بسم الله الرحمن الرحيم

King Fahd University of Petroleum & Minerals

DEPARTMENT OF CIVIL ENGINEERING

First Semester 1432-33 / 2011-112 (111)

CE 203 STRUCTURAL MECHANICS I

Major Exam II

Note to Students:

Even though the course is not "standard grading", being around the average does not indicate C performance, since there is a minimum amount of course comprehension needed to pass the course satisfactorily, irrespective of the exam average and the performance of other students. Therefore, students who did poorly should do double effort in the remaining of the semester to avoid disappointing grade.

Key Solution

Problem	Instructor
1	Dr. Saeid
2	Dr. Ali
3	Dr. Hamdan
4	Dr. Abdulrahman
5	Dr. Mohammad

Notes:

If you have question or concern regarding your grade in a specific problem, you may see the concerned instructor and discuss it with him directly. The deadline for this process is Wednesday 21 Dec.

Problem # 1

Shaft ABC has a solid circular cross section with diameter d = 4 cm. and is subjected to a torque **T** applied at B. The shaft is held fixed at end A while end C allows a rotation angle ϕ of *not* more than 0.02 radians applied at B.

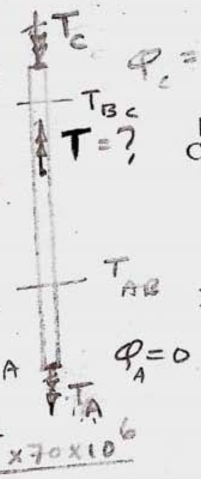
a) Determine the maximum allowable *torque* **T** *that may* safely be applied.
b) Determine the relative angle of twist $\phi_{A/B}$ corresponding to **T**.

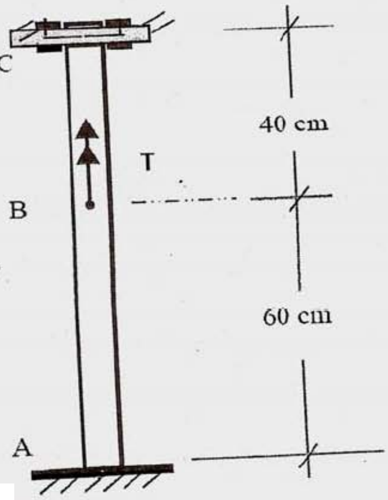
Given: Allowable shear stress $\tau = 50$ **MPa**; $G = 70$ **GPa.**

$J = \frac{\pi}{2} r^4 = 2.5133 \times 10^{-7}\ m^4$

$GJ = 1.7593 \times 10^{4}\ N.m^2$

FBD





Since the allowable T must be such that $\varphi_c > 0.02$ rad and $\tau_{max} > \tau_{all} = 50$ MPa,

hen for $\varphi_c = \varphi_B = 0.02$ rad

$T_c = 0 \Rightarrow 0.02 = \frac{T_{AB}(0.6)}{GJ}$

$\therefore T_{AB} = T = \frac{0.02\ GJ}{0.60}$

$= \frac{0.02 \times \pi/2\,(0.02)^4 \times 70 \times 10^6}{0.60}$

$= 0.5864$ kN.m

$\tau_{max} = T_{AB}\, r/J = 0.5864 \times 0.02/J$

$= 4.67 \times 10^4$ kPa $= 46.7$ MPa $< \tau_{all}$ (2)

$\therefore \tau_{all}$ controls $\Rightarrow \therefore T_c \neq 0 \Rightarrow T - T_A - T_C = 0$ ... (1) (3)

$0.02 = \frac{T_{AB}(0.6)}{GJ} + \frac{T_{BC}(0.4)}{GJ}$ ... (2) (3)

$0.02\ GJ = 0.6 T_A + (T_A - T)\, 0.4$ (3)

$= T_A - 0.4T \Rightarrow T = \frac{T_A - 0.02\,GJ}{0.40}$ ...(3) (3)

and in terms of $T_c \Rightarrow T = \frac{T_c + 0.02\,GJ}{0.6}$ ....(4)

$\because T_{all} = \tau_{all}\, J/r = 50 \times J/0.02 = 628.3$ N.m

Using Eqns. (3) and (4) $\Rightarrow T = \min[691.1 ; 1633.6]$ N.m (3)

$\therefore$ max. safe value of torque T = 691.1 N.m

b) $\varphi_B = \varphi_A + \varphi_{B/A} = 0 - \varphi_{A/B} \Rightarrow \varphi_{A/B} = -\varphi_B = -\frac{T_{AB}(0.6)}{GJ}$

$\therefore \varphi_{A/B} = -\frac{T_A(0.6)}{GJ} = -\frac{(0.02GJ + 0.4T)\,0.6}{GJ}$ (3)

$= -(0.012 + 0.24T/GJ) \Rightarrow \varphi_{A/B} = -0.02143$ rad

Problem # 2

Determine the required thickness, t for the shaft shown below to carry the applied load (P = 12 kN) safely. The shaft is made from a material for which the allowable shear stress, τ_{all} =350 MPa and the allowable angle of twist, ϕ_{all} is 2.

Given G= 80 GPa

Key equations:

$$\tau_{all} = \tau_{cal.}^{max} \quad \boxed{1}$$

$$\phi_{all} = \phi_{cal.}^{max} \quad \boxed{1}$$

(A) For thin-walled tube,

$$\tau_{all} = \frac{T}{2 A_m t_{min}} \quad \boxed{2}$$

$\tau_{all} = 350 \times 10^6$ Pa.

$T = 12 \times 10^3 (0.6) = 7.2 \times 10^3$ N-m $\boxed{2}$

$A_m = (0.1)(0.05) = 5 \times 10^{-3}$ m^2 $\boxed{2}$

$$\therefore \quad 350 \times 10^6 = \frac{(7.2 \times 10^3)}{2(5 \times 10^{-3}) t} \Rightarrow t \simeq 2 \text{ mm} \quad \boxed{1} \quad \boxed{1}$$

(B) For thin-walled tube,

$$\phi_{all} = \frac{TL}{4 A_m^2 G} \sum_{i=1}^{4} \frac{s_i}{t_i} \quad \boxed{2}$$

$\phi_{all} = 2\left(\frac{\pi}{180}\right)$ rad. $\boxed{2}$

$$\therefore \quad 2\left(\frac{\pi}{180}\right) = \frac{(12 \times 10^3)(.6)(2)}{4(5 \times 10^{-3})^2 (80 \times 10^9)} \underbrace{\left[2\left(\frac{.1}{2t} + \frac{0.05}{t}\right)\right]}_{\boxed{3}}$$

$\therefore \quad t \simeq 10$ mm $\boxed{1}$

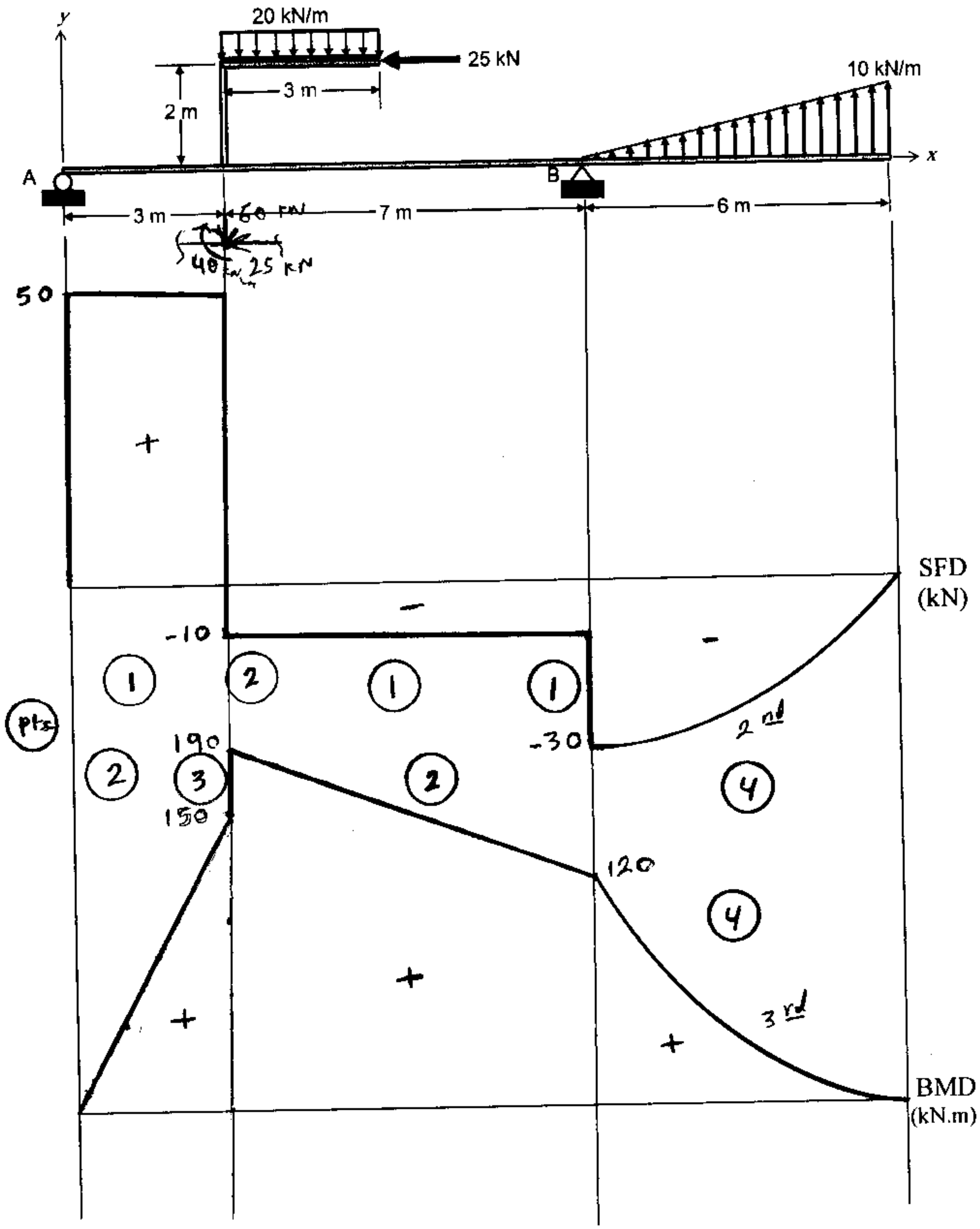
$\therefore$ Use the largest thickness

$\Rightarrow \quad t_{req.} \simeq 10$ mm $\boxed{2}$

Problem # 3

Draw the **shear force and bending moment diagrams** for the beam shown below using the <u>summation (graphical) method</u>. *Write the degree of the curve on each one.*

The reactions are: **A_y = 50 kN↑ ; B_y = 20 kN↓**



Problem # 4

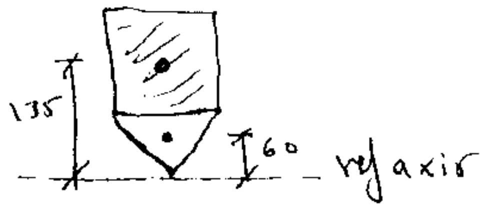
The given beam is subjected to a downward uniformly distributed load **W (kN/m)** as shown.

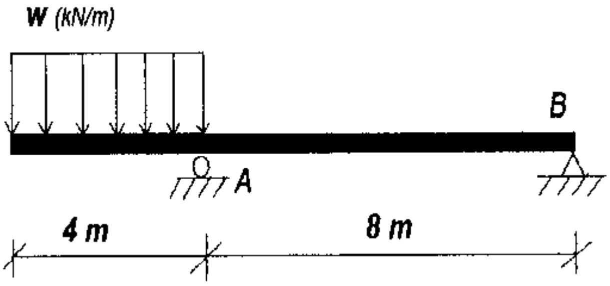
a) Determine the moment of inertia of the beam's cross section about the Neutral Axis.

b) Determine the maximum value of **W** that can be applied given the following information :

Safety Factor = 2
For tension $\sigma_{ult} = 30$ MPa
For compression $\sigma_{ult} = 40$ MPa.

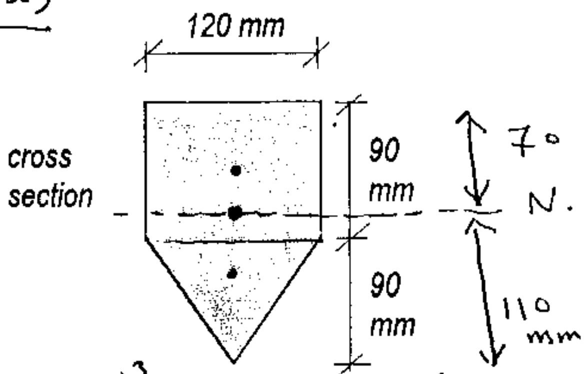
a) Find location of Centroid





$$\bar{y} = \frac{\Sigma A\tilde{y}}{\Sigma A} = \frac{\frac{1}{2}(120)(90)(60) + (120)(90)(135)}{\frac{1}{2}(120)(90) + (120)(90)}$$

$\boxed{\bar{y} = 110 \text{ mm}}$ from the bottom

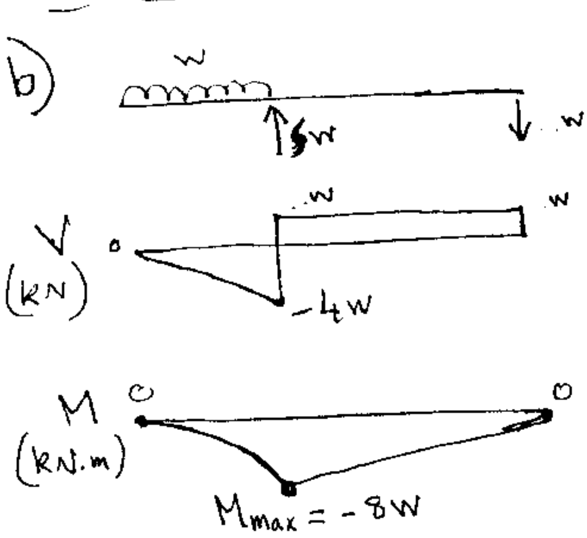


$$I_{NA} = (I_1)_{NA} + (I_2)_{AA}$$

$$I_{NA} = \left[\frac{(120)(90)^3}{36} + \frac{1}{2}(120)(90)(50)^2\right] + \left[\frac{(120)(90)^3}{12} + (120)(90)(25)^2\right]$$

$$I_{NA} = 15.93 \times 10^6 + 14.04 \times 10^6 = \boxed{29.97 \times 10^6 \text{ mm}^4}$$

b)



$M_{max} = -8W$ (kN.m), So $\left\{ \begin{array}{l} T \ \text{u} \\ C \ \text{d} \end{array} \right.$

* Check tension $\sigma_{all} = \frac{30}{2} = 15$ M

$15 = \frac{(8W)\times 10^6 (70)}{I} \rightarrow W = .8c$

* check compression $\sigma_{all} = \frac{40}{2} = 20$ M

$20 = \frac{(8W)\times 10^6 (110)}{I} \rightarrow W = .681$ kN

$\therefore \boxed{\text{Max } W = 0.681 \text{ kN/m}}$

Problem # 5

The bending moment diagram (BMD) and the cross-section of a beam are shown.

a) Sketch the bending stress variation along the y-axis at location B indicating critical values. 6
b) Determine the resultant force that the bending stresses produce on the flange at location B. 6
c) Determine the maximum tensile stress and compressive stress in the whole beam and indicate where each one acts. 8

Take I = 3 x 10⁴ mm⁴

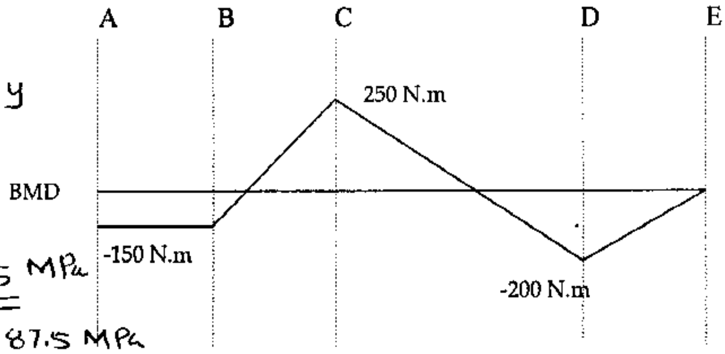
a) $\sigma_{(y)} = -\frac{My}{I}$ (2)

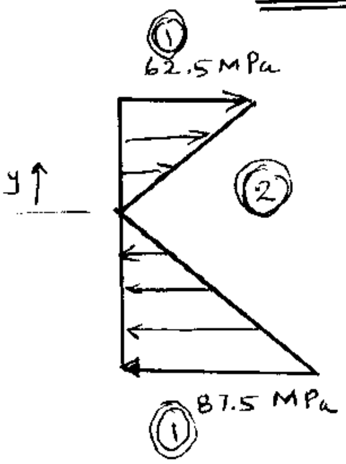
$\sigma_{(y)} = -\frac{(-150\times10^3)\,y}{3\times10^4}$

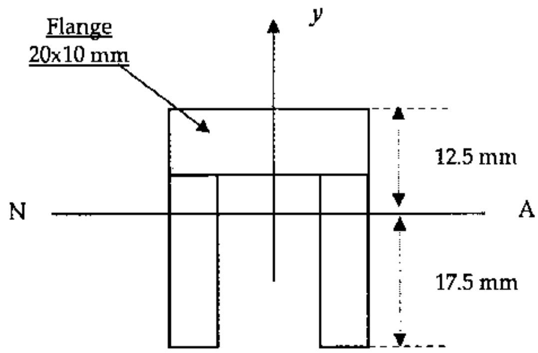
$\sigma_{(y)} = 5y$ MPa

$\sigma_{top} = 5(12.5) = \underline{62.5}$ MPa

$\sigma_{bottom} = 5(-17.5) = \underline{-87.5}$ MPa







Cross section

b) $F = \sigma_{avg} * A_{flew} = \frac{(\sigma_{top} + \sigma_{bot})}{2}(20\times10)$ (4)

$\sigma_{bottom} = 5y\Big|_{y=12.5-10} = 12.5$ MPa

$\therefore F = \frac{62.5 + 12.5}{2} * (20*10) = \underline{7500}$ N (2)

c) (2) Consider two locations: max +ve moment (C) & max -ve moment (D)
Since section is not symmetric wrt N.A.

At C

$\sigma_{top} = -\frac{(250\times10^3)\times12.5}{3\times10^4} = -104.2$ MPa (2)

$\sigma_{bottom} = -250\times10^3\times(-17.5)/3\times10^4 = 145.8$ MPa max T (1) bottom C

At D

$\sigma_{top} = -\frac{(-200\times10^3)(12.5)}{3*10^4} = +83.3$ MPa (2) max C

$\sigma_{bottom} = -(-200\times10^3)(-17.5)/3\times10^4 = -116.7$ MPa (1) bottom D