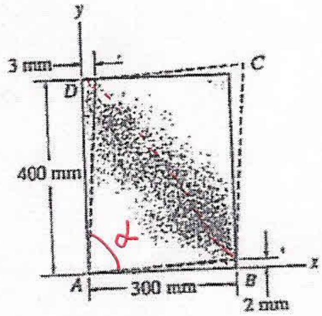
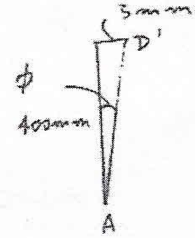
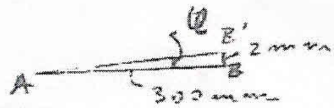
2-26 The piece of rubber is originally rectangular and subjected to the deformation shown by the dashed lines. Determine the average normal strain along the diagonal DB and side AD.

Shear strain in the plate relative to x-y axis

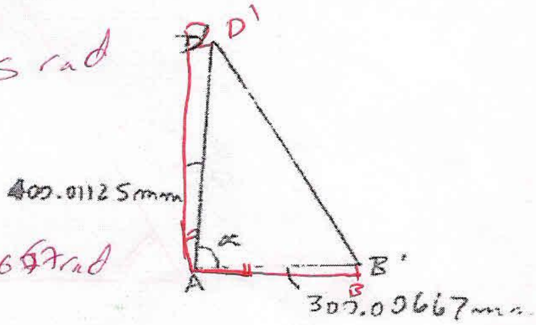






$$AD' = \sqrt{(400)^2 + (3)^2} = 400.01125 \text{ mm}$$

$$\phi = \tan^{-1}\left(\frac{3}{400}\right) = 0.42971^\circ = .0075 \text{ rad}$$



$$AB' = \sqrt{(300)^2 + (2)^2} = 300.00667$$

$$\varphi = \tan^{-1}\left(\frac{2}{300}\right) = 0.381966^\circ = 0.0066\cancel{7}\text{rad}$$

$$\alpha = 90^\circ - 0.42971^\circ - 0.381966^\circ = 89.18832^\circ$$

$$D'B' = \sqrt{(400.01125)^2 + (300.00667)^2 - 2(400.01125)(300.00667)\cos(89.18832^\circ)}$$

$$D'B' = 496.6014 \text{ mm}$$

$$DB = \sqrt{(300)^2 + (400)^2} = 500 \text{ mm}$$

$$\varepsilon_{DB} = \frac{496.6014 - 500}{500} = -0.00680 \text{ mm/mm} \qquad \text{Ans}$$

$$\varepsilon_{AD} = \frac{400.01125 - 400}{400} = 0.0281(10^{-3}) \text{ mm/mm} \qquad \text{Ans}$$

$$\gamma_{xy} = \cancel{\alpha = 89.18832^\circ}$$

$$= 0.0075 + .00667$$

$$= 0.0142 \text{ rad}$$

Problem 2: (20 points)

The stress-strain diagram for a specimen having a length of 300 mm and a diameter of 25 mm is shown below.

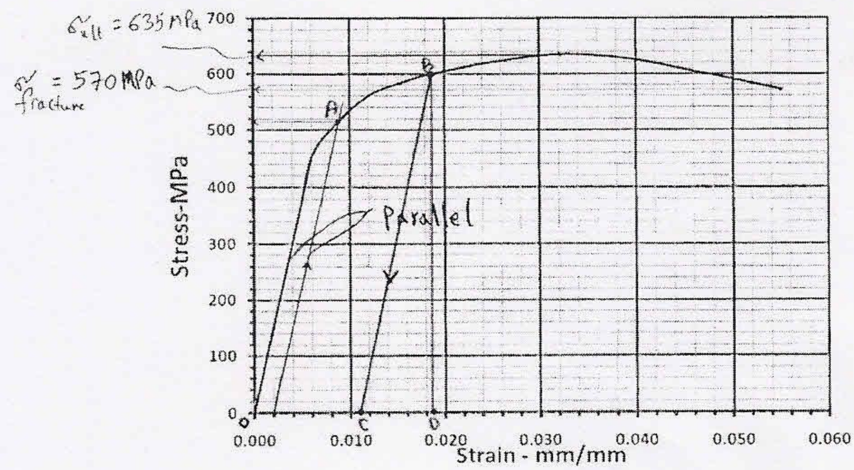
a. Determine the modulus of elasticity, the ultimate stress and the fracture stress.

b. Determine the yield strength using the 0.2% offset method.

c. Determine the new length and diameter when the specimen is stressed to 400 MPa.

d. Determine the final length when the specimen is stressed to 600 MPa and then unloaded.

$\underline{\nu = 0.35}$



The modulus of elasticity, $E = \dfrac{300\text{ MPa} - 0}{0.004\frac{mm}{mm} - 0} = 75 \times 10^9$ Pa. (3)

The ultimate stress, $\sigma_{ult} = 635$ MPa. (1)

The fracture stress, $\sigma_{fracture} = 570$ MPa. (1)

Using 0.2% offset method, the line parallel to the initial straight line of the stress-strain diagram starting from $\epsilon = 0.002\frac{mm}{mm}$ as shown in the diagram. The intersection point on the curve represents the yield strengthe which is, $\sigma_{ys} = 515$ MPa. (3)

When the specimen is stressed to 400 MPa $\Rightarrow$ $\epsilon = 0.005333\frac{mm}{mm}$ (2)

Using the modulus of elasticity.

- New length of the specimen $= 300\text{mm} + (0.005333\frac{mm}{mm} \times 300\text{ mm})$ (2)
$= 301.600$ mm.

- New diameter of the specimen $= 25\text{ mm} - (0.35 \times 0.005333 \times 25\text{ mm})$
$= \underline{24.953\text{ mm}}$. (2)

d) when the specimen is stressed to 600 MPa and then unloaded.

(1)

At 600 MPa $\Rightarrow$ the strain, $\epsilon = 0.0185 \frac{mm}{mm}$

We draw straight line BC from B which is paralled to the straigh portion of the curve (elastic portion).

From the triangle CBD:

(2)

$$E = \frac{BD}{CD} \Rightarrow CD = \frac{600 \times 10^{6}\ Pa}{75 \times 10^{9}\ Pa} = 0.008 \frac{mm}{mm}$$

This represents the recovered elastic strain.

(2)

$\therefore$ The permenant strain $= \epsilon_{perm} = \epsilon_{CD} = 0.0185 - 0.008$

$$= 0.0105 \frac{mm}{mm}.$$

$\therefore$ The final length of the specimen $= L_o + (\epsilon_{perm} * L_o)$

$$= 300\ mm + \left(0.0105 \frac{mm}{mm}\right) \cdot (300\ mm)$$

$$= \underline{303.15\ mm.}$$

(1)

4) In a tensile test of a bar with rectangular cross section 200 mm × 300 mm, the axial force at the proportional limit is 1200 kN. The 900-mm gage length is observed to increase by 0.45 mm, and the 300-mm dimension decreases by 0.015 mm. Calculate

   a) the proportional limit,
   b) the modulus of elasticity,
   c) Poisson's ratio,
   d) the new value of the 200-mm dimension. [Secs. 3.1 - 3.4; 3.6] (15 pts.)

5) In Fig. P5 shown, determine the displacement of point E. [Secs. 4.1-4.3 (Introd.)] (15 pts.)

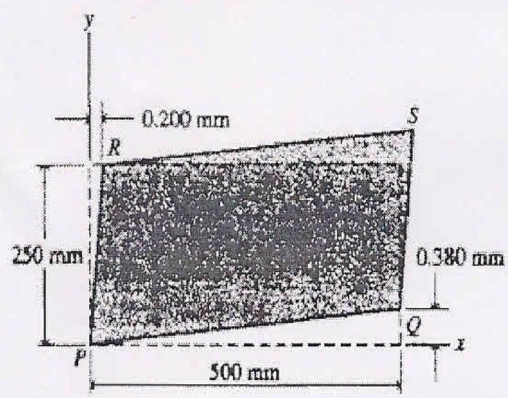
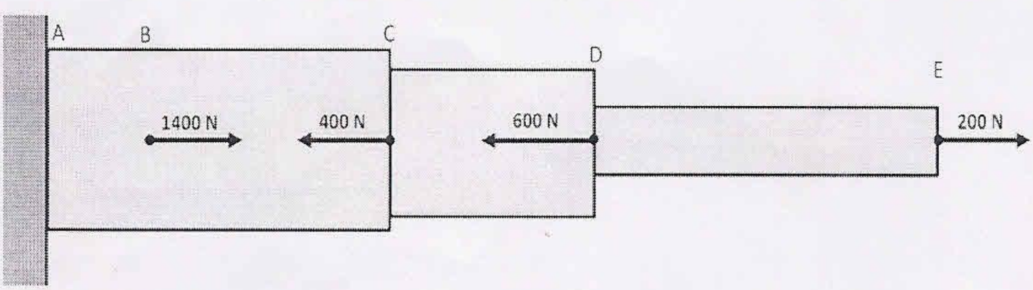


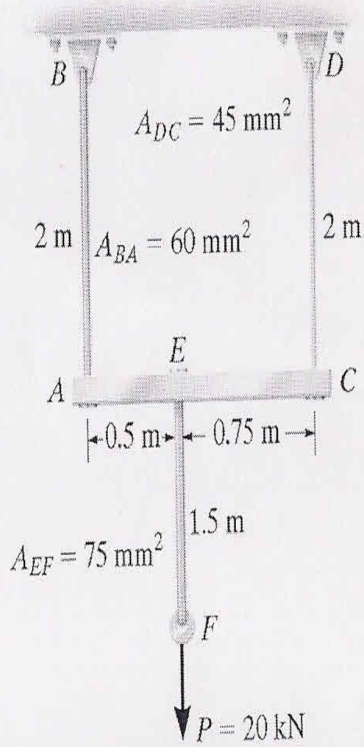
Fig. P2



Member	*Properties*		
	L (m)	*A (mm²)*	*E (GPa)*
AB	0.5	50	250
BC	1.5	50	250
CD	2	32	400
DE	3	10	600

Fig. P5

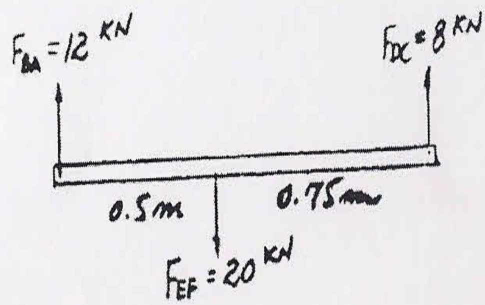
4-15. The assembly consists of three titanium rods and a rigid bar AC. The cross-sectional area of each rod is given in the figure. If a vertical force $P = 20$ kN is applied to the ring F, determine the vertical displacement of point F. $E_{ti} = 350$ GPa.



$$\delta_A = \frac{PL}{AE} = \frac{12(10^3)(2000)}{(60)(10^{-6})(350)(10^9)} = 1.1429\ \text{mm}$$

$$\delta_C = \frac{PL}{AE} = \frac{8(10^3)(2000)}{45(10^{-6})(350)(10^9)} = 1.0159\ \text{mm}$$

$$\delta_{F/E} = \frac{PL}{AE} = \frac{20(10^3)(1500)}{75(10^{-6})(350)(10^9)} = 1.1429\ \text{mm}$$

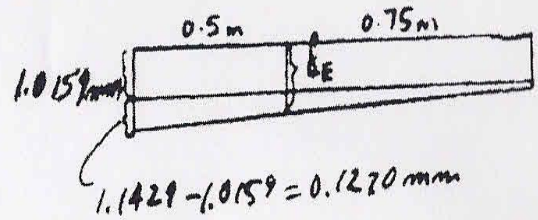


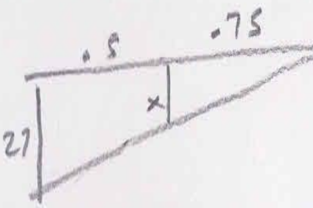
$$\delta_E = 1.0159 + \frac{0.75}{1.25}(0.1270) = 1.092\ \text{mm}$$

$$\delta_F = \delta_E + \delta_{F/E}$$

$$= 1.092 + 1.1429$$

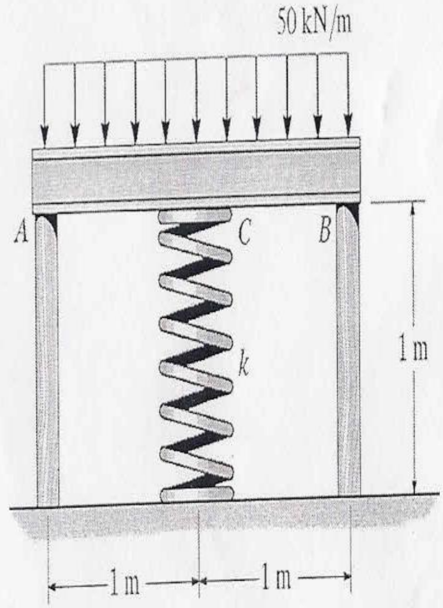
$$= 2.23\ \text{mm} \qquad \textbf{Ans}$$





$$\frac{.127}{1.25} = \frac{x}{0.7\ldots}$$

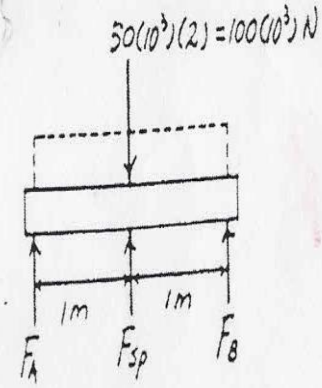
4-63. The rigid bar is supported by the two short white spruce wooden posts and a spring. If each of the posts has an unloaded length of 1 m and a cross-sectional area of 600 mm^2, and the spring has a stiffness of $k = 2$ MN/m and an unstretched length of 1.02 m, determine the force in each post after the load is applied to the bar.



Equations of Equilibrium :

$$\curvearrowleft + \Sigma M_C = 0; \quad F_B(1) - F_A(1) = 0 \quad F_A = F_B = F$$

$$+\uparrow \Sigma F_y = 0; \quad 2F + F_{sp} - 100(10^3) = 0 \qquad [1]$$

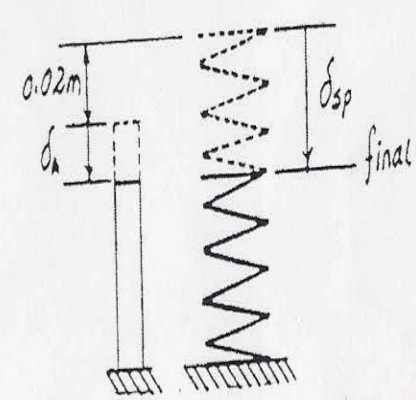


Compatibility :

$$(+\downarrow) \qquad \delta_A + 0.02 = \delta_{sp}$$

$$\frac{F(1)}{600(10^{-6})9.65(10^9)} + 0.02 = \frac{F_{sp}}{2.0(10^6)}$$

$$0.1727F + 20(10^3) = 0.5\,F_{sp} \qquad [2]$$



Solving Eqs. [1] and [2] yields :

$$F_A = F_B = F = 25581.7 \text{ N} = 25.6 \text{ kN} \qquad \textbf{Ans}$$

$$F_{sp} = 48\,836.5 \text{ N}$$