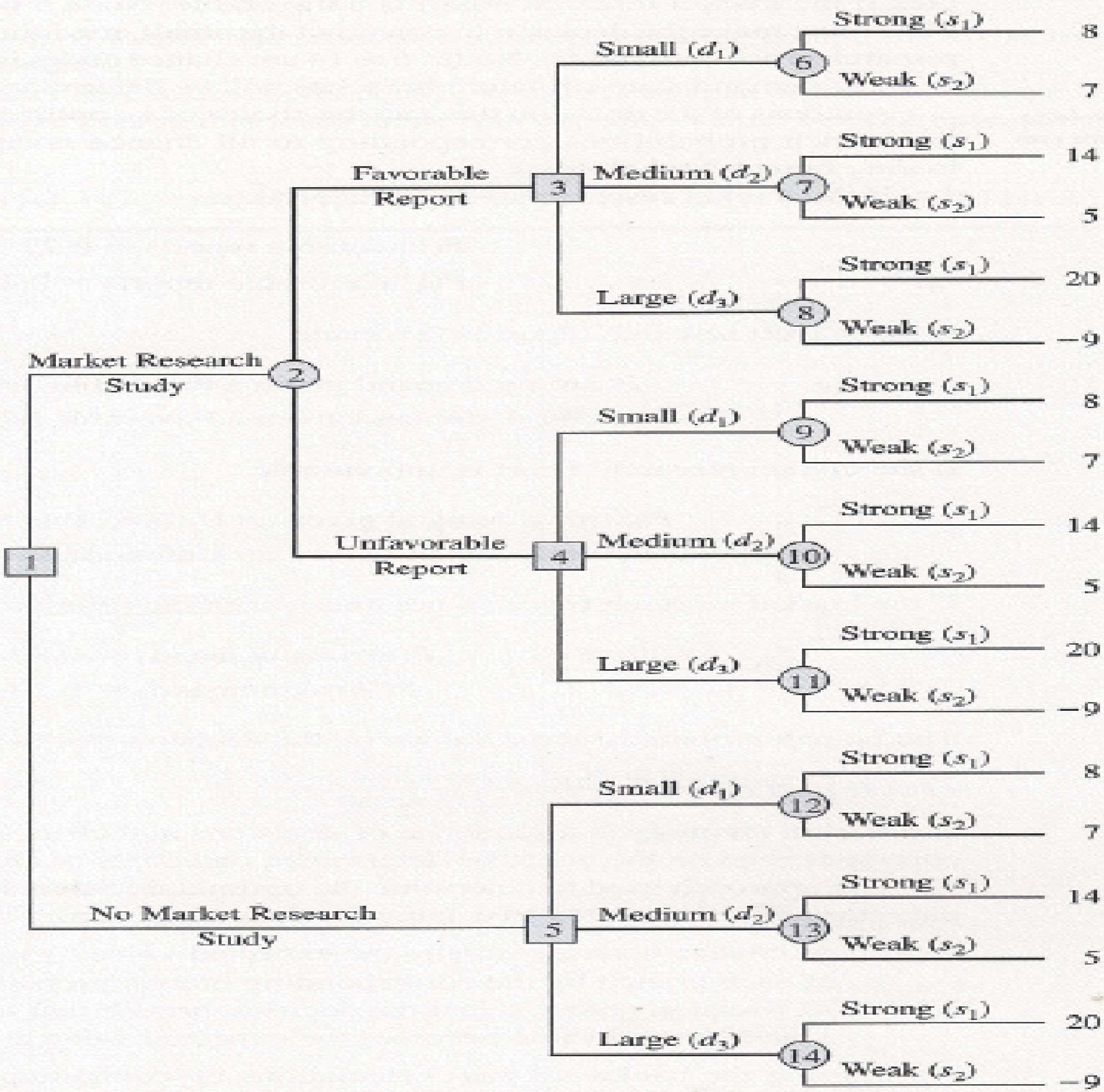
- Decision maker starts with initial probability assessments (*prior probabilities*)
- Often the decision maker will decide to do additional studies in order to arrive at better estimates of the probabilities (*posterior probabilities*)
- The new information is obtained through sampling from the population of concern and thus referred to as *sample information*.

PDC Decision Considering Sample Information

- PDC wishes to consider a market research study to improve its knowledge of the demand.
- The results of the study will be one of two:
  - Favorable: indicating interest of a large number of people in purchasing condominiums
  - Unfavorable: indicating interest of only few people in purchasing condominiums

Decision Trees

FIGURE 4.8 THE PDC DECISION TREE INCLUDING THE MARKET RESEARCH STUDY



Probabilities of Events Associated with Study

Event	Probabilities
Favorable	0.77
Unfavorable	0.23
Strong Demand / Favorable	0.94
Weak Demand / Favorable	0.06
Strong Demand / Unfavorable	0.35
Weak Demand / Unfavorable	0.65

FIGURE 4.9 THE PDC DECISION TREE WITH BRANCH PROBABILITIES

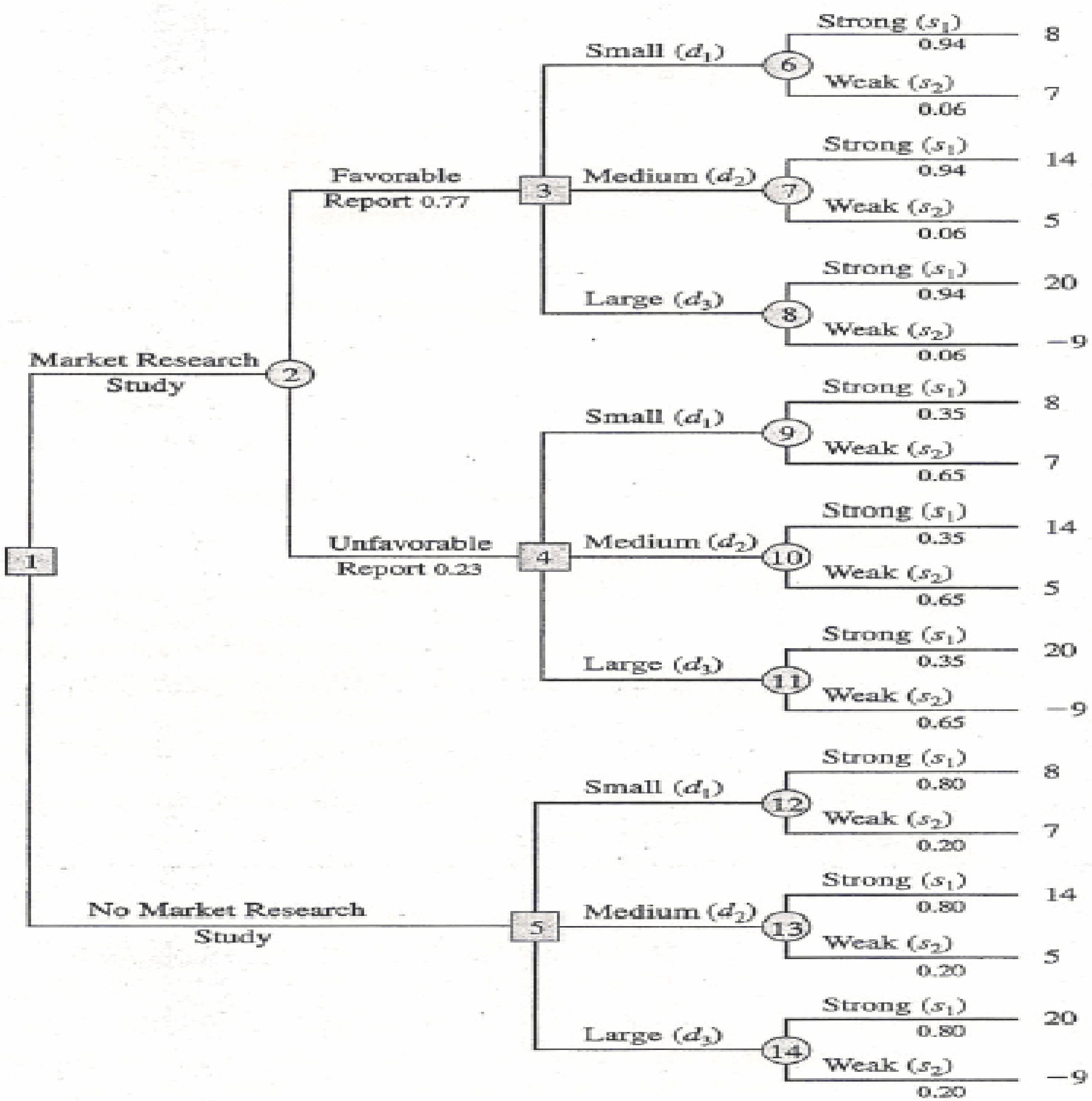


FIGURE 4.10 PDC DECISION TREE AFTER COMPUTING EXPECTED VALUES AT CHANCE NODES 6 TO 14

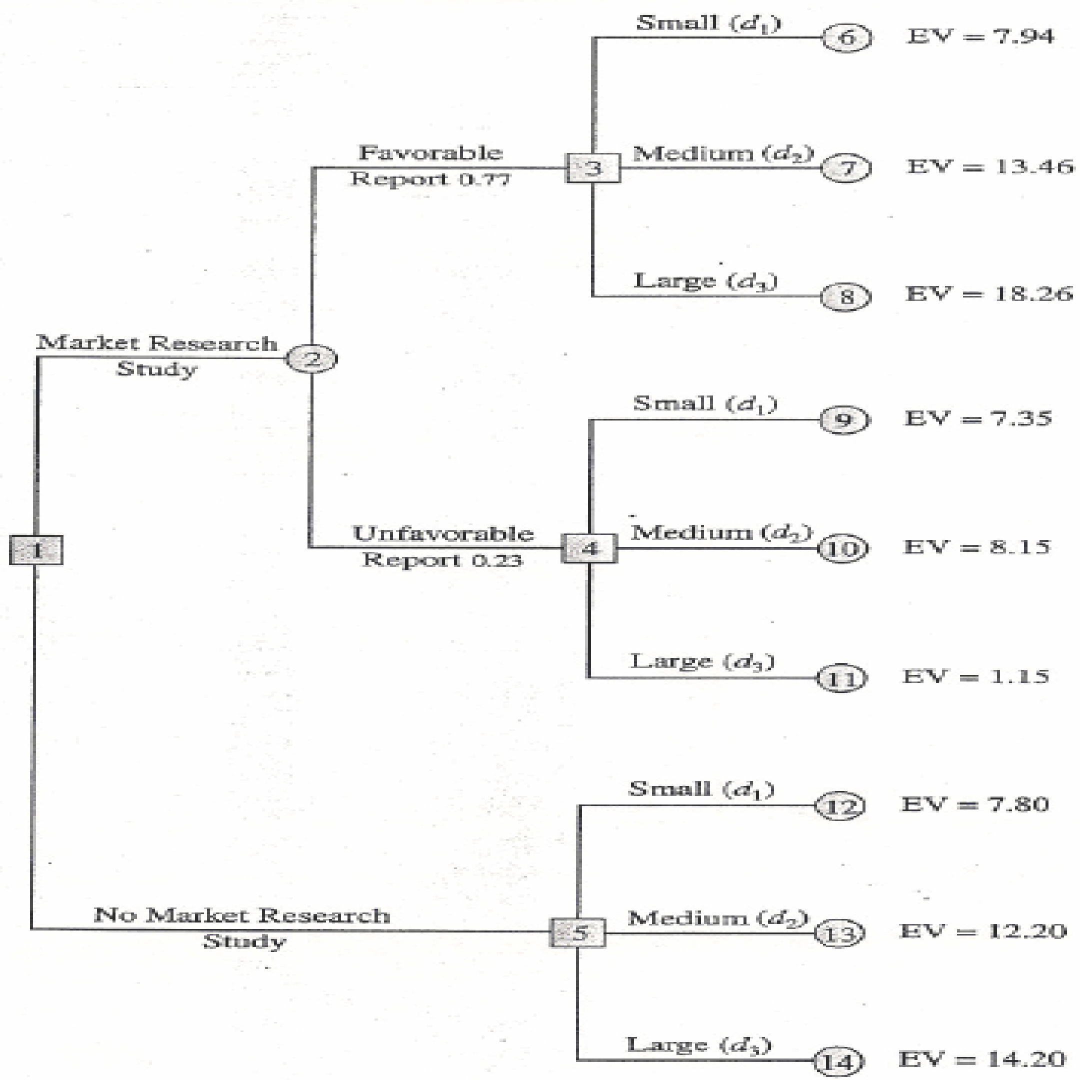


FIGURE 4.11 PDC DECISION TREE AFTER CHOOSING BEST DECISIONS AT NODES 3, 4, AND 5

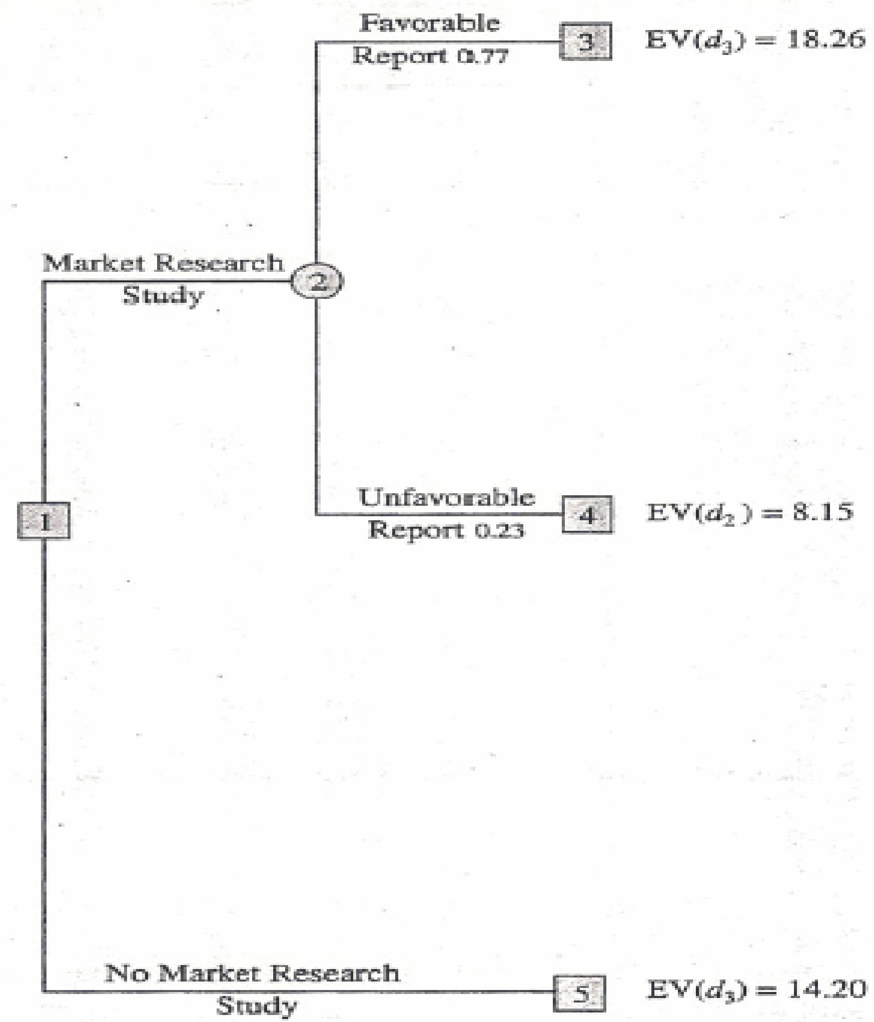


FIGURE 4.12 PDC DECISION TREE REDUCED TO 2 DECISION BRANCHES

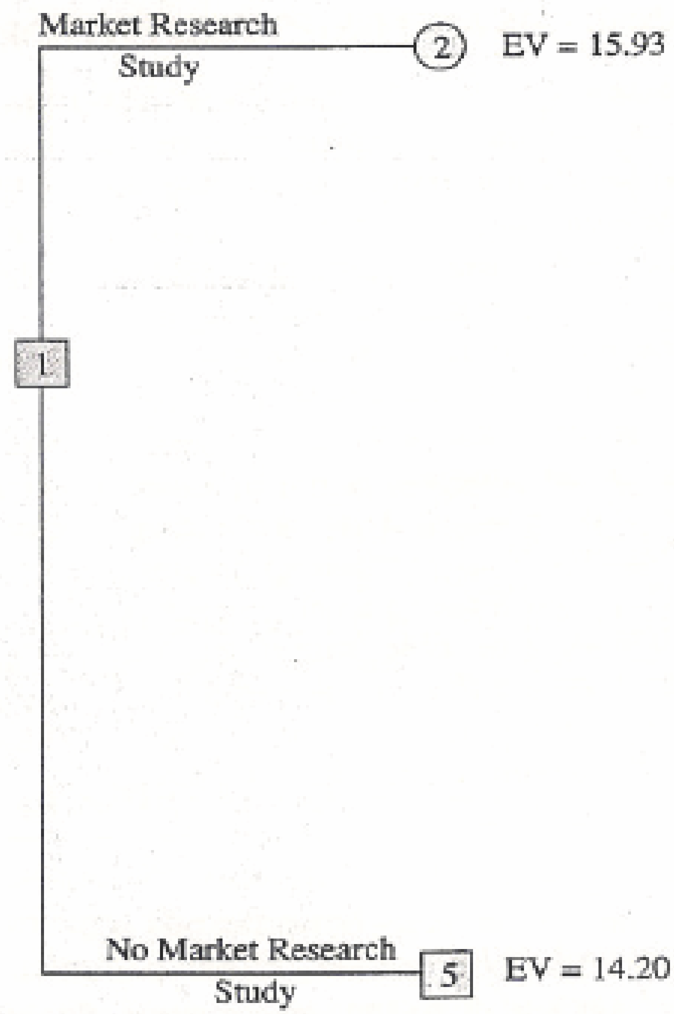
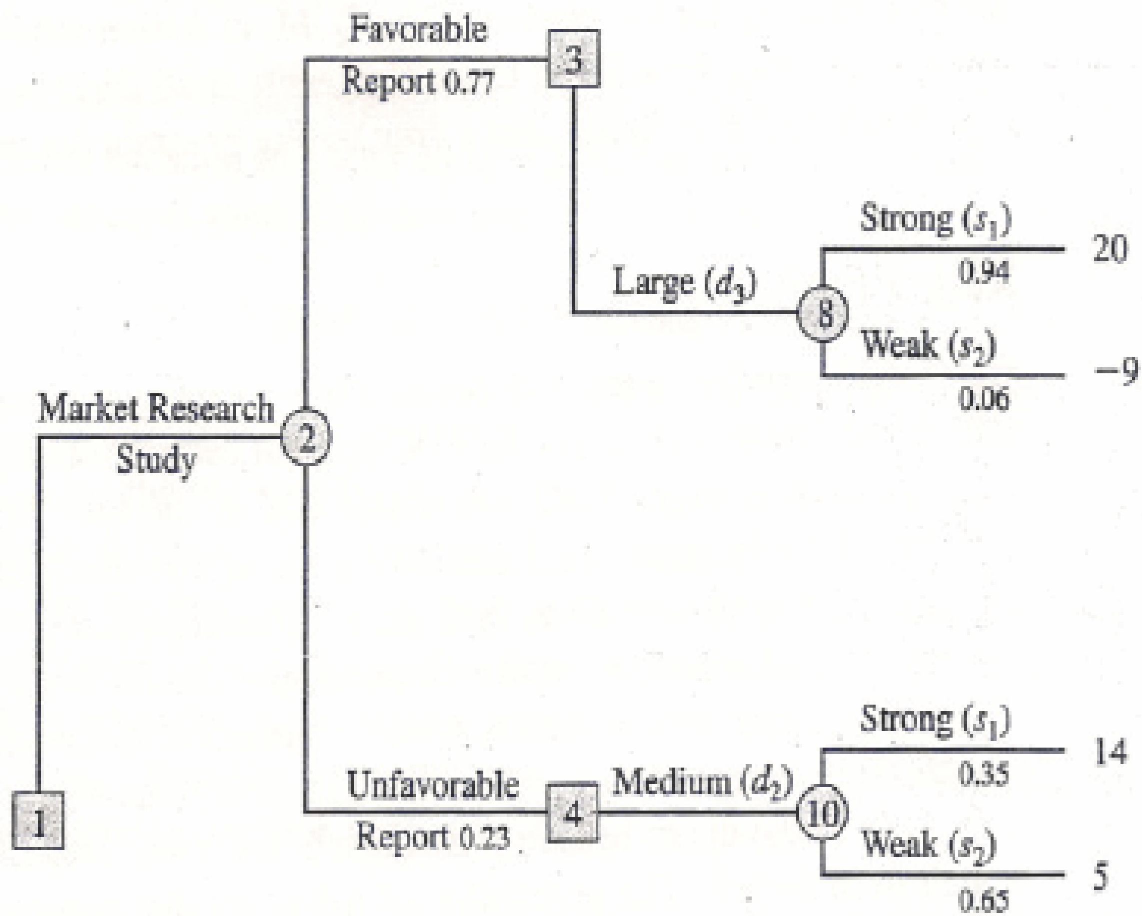
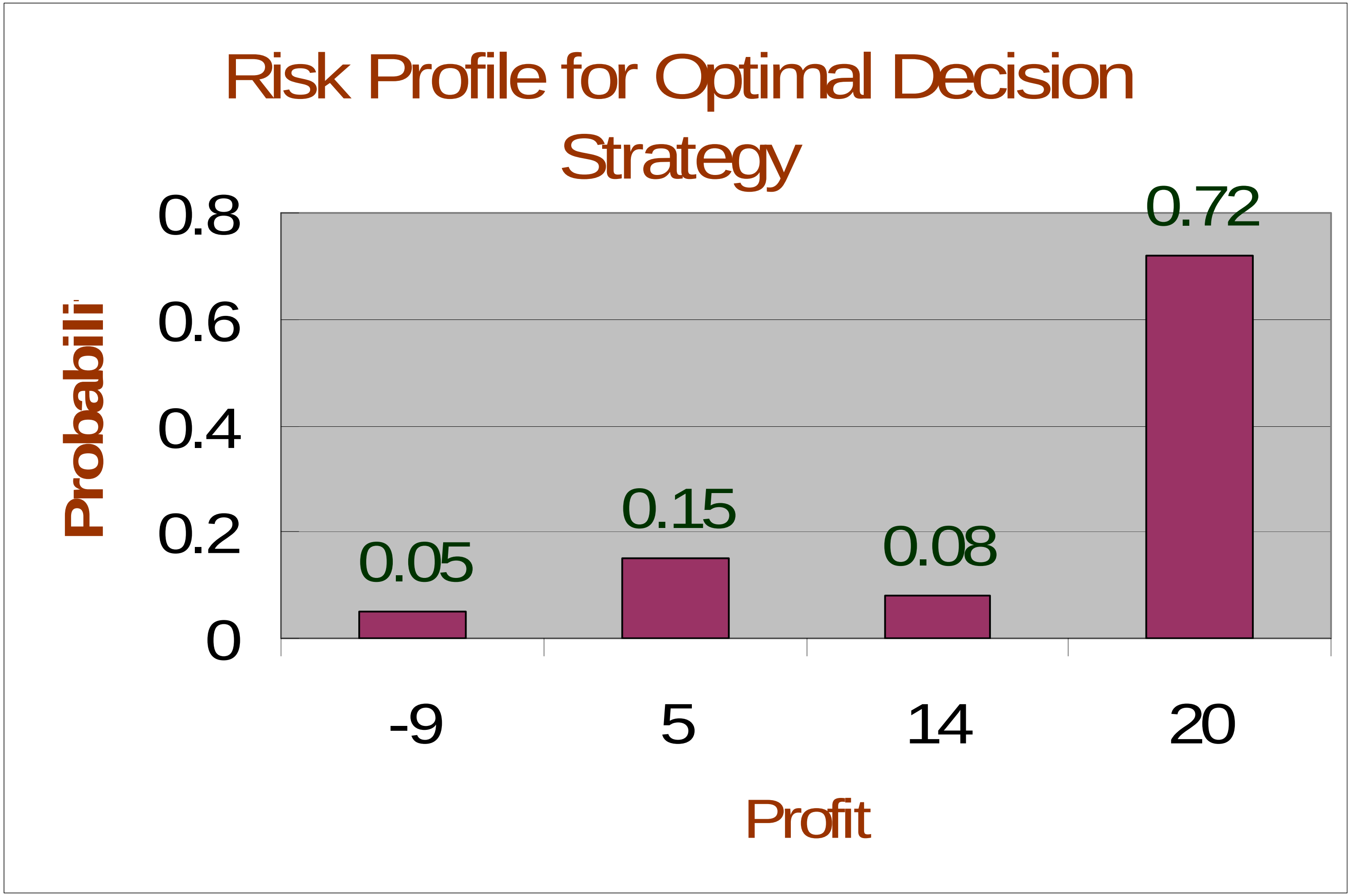


FIGURE 4.13 PDC DECISION TREE SHOWING ONLY BRANCHES ASSOCIATED WITH OPTIMAL DECISION STRATEGY




Risk Profile for Optimal Decision Strategy
Probabili
0.8
0.6
0.4
0.2
0
0.05
0.15
0.08
0.72
-9
5
14
20
Profit

Expected Value of & Efficiency of Sample Information

- EVSI = EVwSI – EVwoSI

  EVwSI = 15.93

  EVwoSI = 14.2

  EVSI = 1.73
- Efficiency of Sample Information

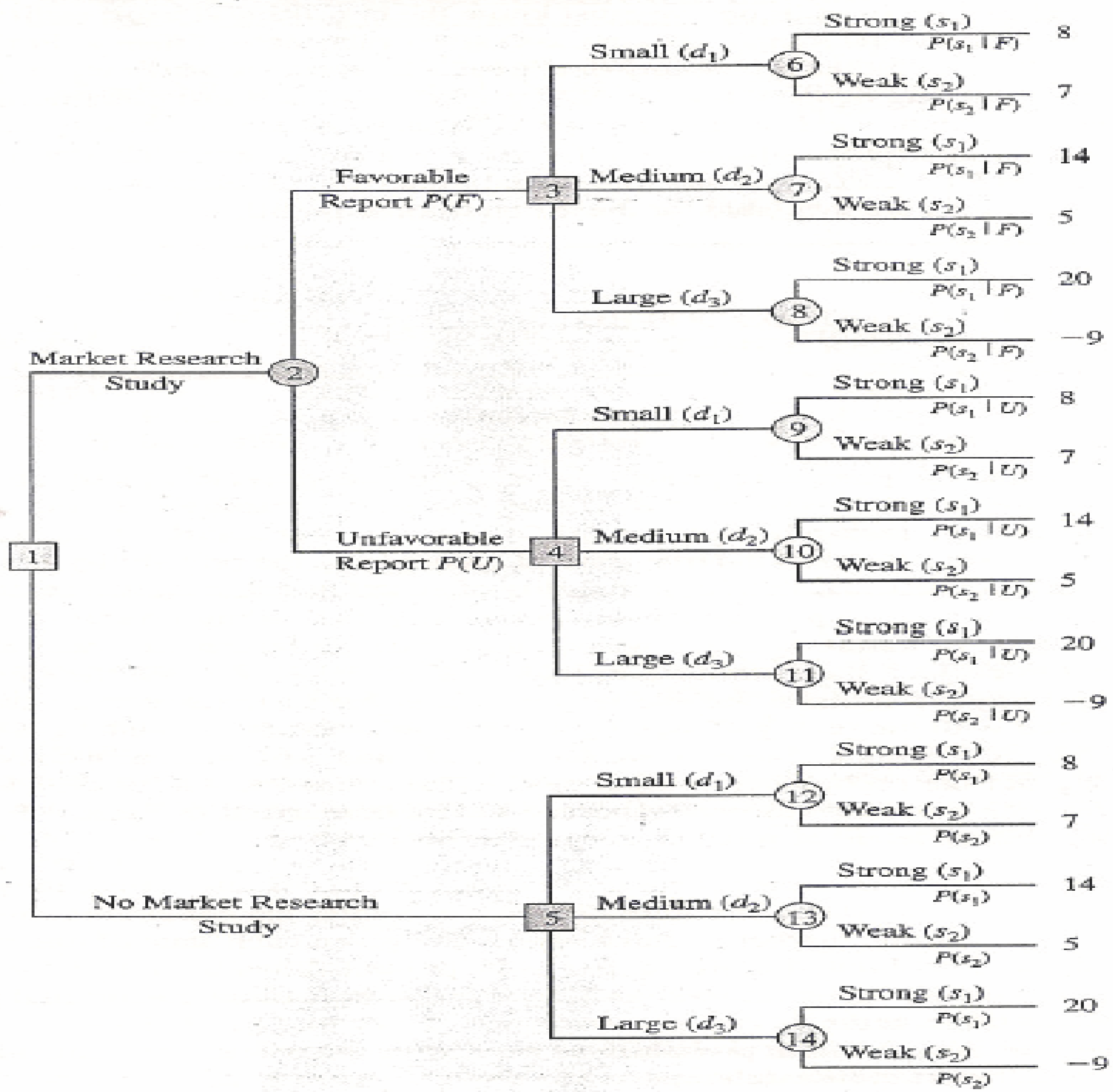
  E(%) = EVSI / EVPI) x 100

  E(%) = 1.73 / 3.2) x 100 = 54.1

Computing Brach Probabilities

- The Probabilities [$P(s_1/F)$, $P(s_1/U)$, $P(s_2/F)$] etc. that were used to solve the tree, are referred to as posterior probabilities.
- In order to determine these probabilities we need to know the conditional probabilities of favorable or unfavorable report given the state of nature of a strong or weak demand; that is [$P(F/s_1)$, $P(U/s_1$, $P(F/s_2)$, $P(U/s_2)$]. This can found from previous results of such studies

FIGURE 4.15 THE PDC DECISION TREE



Probabilities of Study Results

	Results of Market Research	
State of Nature	Favorable, F	Unfavorable, U
Strong Demand, s_1	$P(F/s_1)=0.90$	$P(U/s_1)= 0.10$
Weak Demand, s_2	$P(F/s_2)=0.25$	$P(U/s_2)=0.75$

Branch Probabilities for Favorable Market

State of Nature s_j	Prior Probability $P(s_j)$	Prior Probability $P(F\|s_j)$	Joint Probability $P(F \cap s_j)$	Posterior Probability $P(s_j\|F)$
s_1	0.80	0.90	0.72	0.94
s_2	0.20	0.25	0.05	0.06
		$P(F)$ =	0.77	1.00

Branch Probabilities for Unfavorable Market

State of Nature s_j	Prior Probability $P(s_j)$	Prior Probability $P(U \mid s_j)$	Joint Probability $P(U \cap s_j)$	Posterior Probability $P(s_j \mid U)$
s_1	0.80	0.10	0.08	0.35
s_2	0.20	0.75	0.15	0.65
		$P(U)$ =	0.23	1.00