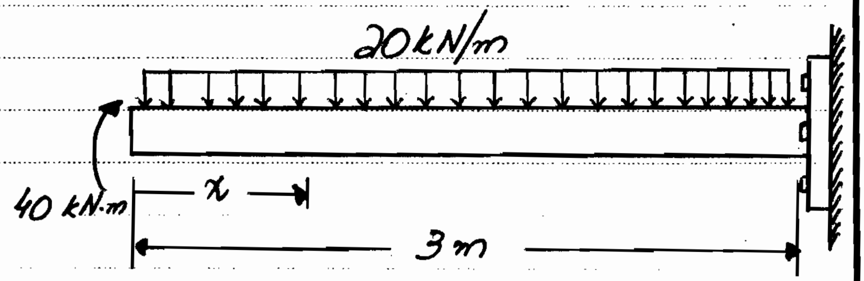
Problem#02

Given:

The beam Shown



Required:

Equations for V and M &

Shear force diagram (SFD) and Bending moment diagram (BMD)

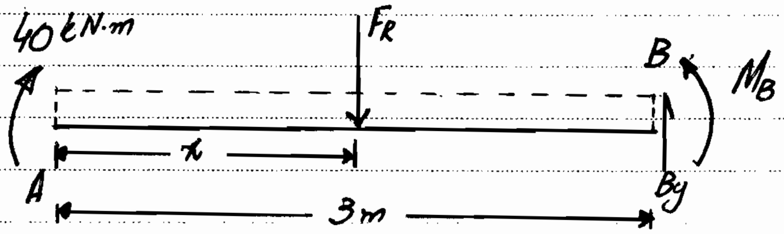
Solution:

Even though the reactions are not needed in this particular problem, we will calculate them for checking. (How?!)

F_R = Area of load

$= 20(3) = 60\ kN$

$@\ x = 1.5m$



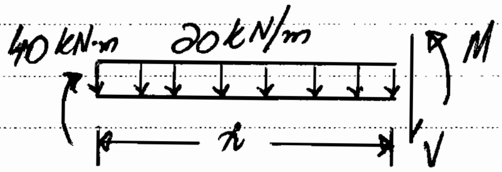
$\uparrow \Sigma F_y = 0 \Rightarrow -60 + B_y = 0 \Rightarrow B_y = 60\ kN \uparrow$

$+\circlearrowleft \Sigma M_B = 0 \Rightarrow -40 + 60(1.5) + M_B = 0$

$\Rightarrow M_B = -50 = 50\ kN\cdot m \circlearrowright$

The beam can be taken as one segment, thus only one section is needed. (Why?!)

In the FBD Shown,



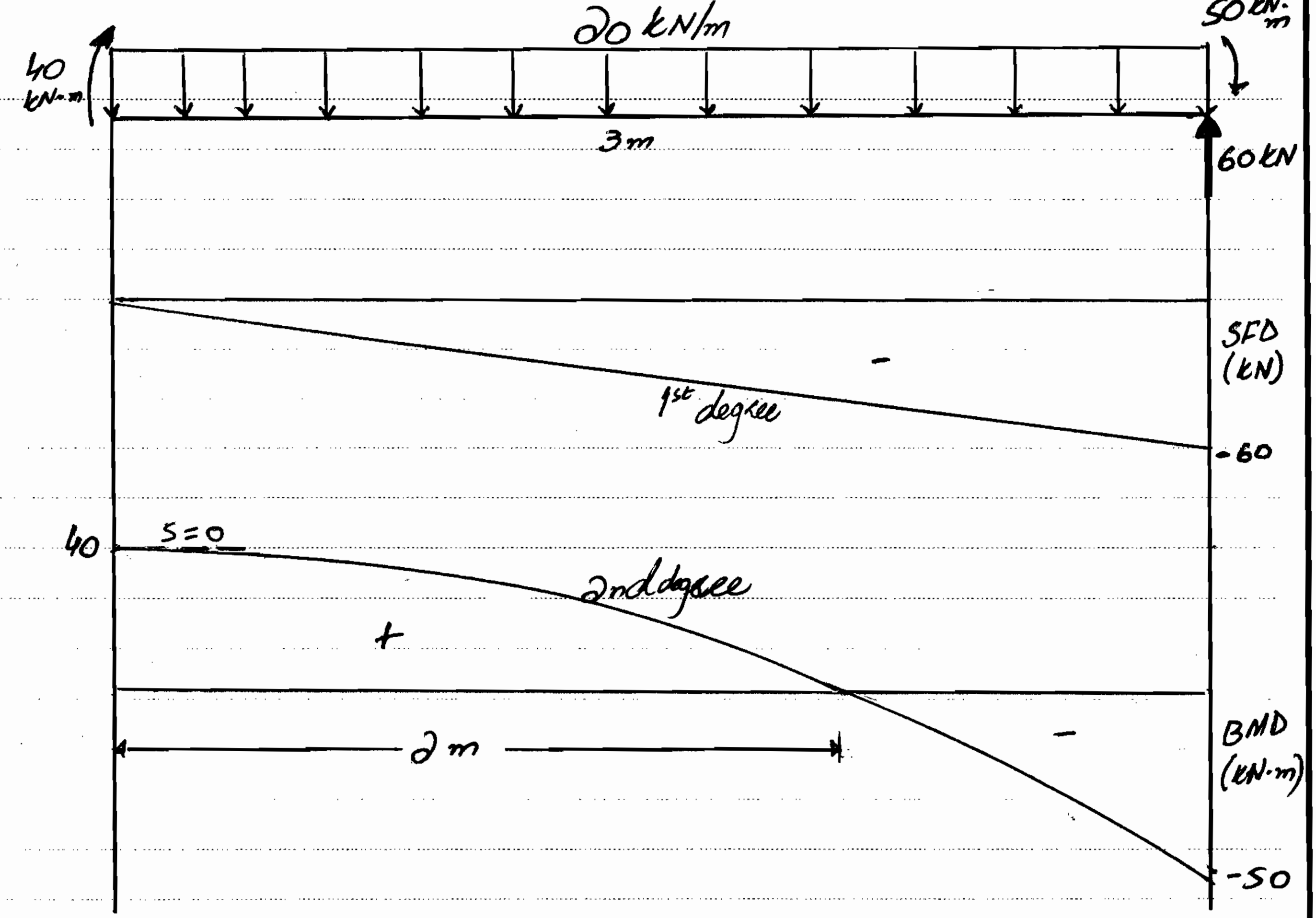
$+\uparrow \sum F_y = 0 \Rightarrow$

$-V - 20(x) = 0 \Rightarrow$ $\boxed{V = -20x \text{ kN}}$

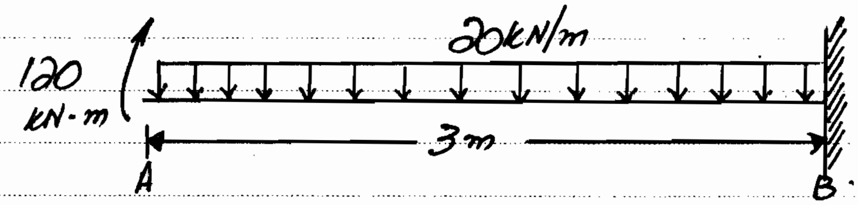
$+\circlearrowleft \sum M = 0 \Rightarrow$

$M - 40 + 20(x)\left(\frac{x}{2}\right) = 0 \Rightarrow$ $\boxed{M = 40 - 10x^2 \text{ kN·m}}$

The SFD and BMD are shown below, plotted from the equations above.



Problem#02:



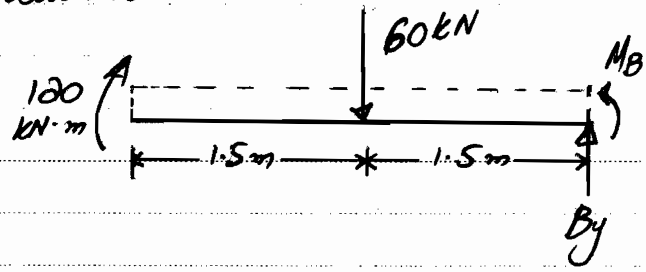
Given:
The beam Shown

Required:

SFD & BMD by graphical Method.

Solution:

As in example 1, the reactions are calculated.



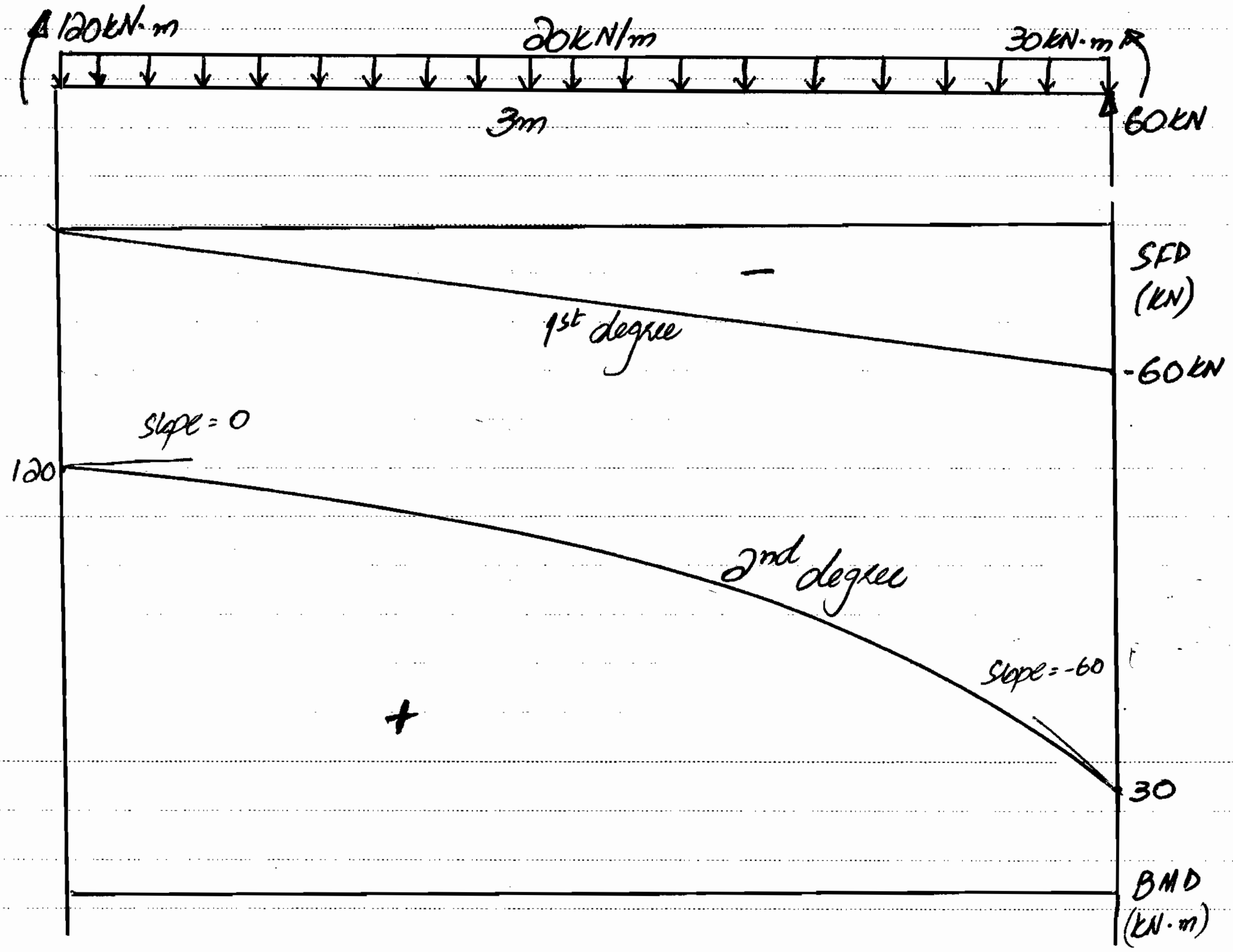
$\uparrow \Sigma F_y = 0$

$B_y = 60 \text{ kN} \uparrow$

$+\circlearrowleft \Sigma M_B = 0 \Rightarrow -120 + 160(1.5) + M_B = 0 \Rightarrow M_B = 30 \text{ kN}\cdot\text{m} \circlearrowleft$

Using the Semi-graphical (summation/Integration/area) method, the SFD and BMD are plotted below.

Solution of HW #9



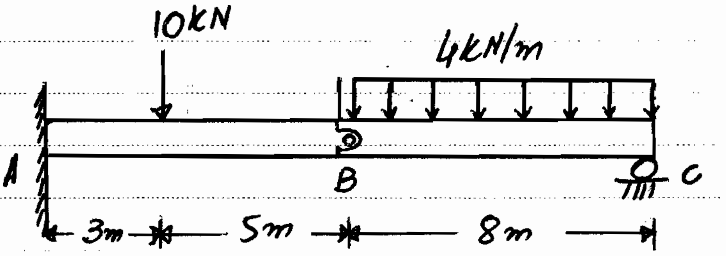
Between A & B

Area of Load = $\Delta V = -20(3) = -60\,kN$

Area of $V = \Delta M = \frac{1}{2}(3)(-60) = -90\,kN\cdot m$

Problem #03:

Given:
The beam Shown



Required:
SFD & BMD by the Semi-graphical Method.

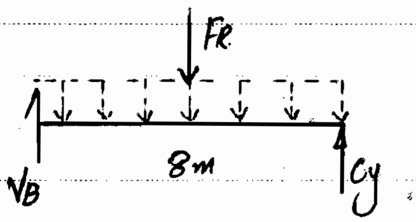
Solution:

First, we need to calculate the reactions. (why?!)

To be able to find the reactions, we need to seperate the beam from the pin at B. (why?!).

without the "internal" pin at B, the problem is statically indeterminate. (How?!)

We take the right part from B (why?!)



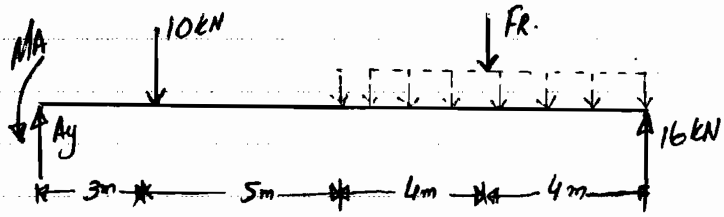
$F_R = 4(8) = 32\,kN$
No M @ B. (Why?!)

$+\circlearrowleft \sum M_B = 0$ (why B?!)

$8C_y - 32(4) = 0 \Rightarrow$

$C_y = 16\,kN \uparrow$

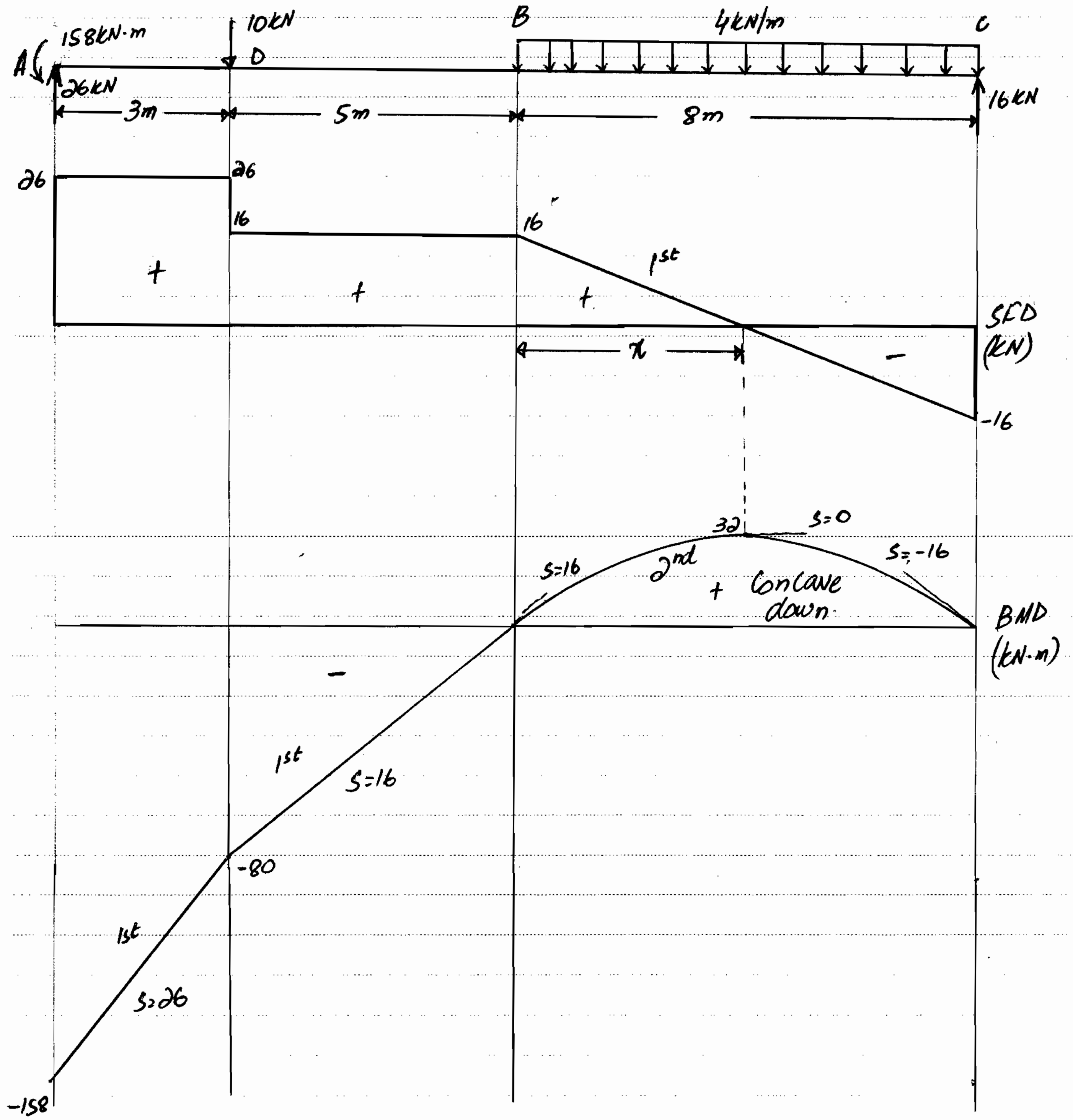
Now, the entire Beam



$+\uparrow \sum F_y = 0 \Rightarrow A_y - 10 - 32 + 16 = 0 \Rightarrow A_y = 26\,kN \uparrow$

$+\circlearrowleft \sum M_A = 0 \Rightarrow M_A - 10(3) - 32(12) + 16(16) = 0 \Rightarrow M_A = 158\,kN \cdot m \circlearrowleft$

Now, the SFD & BMD are drawn.

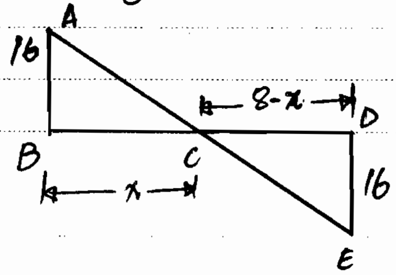


Explanation:

S = Slope ; 1st, 2nd, etc. = degree of curve

Even though, it is clear that $x = 4m$, we will use the general method to find it.

Method①: We take ΔABC & ΔCDE "Similar triangles"

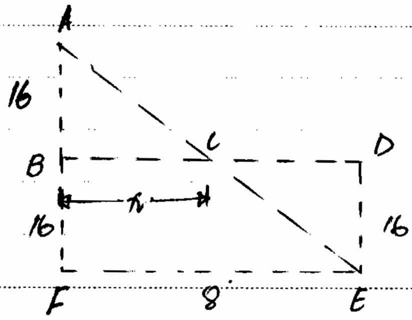


$$\frac{x}{16} = \frac{8-x}{16}$$

$x = 8-x \Rightarrow 2x = 8 \Rightarrow x = 4m$

Method②: More general and easier

We take the Similar triangles ABC and AFE



$$\Rightarrow \frac{x}{16} = \frac{8}{16+16} \Rightarrow x = 4m$$

Note that x appears only in one side of the equation.

Areas:-

$\Delta V_{A\to D} = A_{load\ A\to D} = 0$

$\Delta V_{D\to B} = A_{load\ D\to B} = 0$

$\Delta V_{B\to C} = A_{load\ B\to C} = -4(8) = -32 \Rightarrow$

$V_C = V_B + \Delta V_{B\to C} = 16 - 32 = -16\,kN$

$\Delta M_{A\to D} = A_{V_{A\to D}} = 26(3) = 78 \Rightarrow$
$M_D = M_A + \Delta M_{A\to D} = -158 + 78 = -80\,kN\cdot m$

$\Delta M_{D\to B} = A_{V_{D\to B}} = 16(5) = 80 \Rightarrow$
$M_B = M_D + \Delta M_{D\to B} = -80 + 80 = 0$

As expected! (why?!)

$$\Delta M_{B\to E} = A_{V_{B\to E}} = \frac{1}{2}(16)(4) = 32$$

$$M_E = M_B + \Delta M_{B\to E} = 0 + 32 = 32 \text{ kN}\cdot\text{m}$$

$$\Delta M_{E\to C} = A_{V_{E\to C}} = \frac{-1}{2}(16)(4) = -32$$

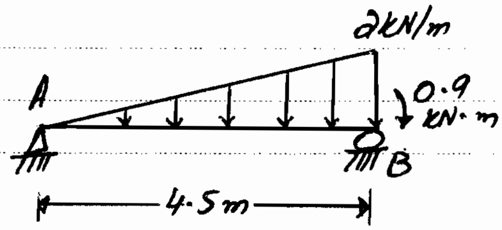
$$M_C = M_E + \Delta M_{E\to C} = 32 - 32 = 0$$

(“As Expected! Why?!)

Problem# 04:

Given:

The beam Shown

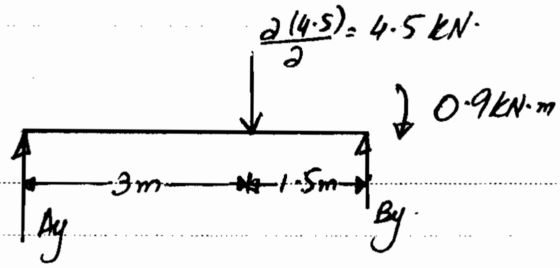


Required:

SFD and BMD by graphical Method

Solution:

First, find the reactions.

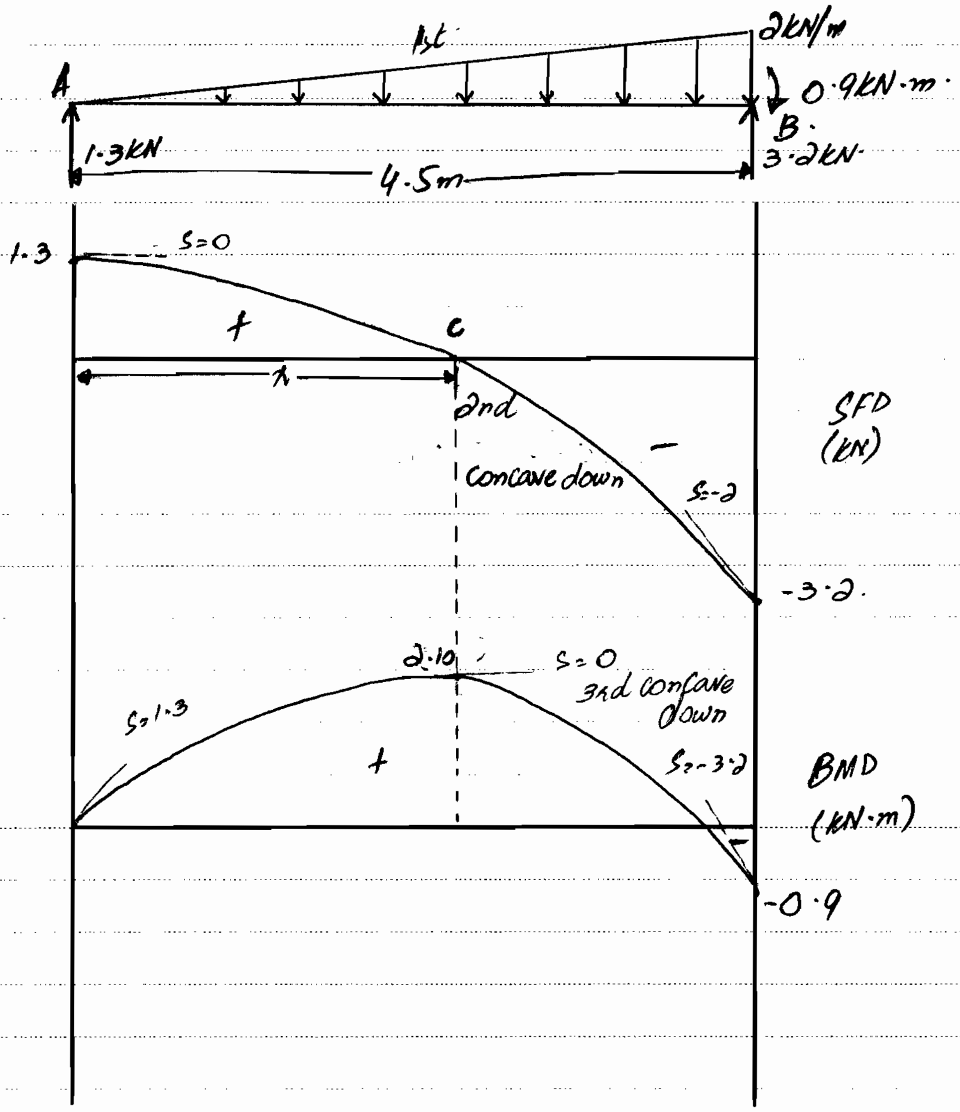


$+\circlearrowleft \Sigma M_B = 0 \Rightarrow$

$0 = 4.5(1.5) - 0.9 - 4.5 A_y \Rightarrow A_y = 1.3 \text{ kN}\uparrow$

$+\uparrow \Sigma F_y = 0 \Rightarrow 1.3 - 4.5 + B_y = 0 \Rightarrow B_y = 3.2 \text{ kN}(\uparrow)$

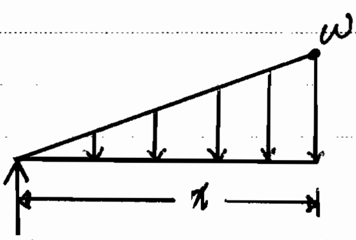
SFD and BMD are drawn Below.



The point C is @ V=0. To get it, we need to determine the distance x at which

$A_{load} = -1.3$. (why?!)

$\frac{w}{2} = \frac{x}{4.5} \Rightarrow$



$w = x/2.25$

$A = \frac{wx}{2} = \frac{x^2}{2(2.25)} = \frac{x^2}{4.5} \equiv 1.3 \Rightarrow$

$x^2 = 5.85 \Rightarrow x = 2.4187m$

Areas:

$\Delta V_{A\to B} = A_{Load\, A\to B} = -2(4.5)/2 = -4.5$

$V_B = V_A + \Delta V_{A\to B} = 1.3 - 4.5 = -3.2$ kN

$\Delta M_{A\to C} = A_{V_{A\to C}} = \frac{2}{3}(1.3)(2.4187)$

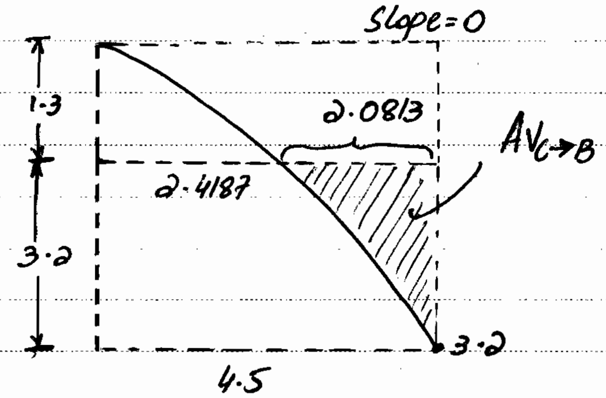
$= 2.0962 \approx 2.10$ kN·m

$M_C = M_A + \Delta M_{A\to C} = 0 + 2.10$ kN·m

$\Delta M_{C\to B} = A_{V_{C\to B}} = \frac{2}{3}(4.5)(1.3+3.2) - 4.5(3.2) - 2.10$

$= -3$

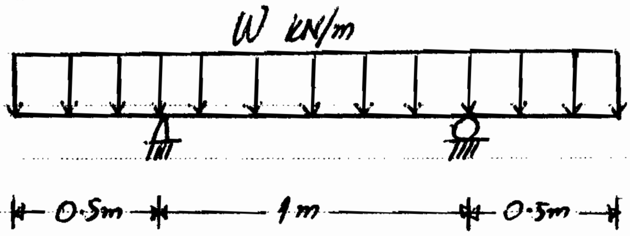
Or $-\left[\frac{1}{3}(4.5)(1.3+3.2) - \frac{1}{3}(2.4187)(1.3) - 2.0813(1.3)\right] = -3$



$M_B = M_C + \Delta M_{C\to B}$

$= 2.10 - 3 = -0.9$ kN·m "as expected"

Problem #05:



Given:

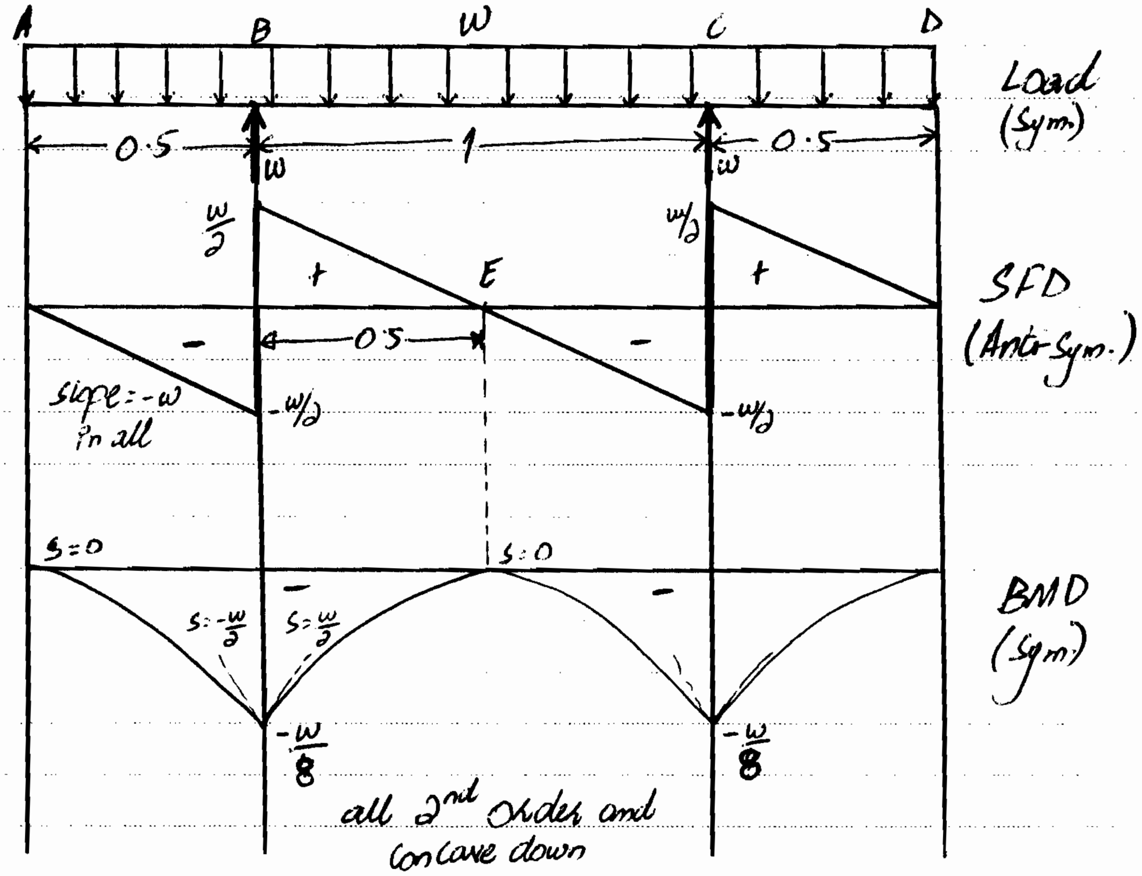
The beam Shown
$V_{\substack{max \\ min}} = \pm 5\,kN$; $M_{\substack{max \\ min}} = \pm 2\,kN\cdot m$

Required:

a) SFD and BMD by the graphical method
b) W_{max}

Solution:

a) Due to Symmetry, $B_y = C_y = \frac{\Sigma W}{2} = \frac{2W}{2} = W\,kN$



$\Delta V = A_{load} = \pm \frac{w}{2}$ (in AB, BE, EC, CD)

$\Delta M = A_V = \pm \frac{w}{2}\left(\frac{0.5}{2}\right) = \pm \frac{w}{8}$ as shown

b)

From SFD, $|V_{max}| = \frac{w}{2} \Rightarrow$

set $\frac{w}{2} \equiv 5 \Rightarrow w_{max}^{①} = 10$ kN/m

From BMD, $|M_{max}| = \frac{w}{8} \Rightarrow$

set $\frac{w}{8} \equiv 2 \Rightarrow w_{max}^{②} = 16$ kN/m

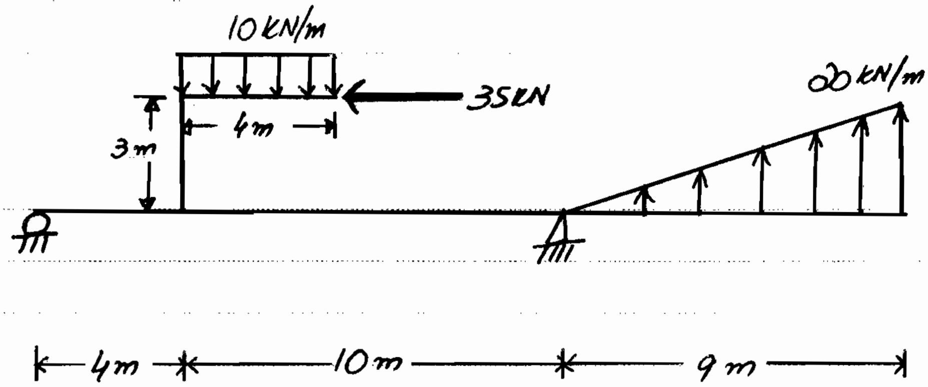
Thus w due to V_{max} controls. (why?!)

$\Rightarrow \boxed{w_{max} = 10 \text{ kN/m}}$

Problem #06:

Given:

The beam shown



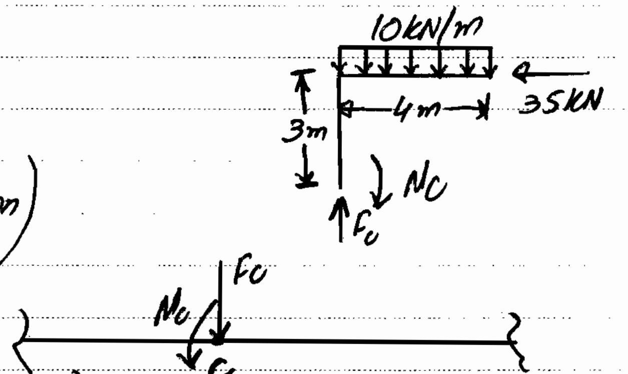
Required:

SFD and BMD by the Graphical method.

Solution:

First we need to determine the effect of the "extension" at C on the beam.

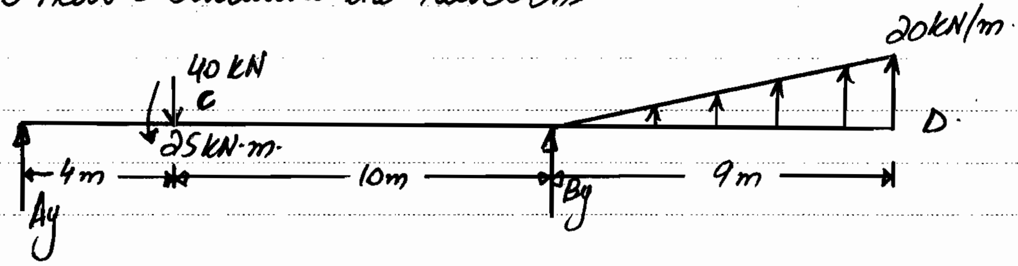
* Remember "equal and opposite"!



$+\uparrow \Sigma F_y = 0 \Rightarrow F_C = 40\ kN$ ($\uparrow$ on Extension, $\downarrow$ on beam)

$+\circlearrowleft \Sigma M = 0 \Rightarrow M = -10(4)(2) + 35(3)$
$= 25\ kN\cdot m$ ($\circlearrowright$ on Extension, $\circlearrowleft$ on beam)

Now we need to calculate the reactions.



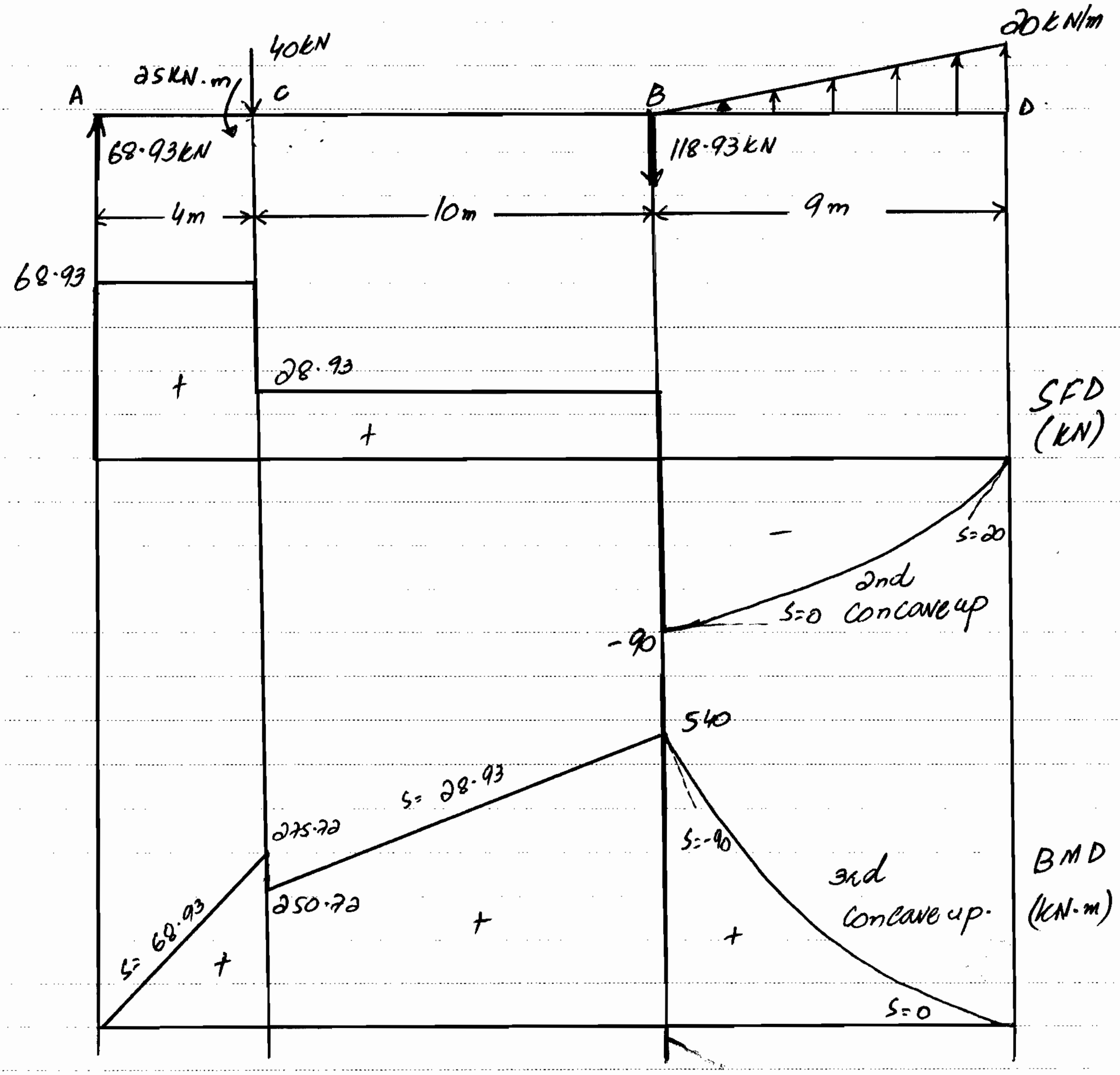
Solution of HW #9

$+\circlearrowleft \Sigma M_B = 0 \Rightarrow -14A_y + 25 + 40(10) + \frac{(20)(9)}{2}\left(\frac{2}{3}\times 9\right) = 0$

$\Rightarrow A_y = 68.929\ kN \approx 68.93\ kN (\uparrow)$

$+\uparrow \Sigma F_y = 0 \Rightarrow 68.929 - 40 + \frac{20(9)}{2} + B_y = 0 \Rightarrow$

$B_y = -118.93\ kN = 118.93\ kN (\downarrow)$



Areas:

$\Delta V_{A \to C} = A_{load\, A \to C} = 0 \Rightarrow$

$V_C = V_A + \Delta V_{A \to C} = 68.93 + 0 = 68.93\ kN$

$\Delta V_{C \to B} = A_{load\, C \to B} = 0 \Rightarrow$

$V_{B-} = 28.93 + 0 = 28.93\ kN$

$\Delta V_{B \to D} = A_{load\, BD} = \frac{1}{2}(20)(9) = 90\ kN$

$V_D = V_{B+} + \Delta_{B \to D} = -90 + 90 = 0$ "as expected"

$\Delta M_{A \to C} = A_{V_{A \to C}} = 68.93(4) = 275.72$

$M_{C-} = M_A + \Delta M_{A \to C} = 0 + 275.72 = 275.72\ kN \cdot m$

$\Delta M_{C \to B} = A_{V_{C \to B}} = 28.93(10) = 289.3$

$M_B = M_{C+} + \Delta M_{C \to B} = 250.78 + 289.3$
$= 540\ kN \cdot m$

$\Delta M_{B \to D} = A_{V_{B \to D}} = \frac{2}{3}(-90)(9) = -540$

$M_D = M_B + \Delta M_{B \to D} = 540 - 540$
$= 0$

"as expected"