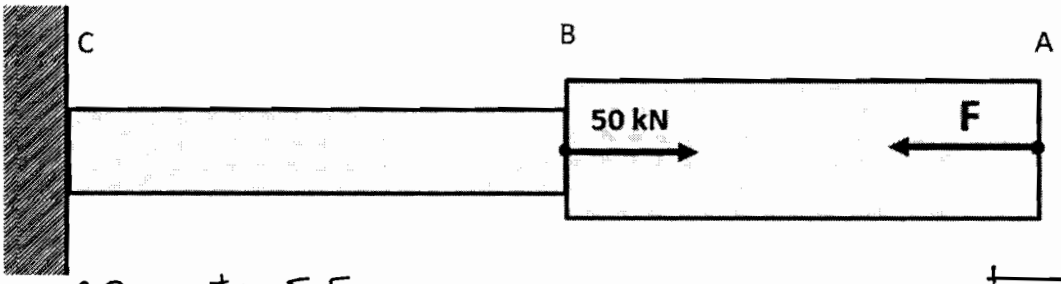
Problem 3: (20 points)

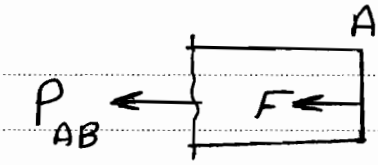
The rods AB and BC are subjected to the *loads and temperature changes* shown in the figure and table below. Determine the **maximum allowable force F** that can be applied (in the shown direction) if

- the maximum allowable normal stress in AB is 150 MPa (tension or compression), and
- the maximum allowable normal stress in BC is 100 MPa (tension or compression), and
- the maximum allowable displacement of point A is $5\,(10)^{-4}$ m.

Member \ Properties	L (m)	A (m^2)	E (GPa)	ΔT (°C)	α ($/°C$)
AB	0.5	$4\,(10)^{-4}$	200	+40	$20\,(10)^{-6}$
BC	0.6	$3\,(10)^{-4}$	100	-60	$15\,(10)^{-6}$



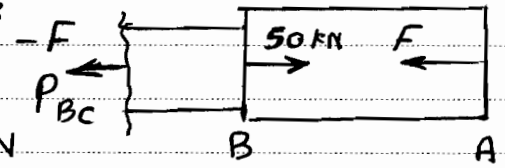
(Pb)



① FBD : AB $\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow$

① $-F - P_{AB} = 0 \Rightarrow P_{AB} = -F$ "C"

① FBD : BC $\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow$



① $-F + 50\,(10)^3 - P_{BC} = 0 \Rightarrow P_{BC} = 50\,(10)^3 - F$

$\sigma^{AB}_{max\,allow} = P_{AB}/A_{AB} \equiv \pm 150\,(10)^6 \Rightarrow$

② $-F/4\,(10)^{-4} = -150\,(10)^6 \Rightarrow F^{①}_{max} = 60$ kN

$\sigma^{BC}_{max\,allow} = P_{BC}/A_{BC} \equiv \pm 100\,(10)^6 \Rightarrow$

② $[50\,(10)^3 - F]/3\,(10)^{-4} = -100\,(10)^6 \Rightarrow F^{②}_{max} = 80$ kN

displ. of A $= \Sigma\delta = (\delta_{mech} + \delta_{therm})_{AB} + (\delta_{mech} + \delta_{therm})_{BC}$

② $\delta^{AB}_{mech} = e_{load} = \dfrac{PL}{AE} = \dfrac{-F(0.5)}{4\,(10)^{-4}\,200\,(10)^9} = -6.25\,(10)^{-9}\,F$ (« ← »)

① $\delta^{AB}_{therm} = e_{\Delta T} = \alpha \Delta T L = 20\,(10)^{-6}(+40)(0.5) = +4\,(10)^{-4}$ m (« → »)

② $\delta^{BC}_{mech} = e_{load} = \dfrac{PL}{AE} = \dfrac{[50\,(10)^3 - F](0.6)}{3\,(10)^{-4}\,100\,(10)^9} = 1\,(10)^{-3} - 2\,(10)^{-8}\,F$

② $\delta^{BC}_{therm} = e_{\Delta T} = \alpha \Delta T L = 15\,(10)^{-6}(-60)(0.6) = -5.4\,(10)^{-4}$ m (« ← »)

displ. of A $= -6.25\,(10)^{-9}\,F + 4\,(10)^{-4} + 1\,(10)^{-3} - 2\,(10)^{-8}\,F - 5.4\,(10)^{-4}$

① $= 8.6\,(10)^{-4} - 2.625\,(10)^{-8}\,F \Rightarrow$

$8.6\,(10)^{-4} - 2.625\,(10)^{-8}\,F \equiv -5\,(10)^{-4}$ [Note the minus sign! Why ?!]

② $\Rightarrow F^{③}_{max} = 1.36\,(10)^{-3}/2.625\,(10)^{-8}$

$= 51.81$ kN

② $F_{max} = \min(F^{①}_{max}, F^{②}_{max}, F^{③}_{max}) \Rightarrow$ **$F_{max} = 51.81$ kN**

(« Why ?! ») { Note that σ_{BC} is still "ok" !! }