بسم الله الرحمن الرحيم

King Fahd University of Petroleum & Minerals

DEPARTMENT OF CIVIL ENGINEERING

Second Semester 1433-34 / 2012-13 (122)

CE 203 STRUCTURAL MECHANICS I

Major Exam II

Tuesday, April 23, 2013 7:00-9:30 P.M.

KEY SOLUTION

Note to Students:

Even though the course is not "standard grading", ***being around the average does not indicate C performance, since there is a minimum amount of course comprehension needed to pass the course satisfactorily*****, irrespective of the exam average and the performance of other students. Therefore, students who did poorly in this exam should do double effort in the remaining of the semester to avoid disappointing grade.**

After reviewing the key solution and still having a concern about your mark, you may consult with the faculty members who prepared each problem.

The deadline for review is Saturday May 11, 2013.

Problem	Solved & Graded by
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4	Dr. Salah Al-Dulaijan
5	Dr. Hamdan Al-Ghamedy

Notes:

1. A sheet that includes selected Basic Formulae and definitions is provided with this examination.
2. Write clearly and show all calculations, FBDs, and units.

Problem 1: (20 points)

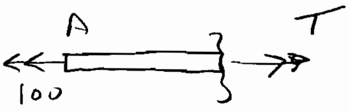
Shafts *ABC* and *EDH* are connected using the shown gear system. Both shafts have circular cross sections **(radius = *20 mm*)** and material shear modulus ***G*= *100 GPa*.**

a) Determine the magnitude of the maximum shear stress ***in the whole structure***. Also, indicate where this maximum stress is located.

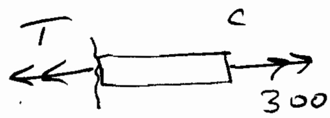
b) Determine the relative angle of twist of point ***C*** with respect to point ***A***.

c) Determine the angle of twist of point ***C***.

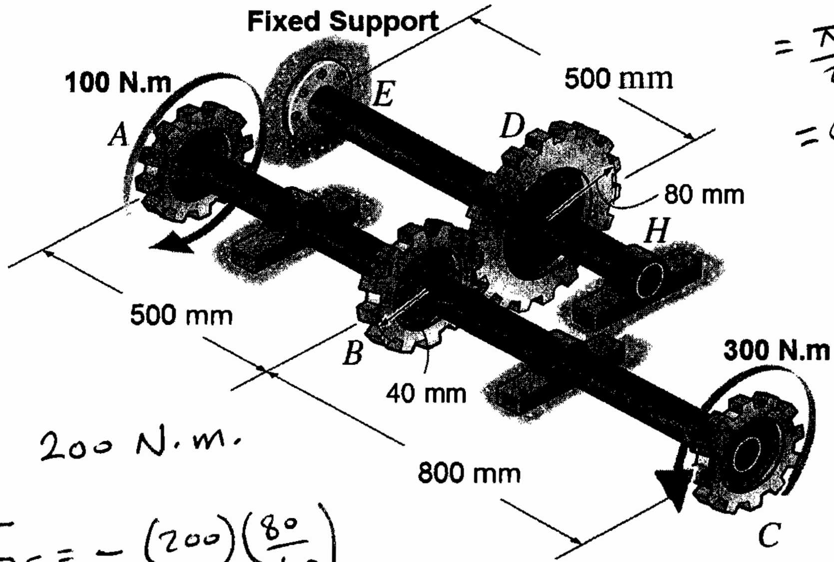
a)



$T_{AB} = +100$ N.m



$T_{BC} = +300$ N.m



$$J = \frac{\pi r^4}{2} = \frac{\pi}{2}(20)^4 = 0.2513 \times 10^6 \text{ mm}^4$$

∴ Net torque at B = 200 N.m.

at gears B & D $\quad T_{DE} = -(200)\left(\frac{80}{40}\right) = -400$ N.m.

Since all 3 segments have the same radius

No need to check τ_{AB} & τ_{BC}

$$\tau_{max} = \tau_{DE} = \frac{(400{,}000)(20)}{J} = 31.83 \text{ MPa}$$

Located at outer surface of shaft DE

b) $$\phi_{C/A} = \phi_{C/B} + \phi_{B/A} = \frac{(+300{,}000)(800)}{(100{,}000)\,J} + \frac{(+100{,}000)(500)}{(100{,}000)\,J}$$

$$\phi = \frac{TL}{GJ}$$

$$= +9.549 \times 10^{-3} + 1.989 \times 10^{-3} = \boxed{+11.54 \times 10^{-3}} \text{ rad}$$

c) $$\boxed{\phi_C = \phi_{C/E} = \phi_B + \phi_{C/B}}$$

find $$\phi_{D/E} = \frac{(-400{,}000)(500)}{(100{,}000)\,J} = -7.958 \times 10^{-3} \text{ rad}$$

at gears $$\phi_B = -\frac{80}{40}\phi_{D/E} = +15.92 \times 10^{-3} \text{ rad}$$

$$\therefore \phi_C = +15.92 \times 10^{-3} + 9.549 \times 10^{-3}$$

$$\boxed{\phi_C = +25.46 \times 10^{-3} \text{ rad}}$$

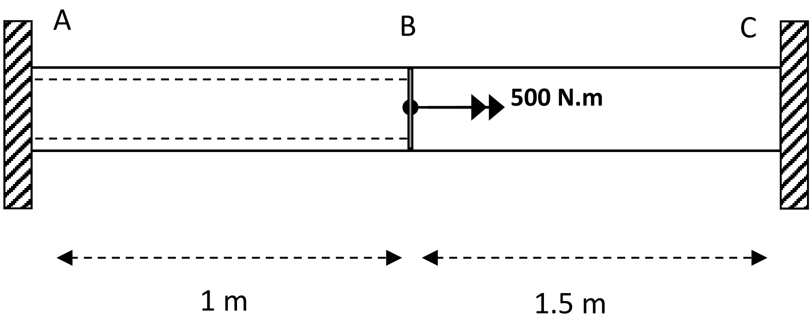
Problem 2: (20 points)

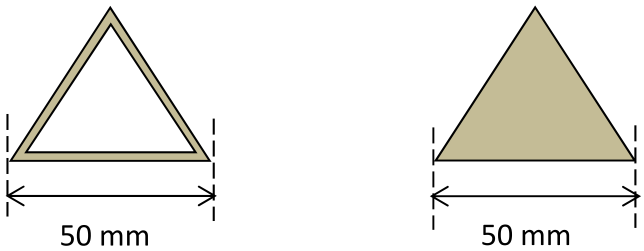
The shown shaft is made by connecting two segments, AB which has a thin hollow equilateral triangular cross section and BC which has an equilateral triangular cross section.

Determine:

a) The maximum shear stress in the whole shaft and indicate its location.
b) The angle of twist of point B.

Given:
Side width = 50 mm
Thickness = 2 mm
G= 30 GPa





Cross section between A and B *Cross section between B and C*

(t=2 mm)

1

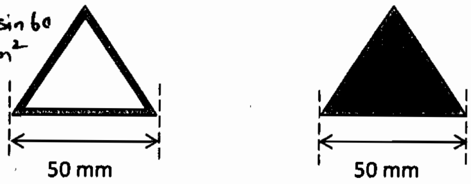
$A_m \doteq 48 \times 48 \sin 60$

$= 998\ mm^2$

1

$\oint \frac{ds}{t} \doteq \frac{48 \times 3}{2}$

$= 72$



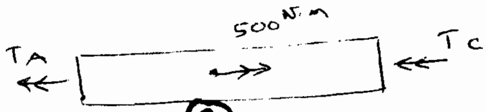
Cross section between A and B (t=2 mm) Cross section between B and C

$a = 50^{mm}$

To be able to calculate τ & φ we must know the reaction at either End

$\Sigma M_x = 0$

$\boxed{T_A + T_C - 500 = 0}$ — (1) 3



2

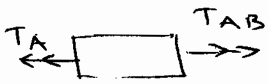
Statically Indeterminate

compatibility Eqn.

$\varphi_{A/C} = 0$ 1

$\varphi_{A/B} + \varphi_{B/C} = 0$

(thin hollow) (Solid triangle)



$\Sigma M_x = 0$

$\boxed{T_{AB} = T_A}$ — (a)

1

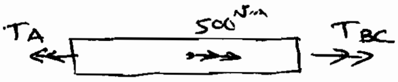
between A-B

using proper formulae

1 2

$$\frac{T_{AB} L_{AB}}{4 A_m^2 G} \oint \frac{ds}{t} + \frac{46\ T_{BC} L_{BC}}{a^4 G} = 0$$

50



$\Sigma M = 0$

between B/C $\boxed{T_{BC} = T_A - 500}$ — (b)

1

Substitute values

$$\frac{T_{AB} \times 1 \times 72}{4\,(998)^2\,(30)} + \frac{46\,(T_A - 500) \times 1.5}{(50)^4\,(30)} = 0$$

1

$1.64\ T_A + T_A - 500 = 0 \implies \boxed{T_A = 190\ N.m}$

max shear stress in thin hollow section A-B

$$\tau = \frac{T}{2 A_m t} = \frac{190 \times 10^3}{2(998)(2)} = 47.6\ MPa$$

2

max shear stress in Solid triangular Section (B-C)

310

$$\tau = \frac{20T}{a^3} = \frac{20\,(500 - 190) \times 10^3}{50^3} = 49.6\ MPa$$

2

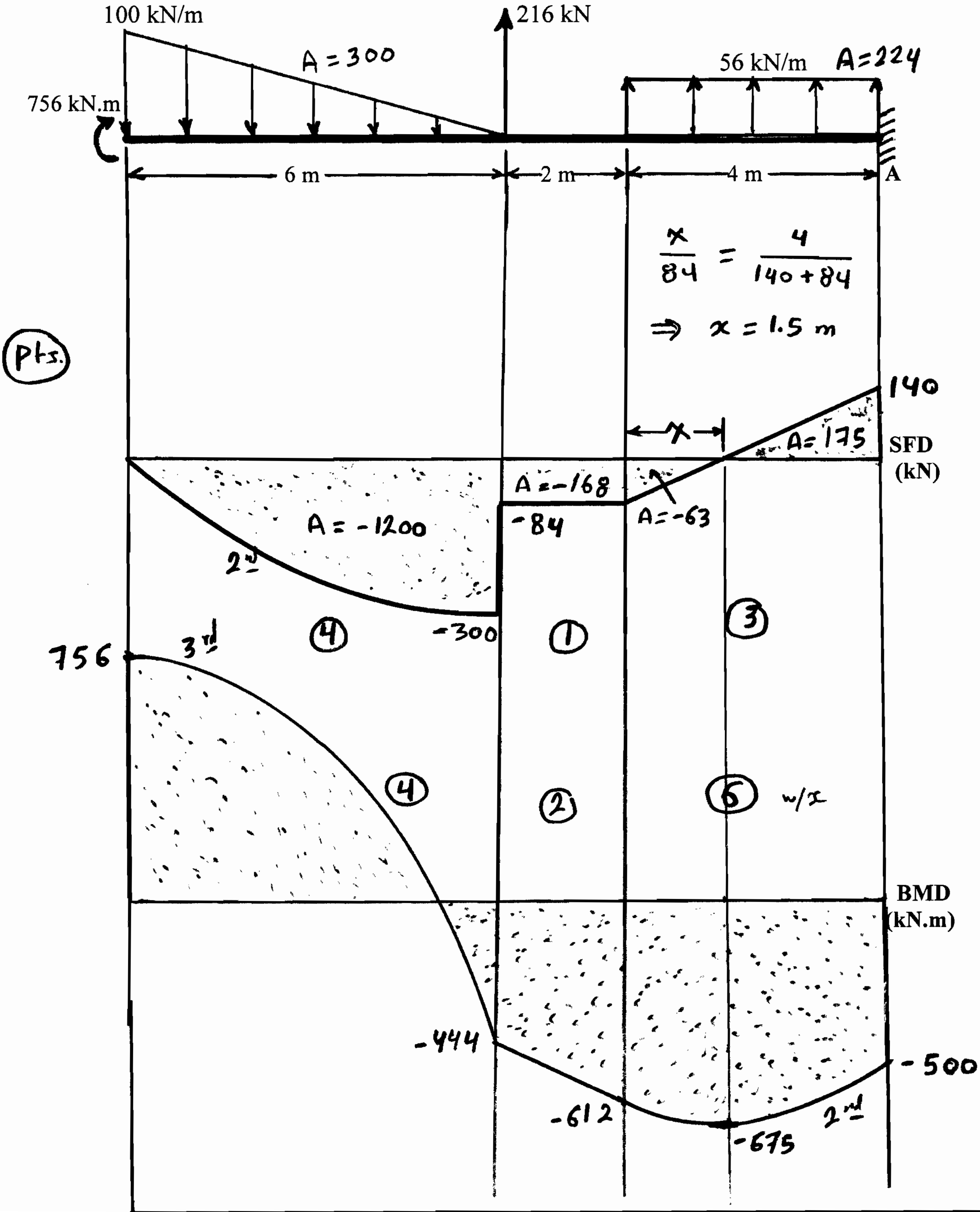
$\therefore\ \tau_{max} = 49.6\ MPa$ at middle of solid triangular sides

$$\varphi_B = \frac{46\,(310)(1500)}{} = 0.11^{rad} = 6.5^{0}$$

Problem 3 (20 pts.)

Draw the **shear force and bending moment diagrams** for the beam shown below using the ***summation (graphical) method***. *Write the degree (2, 3, etc.) of the* ***curve on each one***. Put all values on the diagrams. *Use appropriate scale. **No credit will be given if another method is used.***

The reactions are: $R_A = 140$ kN ↓ $M_A = 500$ kN.m



Problem 4: (20 points)

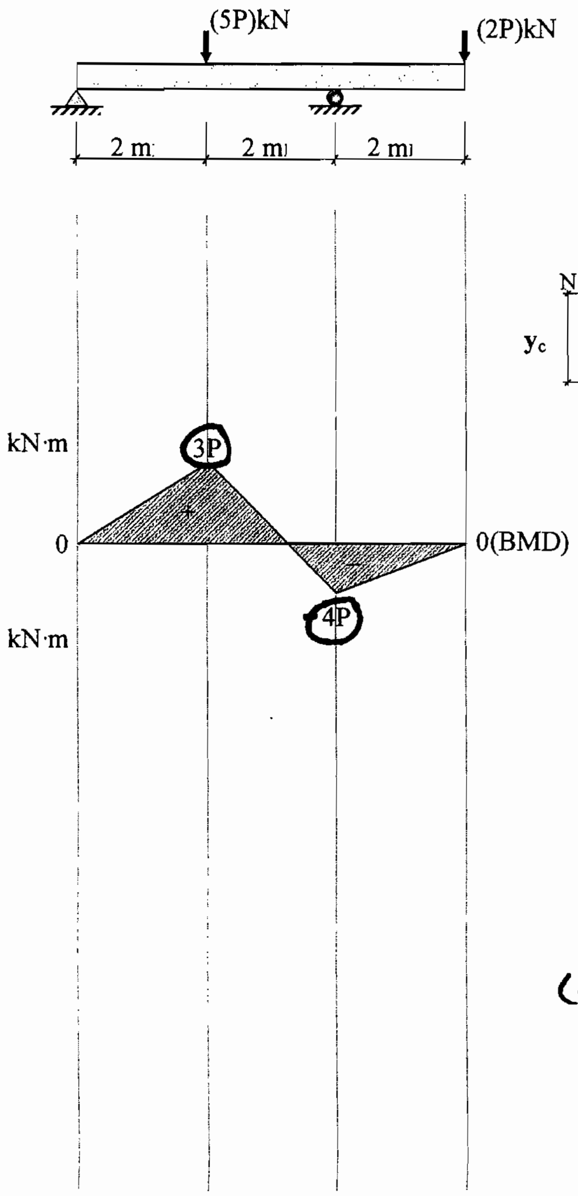
Beam ABCD is shown with the bending moment diagram and the cross-section details.

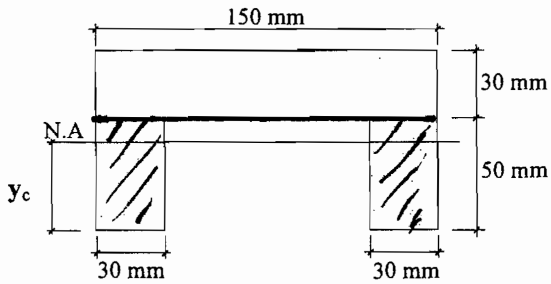
If for the used material

$(\sigma_{\text{allowable}})_{\text{Tension}} = 20$ MPa and
$(\sigma_{\text{allowable}})_{\text{Compression}} = 15$ MPa.

a) Verify that $y_c = 49$ mm from the bottom of the cross-section and that $I_{N.A.} = 3.8425 \times 10^6$ mm^4.

b) Using the values of y_c and $I_{N.A}$ given in part (a) and the given bending moment diagram, compute the maximum load P that can be safely applied to the beam.





(a) $\bar{y}_c = \dfrac{\Sigma A\bar{y}}{\Sigma A}$

$\dfrac{[(30)(150)(65)] + 2[(30)(50)(25)]}{(150)(30) + 2(50\times 30)} =$

$= \dfrac{367500}{7500} = 49$ mm (3)

$\bar{I}_{N.A} = \bar{I} + Ad^2$

$\dfrac{(150)(30)^3}{12} + (150)(30)(16)^2$

$+ \dfrac{(30)(50)^3}{12} + (30)(50)(24)^2 =$

$= 3.8425\times10^6$ mm^4 $= 3.8425\times10^{-6}$ m^4 (4)

(b) + moment = 3P

$\sigma = -\dfrac{Mc}{I}$, $-15\times10^6 = -\dfrac{(3P)(.031)}{3.8425\times10^{-6}}$

(3) $\boxed{P = 619.76\ N}$ comp

$20\times10^6 = -\dfrac{(3P)(-0.49)}{3.8425\times10^{-6}}$

(3) $\boxed{P = 522.78\ N}$ Ten

− moment (−4P)

$20\times10^6 = -\dfrac{(-4P)(.031)}{3.8425\times10^{-6}}$

(3) $\boxed{P = +619.76\ N}$

$-15\times10^6 = -\dfrac{(-4P)(-0.049)}{3.8425\times10^{-6}}$ $\Rightarrow$ (3) (1) (max P) $\boxed{P = 294.1\ N}$

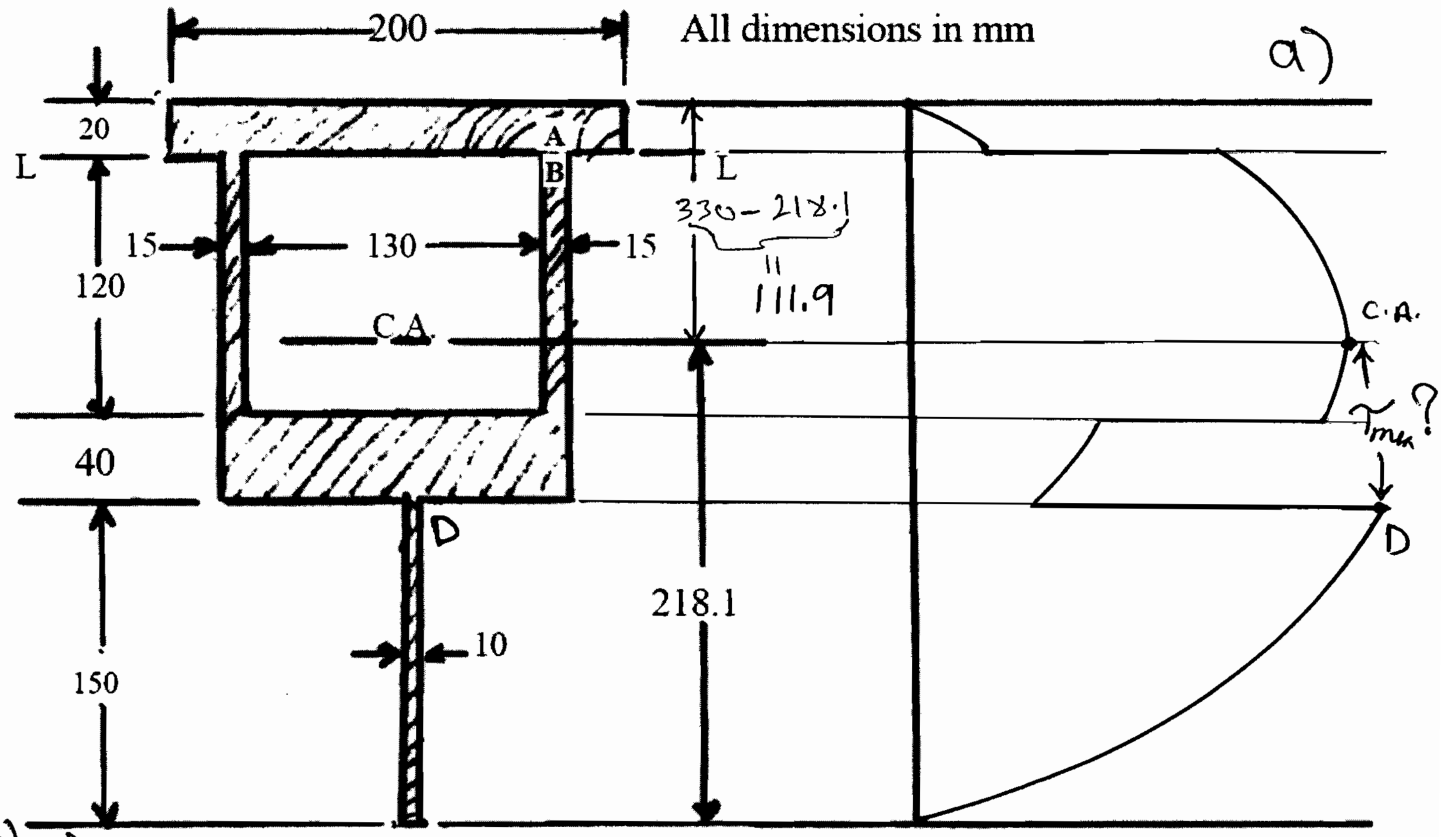
The vertical shear force in a beam, with the cross-section shown below, is 500 kN.

a) *Qualitatively* (without numbers), **draw the vertical shear stress distribution** (τ) on the cross section in the provided space.

b) Determine the **shear stresses** at points A and B (just above and just below line LL).

c) Determine the **value and location of the maximum shear stress.**

***The Centroidal Axis* (C.A.) *is located as shown on the cross section and* $I_{C.A.}$ = 9.884(10)^7 mm^4.**

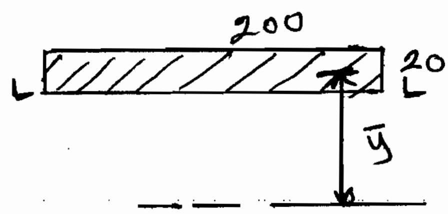


τ Distribution

(pts.) a)

(3) τ-distribution as shown

b) $Q_{LL} = A\bar{y} = 200(20)(111.9 - 10)$
(2) $= 407{,}600$ mm^3



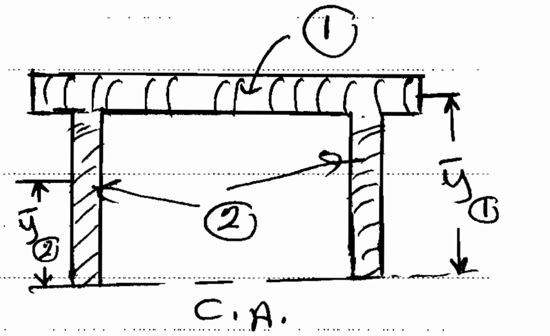
(2) $q_{LL} = \dfrac{VQ}{I} = \dfrac{500(10^3)(407\,600)}{9.884(10)^7}$

$= 2061.92$ N/mm

(1) $\tau_A = \dfrac{q_{LL}}{t_A} = \dfrac{2061.92}{200} \Rightarrow$ $\boxed{\tau_A = 10.31 \text{ MPa}}$

(1) $\tau_B = \dfrac{q_{LL}}{t_B} = \dfrac{2061.92}{15+15} \Rightarrow$ $\boxed{\tau_B = 68.73 \text{ MPa}}$

c) τ_{max} is at the C.A. or at D as shown on the τ-dist. above. Thus we need to check both locations.



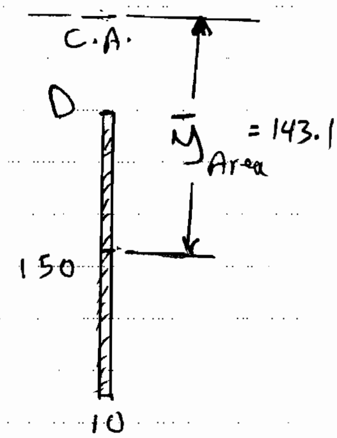
@ C.A.: Take the upper area as it easier.

We divide it into two areas 1 and 2

$$Q_{C.A.} = Q_1 + Q_2 = 407,600 + 2(15)(111.9-20)\left(\frac{111.9-20}{2}\right)$$
$$= 407,600 + 126,684 = 534,284 \text{ mm}^3$$

5 $$\tau_{C.A.} = \frac{VQ}{It} = \frac{500(10)^3(534,284)}{9.884(10)^7(15+15)} \Rightarrow \tau_{C.A.} = 90.09 \text{ MPa}$$

@ D: It is easier to take the lower area as shown $\Rightarrow$



$$Q_D = A\bar{y} = 10(150)\left(218.1 - \frac{150}{2}\right)$$
$$= 214,650 \text{ mm}^3$$

$$\tau_D = \frac{500(10)^3(214,650)}{9.884(10)^7(10)}$$

4 $$= 108.6 \text{ MPa}$$

1 Thus $\boxed{\tau_{max} = 108.6 \text{ MPa @ D shown}}$

1 units