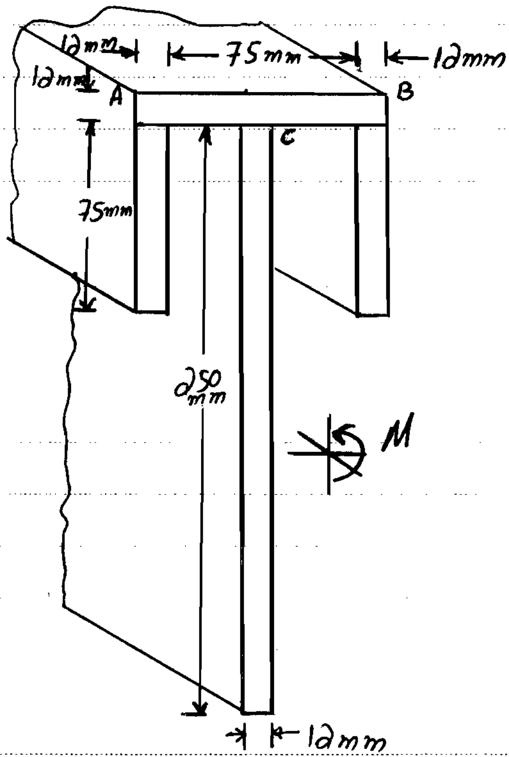
Solution of HW #10

Problem #01:



Given:

The cross-section of a beam shown
$M = -10\ kN\cdot m$

Required:

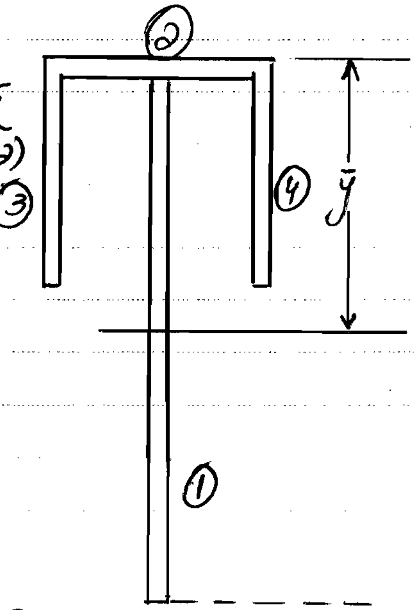
σ^T_{max} ; σ^C_{max} ; σ-distribution

Solution:

Since $\sigma = -\frac{My}{I}$, we, first need to locate the centroid and calculate the moment of inertia (I) of the cross-section.

$$\bar{y} = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i}$$

$$= \frac{12(250)\underbrace{(125+12)}_{137} + \underbrace{(75+12+12)}_{99}(12)(6) + 2(12)(75)\underbrace{(37.5+12)}_{49.5}}{12(250) + (75+12+12)(12) + 2(12)(75)}$$



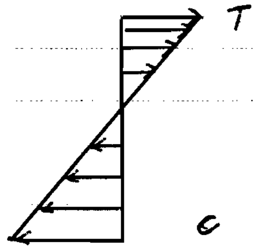
$= 84.7074\ mm$

$$\bar{I} = \Sigma(\bar{I} + Ad^2)_i \quad ; \quad \bar{I}_{rect.} = \frac{1}{12} bh^3$$

$$= \left[\frac{1}{12}(12)(250)^3 + 12(250)(137-84.7074)^2\right]$$
$$+ \left[\frac{1}{12}(99)(12)^3 + 99(12)(84.7074-6)^2\right]$$
$$+ 2\left[\frac{1}{12}(12)(75)^3 + 12(75)(84.7074-49.5)^2\right]$$

$$= 3.42773(10)^7\ mm^4 = 3.42773(10)^{-5}\ m^4$$

For negative moment, ($-M$) the σ-distribution looks as shown; that is, tension up, compression down. Thus,



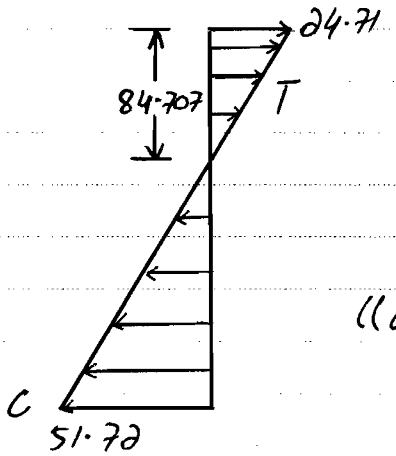
$$\sigma_{max}^{T} = -\frac{M y_{top}}{I} = -\frac{(-10)(10)^3(0.0847074)}{3.42773\,(10)^{-5}} \Rightarrow$$

$$\boxed{\sigma_{max}^{T} = 24.71 \text{ MPa @ Top of Section}}$$

$$\sigma_{max}^{C} = -\frac{M y_{bottom}}{I} = -\frac{(-10)(10)^3(0.0847074-0.262)}{3.42773\,(10)^{-5}} \Rightarrow$$

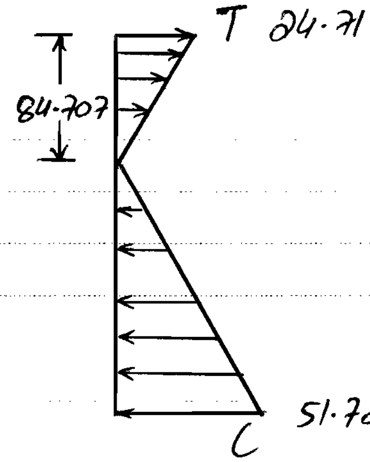
$$\boxed{\sigma_{max}^{C} = 51.72 \text{ MPa @ Bottom of section}}$$

σ-distribution is shown below. (MPa)



Or this way ⇒

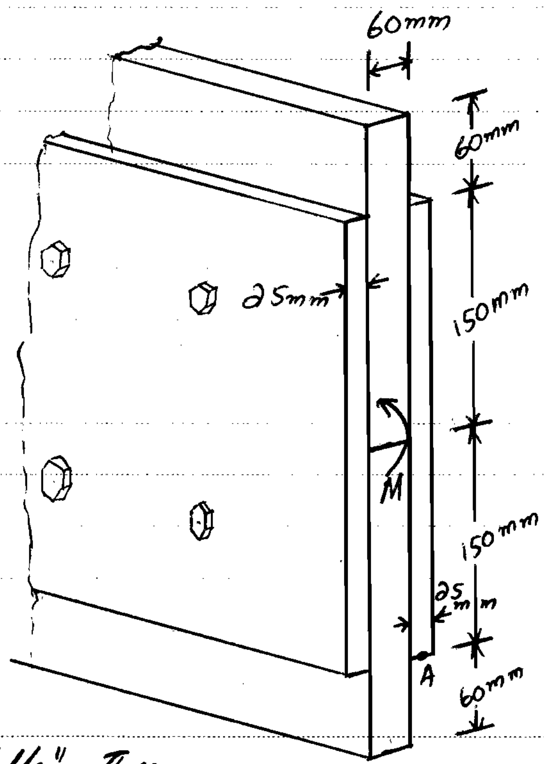
((as your text-book))



Problem#02:

Given:

The beam cross-section shown
$M = +8\ kN.m$

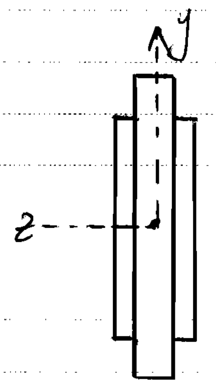


Required:

σ_{Top}, σ_{Bottom}, σ-distribution

Solution:

First, we need to locate the centroid and I for the cross-section. Due to the double symmetry (symmetry about the horizontal and vertical axis), the centroid is in the "middle". Thus,

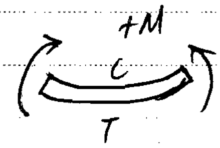


$y_{Top} = 210\ mm$, $y_{bottom} = -210\ mm$

$$I = \frac{1}{12}\left[60(420)^3 + 2(25)(300)^3\right] = 4.8294(10)^8\ mm^4$$
$$= 4.8294(10)^{-4}\ m^4$$

Note no Ad^2. Why?!

$$\sigma = -\frac{My}{I}$$

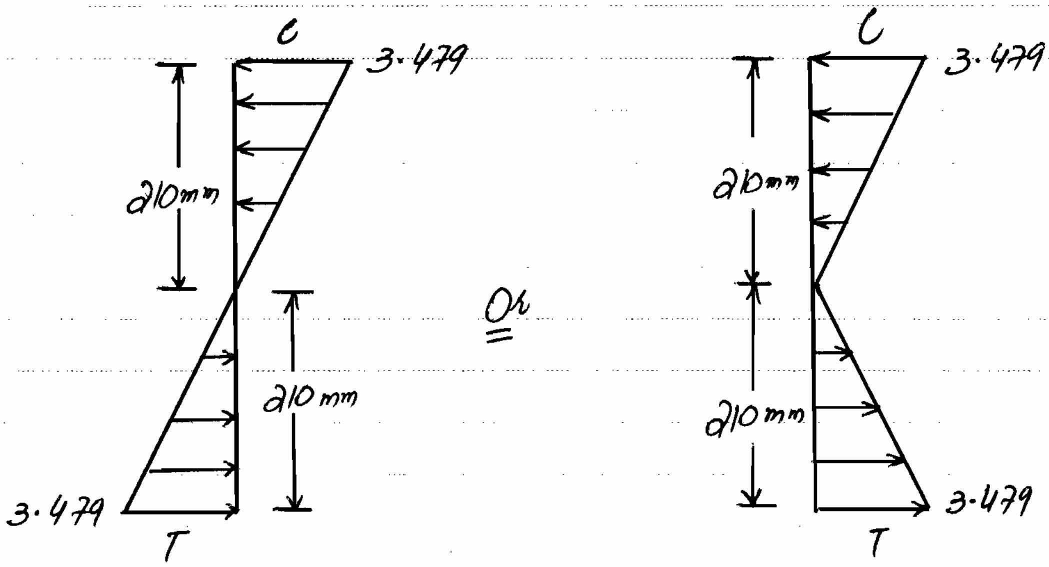


$$\sigma_{Top} = -\frac{8(10)^3(0.21)}{4.8294(10)^{-4}} \Rightarrow \boxed{\sigma_{Top} = 3.479\ MPa\ \text{"C"}}$$

$$\sigma_{bottom} = \frac{-8(10)^3(-0.21)}{4.8294(10)^{-4}} \Rightarrow \boxed{\sigma_{bottom} = 3.479\ MPa\ \text{"T"}}$$

As Expected, Same magnitude as σ_{Top}. Why?!

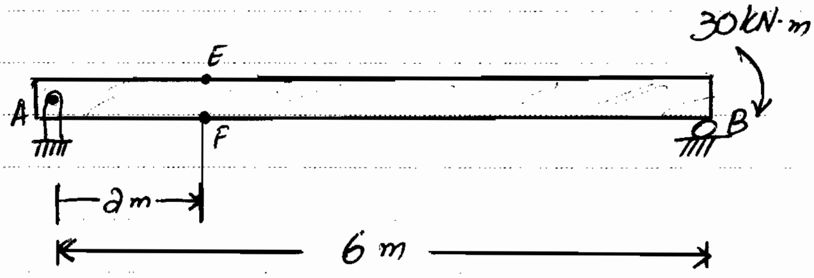
σ- distribution (MPa)



Solution of HW # 10

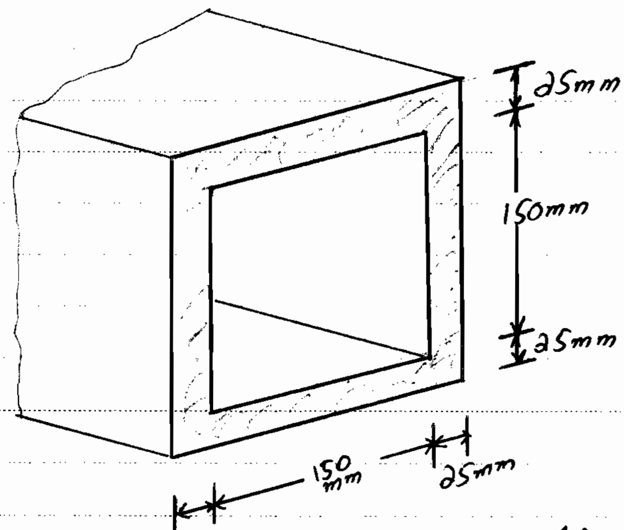
Problem#03:

Given:
The beam with the cross-section shown.



Required:
$\sigma_{Top(E)}$ & $\sigma_{bottom(F)}$ @ Section 2 m from left (A).

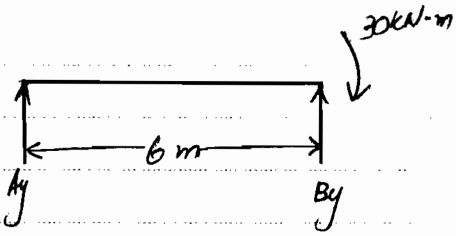
Solution:



Cross-section

Since $\sigma = -\frac{My}{I}$ we need to calculate M @ that section as well as the centroid and I. To find M, we need to determine the reaction @ A. (Why?!)

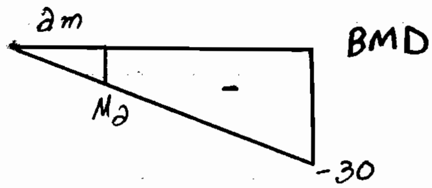
In the FBD shown



$\overset{+}{\circlearrowleft} \Sigma M_B = 0 \Rightarrow$

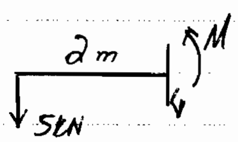
$-6A_y - 30 = 0 \Rightarrow A_y = -5 \Rightarrow A_y = 5\,kN \downarrow$

We can get M @ 2 m by either the moment diagram:



$$\frac{M_2}{2} = \frac{-30}{6} \Rightarrow M_2 = -10\,kN\cdot m$$

Or Section through EF & FBD:



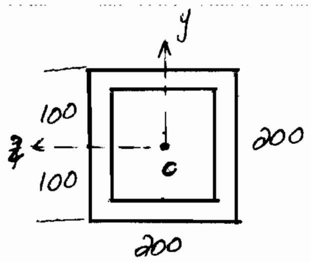
$\overset{+}{\circlearrowleft} \Sigma M = 0 \Rightarrow 5(2) + M = 0 \Rightarrow$

$M = -10\ kN\cdot m\ \ (10\ kN\cdot m \circlearrowright)$

Due to the double symmetry, the centroid is at the "center/middle":

$y_E = y_{top} = 100\,mm$

$y_F = y_{bottom} = -100\,mm$

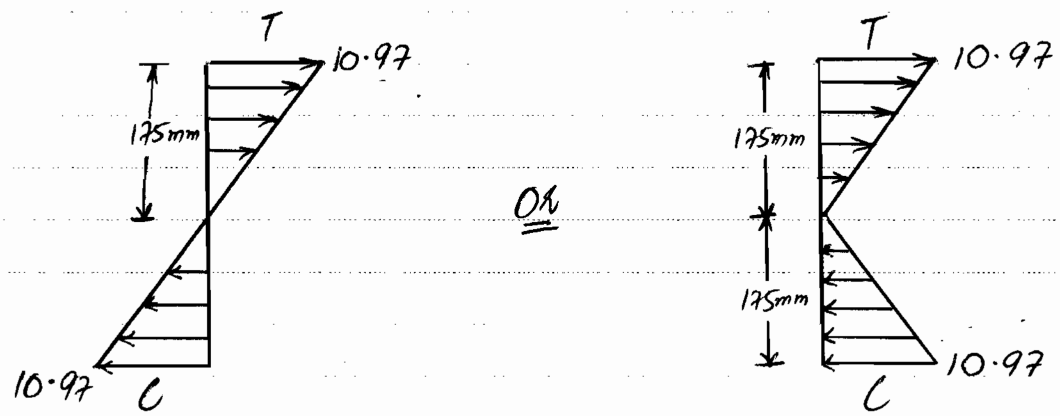


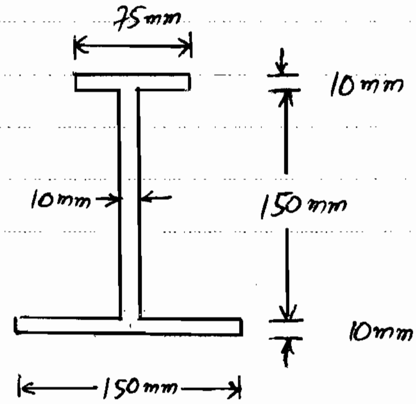
$$\bar{I} = \frac{1}{12}\left[200(200)^3 - (150)(150)^3\right]$$
$$= 9.11458(10)^7\,mm^4 = 9.11458(10)^{-5}\,m^4 \ ((\text{no } Ad^2))$$

$$\sigma_E = -\frac{-10(10)^3(0.1)}{9.11458(10)^{-5}} \Rightarrow \boxed{\sigma_E = 10.97\ MPa\ \text{"T"}}$$

$$\sigma_F = -\frac{-10(10)^3(-0.1)}{9.11458(10)^{-5}} \Rightarrow \boxed{\sigma_F = 10.97\ MPa\ \text{"C"}}$$

<u>Extra</u> "not required": σ- distribution





Problem#04:

Given:

The beam cross-section shown

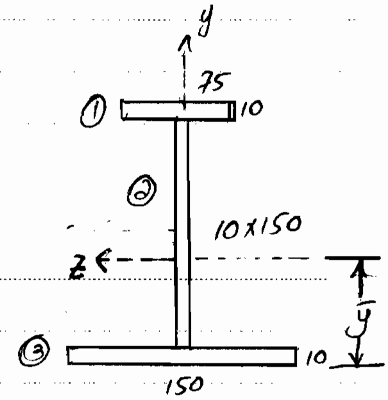
$\sigma^{T}_{allow} = 50$ MPa ; $\sigma^{c}_{allow} = 80$ MPa

Required:

M^{+}_{max}, then M^{-}_{max}

Solution:

First, we need to find $\bar{y}$ and I



We may now try to use a table. We need to first locate $\bar{y}$, then calculate I. (why?!)

Segment/Part	A_i (mm²)	$\bar{y}_i$ (mm)	$A_i\bar{y}_i$ (mm³)
①	750	10+150+5 = 165	123,750
②	1500	10+75 = 85	127,500
③	1500	5	7,500
Σ	3750		258,750

$$\bar{y} = \frac{\Sigma (A_i \bar{y}_i)}{\Sigma A_i} = \frac{258,750}{3750}$$

$= 69$ mm (Reasonable?!)

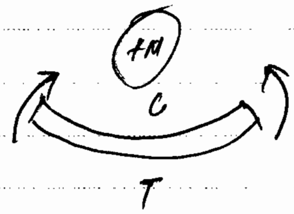
⊛ what is "d"?! Review Statics!!!

Segment (Part)	A_i (mm²)	$d_i^{⊛}$ (mm)	$A_i d_i^2$ (mm⁴)	$\bar{I}_c^{⊙}$ (mm⁴)	$\bar{I}_c + (Ad^2)$ (mm⁴)
①	750	160-69+5 = 96	6,912,000	6,250	6,918,250
②	1500	10+75-69 = 16	384,000	2,812,500	3,196,500
③	1500	69-5 = 64	6,144,000	12,500	6,156,500
Σ	3750		13,440,000	2,831,250	16,271,250

⊙ $I = \frac{1}{12} bh^3$

Solution of HW #10

$\sigma = \dfrac{-My}{I}$



For pos. M, $\sigma^{T}_{max} = -\dfrac{M\, y_{bottom}}{I} \Rightarrow$

$$50(10)^6 = -\frac{M^{①}_{max⊕}(-0.069)}{1.627125(10)^{-5}} \Rightarrow$$

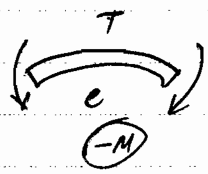
$$\underline{\underline{M^{①}_{max⊕} = 11{,}791\ N\cdot m}}$$

$$\sigma^{C}_{max} = -\frac{M y_{top}}{I} \Rightarrow -80(10)^6 = -\frac{M^{②}_{max⊕}(0.17-0.069)}{1.627125(10)^{-5}} \Rightarrow$$

$$\underline{M^{②}_{max⊕} = 12{,}888\ N\cdot m.}$$

From $M^{①}$ and $M^{②}$, we choose the smaller one for M_{max}. (Why?!)

$$\boxed{\Rightarrow M^{⊕}_{max} = 11.791\ kN\cdot m}$$



For the neg. M, $\sigma^{T}_{max} = -\dfrac{-M y_{top}}{I} \Rightarrow$

$$50(10)^6 = -\frac{-M^{③}_{max⊖}(0.17-0.069)}{1.627125(10)^{-5}} \Rightarrow \underline{\underline{M^{③}_{max⊖} = 8{,}055.1\ N\cdot m}}$$

Clearly, the other criterion (i.e. σ^{C}_{max}) does not control. (How & why?!)

However, we will show that. $\Rightarrow$

$$-80(10)^6 = -\frac{-M^{④}_{max⊖}(-0.069)}{1.627125(10)^{-5}} \Rightarrow \underline{\underline{M^{④}_{max⊖} = 18{,}865\ N\cdot m}} > M^{③}_{max⊖}$$

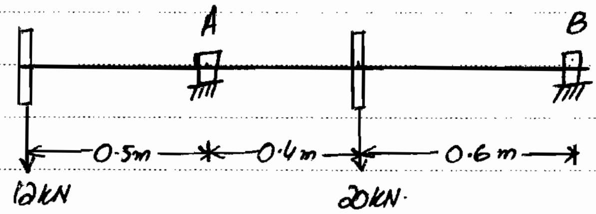
as stated above.

$$\boxed{\Rightarrow M^{⊖}_{max} = 8.055\ kN\cdot m}$$

It is expected that $M^{⊖}_{max} < M^{⊕}_{max}$ (How & why?!)

σ^{T}_{max} controls here in both $M^{⊕}$ and $M^{⊖}$. This is not always the case, even though $\sigma^{T}_{max} < \sigma^{C}_{max}$. (How & why?!)

Problem#05:



Given:

The beam shown

Cross-section: Square ($a \times a$)

$\sigma_{ult.} = 300 MPa$; Safety factor = 3

Required:

a_{min} (the cross-section dimension)

Solution:

$$\sigma = -\frac{My}{I}$$

$$y_{max/min} = \pm \frac{a}{2} \quad ; \quad I = \frac{1}{12}(a)(a^3) = \frac{a^4}{12}$$

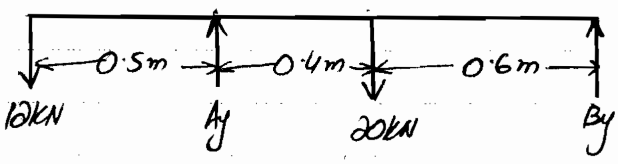
For, M, we need the max. value, and to know it, we need to draw the BMD, and to do that, it is better to draw the SFD first, and to be able to do that we must find the reactions.

In the FBD below,

$+\circlearrowleft \Sigma M_B = 0$ (why start with ΣM?!)

$\Rightarrow 12(1.5) - A_y(1) + 20(0.6) = 0$

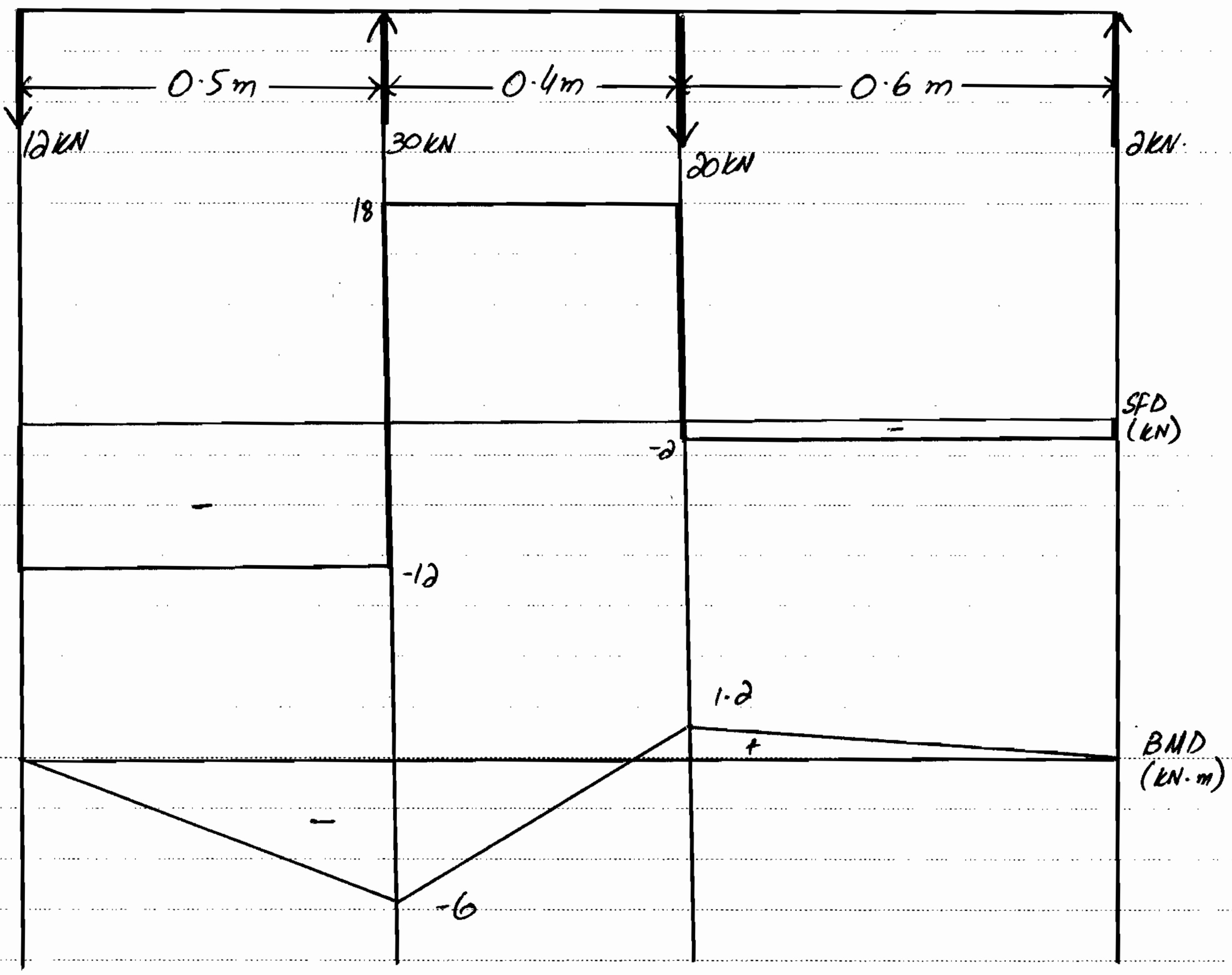
$\Rightarrow A_y = 30$ kN ($\uparrow$)



$+\uparrow \Sigma F_y = 0 \Rightarrow$

$-12 + 30 - 20 + B_y = 0 \Rightarrow B_y = 2 kN (\uparrow)$

Now, the SFD and BMD are drawn as shown.



Since the cross-section has double Symmetry we need to take one value for M_{max} only. It is the absolute max. positive and negative (Why?!)

Then,

$$|M_{max}| = 6\,kN\cdot m$$

$$\sigma_{allow} = \frac{\sigma_{ult.}}{S.F.} = \frac{300}{3} = 100\,MPa$$

$$\sigma_{max} = \left|\frac{M_{max}\,y_{max}}{I}\right| \Rightarrow \sigma_{max} = \frac{6(10)^3\left(\frac{a}{2}\right)}{\frac{a^4}{12}}$$

Now we set $\sigma_{max} = \sigma_{allow}$

$$100(10)^6 = \frac{6(10)^3\left(\frac{a}{2}\right)}{a^4/12} \Rightarrow \boxed{a_{min} = 0.07114\,m = 71.14\,mm}$$