CE 203-121

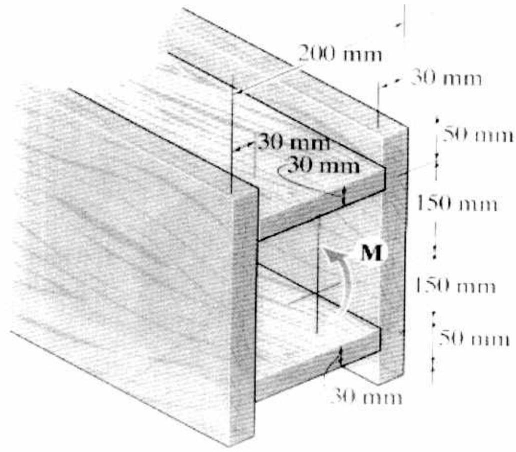
SOLUTION FOR HW #10

PROBLEM #1

Given:

The beam cross-section shown

$M = 10\,kN.m$

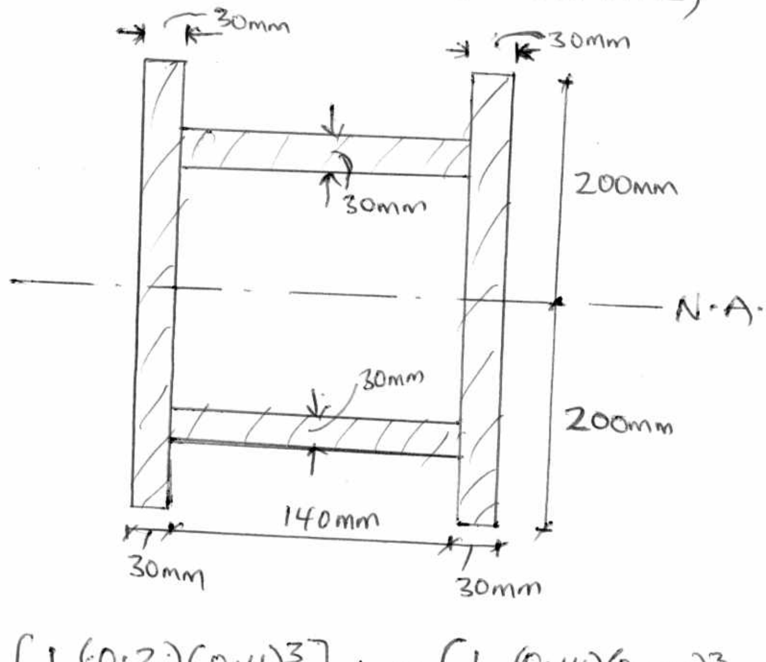


Required:

$\sigma_{max.}$ in the beam.

SOLUTION

Since the beam has symmetrical cross-section, it implies that the N.A. is at the centre,



$$I_{N.A.} = 2\left[\frac{1}{12}(0.3)(0.4)^3\right] + 2\left[\frac{1}{12}(0.14)(0.03)^3 + 0.14(0.03)(0.15)^2\right]$$

$$= 0.50963 \times 10^{-3}\, m^4$$

$$\sigma_{max.} = \frac{Mc}{I}$$

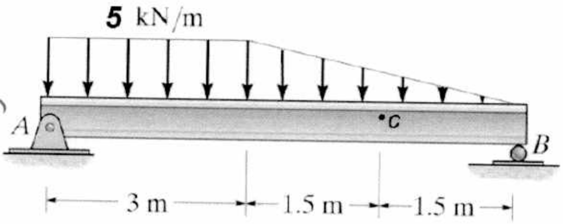
$$= \frac{(10\times10^3)(0.2)}{0.50963\times10^{-3}} = 3.92\,MPa$$

Location: $\sigma_{max.}$ is at the top or bottom of the cross-section.

PROBLEM #2

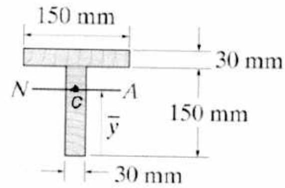
Given:

The beam and its Cross-section Shown



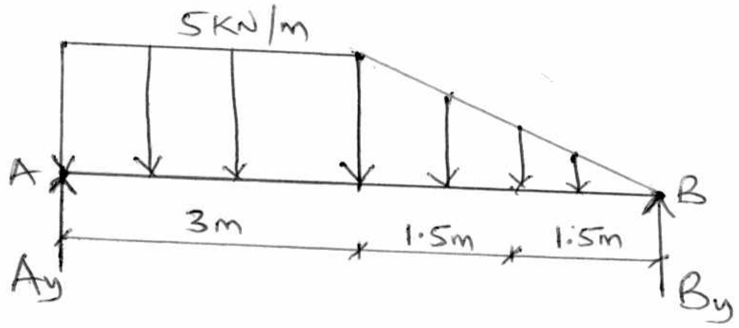
Required:

- Stress at the top and bottom
- Stress distribution along the vertical axis.



SOLUTION

FBD:



Support reactions:

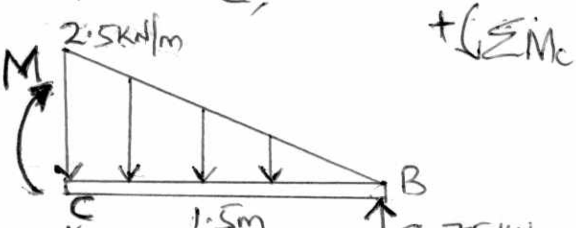
$+\circlearrowright \Sigma M_A = 0; \; -15(1.5) - 7.5(4) + B_y(6) = 0$

$\therefore B_y = 8.75 \text{KN} (\uparrow)$

$+\uparrow \Sigma F_y = 0; \; -15 - 7.5 + 8.75 + A_y = 0$

$\therefore A_y = 13.75 \text{KN} (\uparrow)$

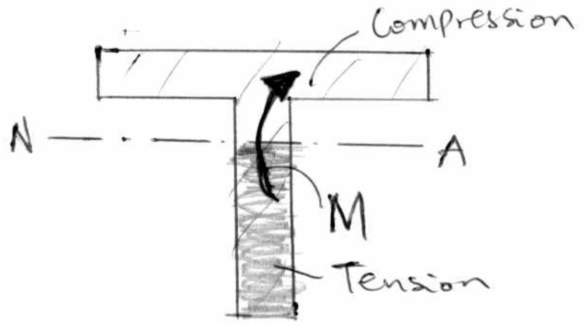
- To find the internal moment at a section passing through Point C,



$+\circlearrowright \Sigma M_C = 0; \; -M - \left(\frac{1}{2} \times 2.5 \times 1.5\right)(0.5) + 8.75(1.5) = 0$

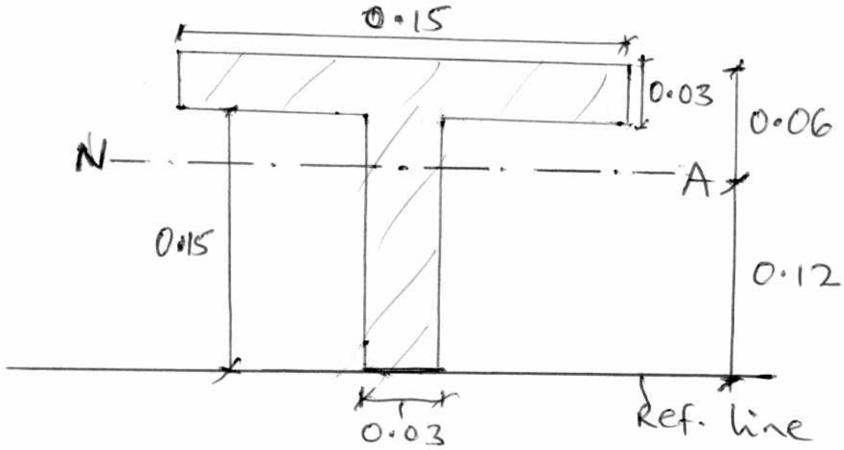
$\therefore M_C = 12.1875 \text{KNm} (\circlearrowleft)$

The moment is shown in the beam Cross-section as follows,



This moment will cause compression above N.A. and tension below the N.A.

- Location of N.A.



(All dimensions in m)

$$\bar{y} = \frac{(0.15)(0.03)(0.165) + (0.03)(0.15)(0.075)}{(0.15)(0.03) + (0.15)(0.03)} = 0.12M$$

$$\Rightarrow I_{N.A} = \left[\frac{(0.03)(0.15)^3}{12} + (0.03)(0.15)(0.045)^2\right]$$

$$+ \left[\frac{(0.15)(0.03)^3}{12} + (0.03)(0.15)(0.045)^2\right]$$

$$= 1.755 \times 10^{-5} + 9.45 \times 10^{-6}$$

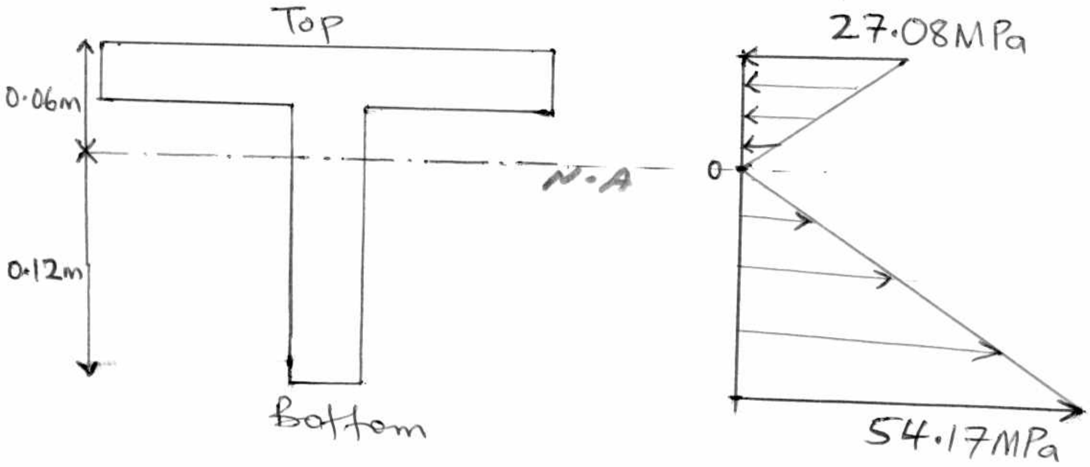
$$= 2.7 \times 10^{-5} \, m^4$$

- Stresses:

$$\sigma_{top} = -\frac{[12.1875 \times 10^3][0.06]}{2.7 \times 10^{-5}} = -27.08 MPa \text{ "Compression"}$$

$$\sigma_{bottom} = -\frac{[12.1875 \times 10^3][-0.12]}{2.7 \times 10^{-5}} = +54.17 MPa \text{ "Tension"}$$

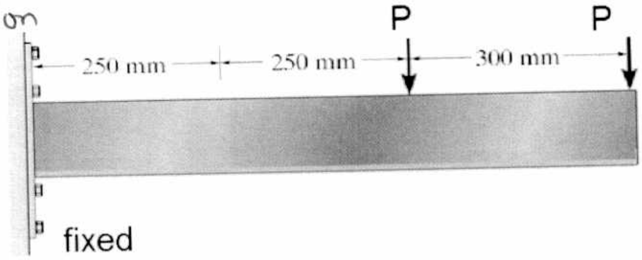
- Stress distribution.



PROBLEM #3

Given:

- The beam and its Cross-section as shown
- $\sigma_{ultimate} = 180\,MPa$
- $F.S. = 3$

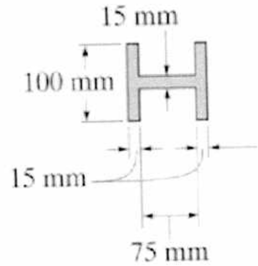


Required:

a) $P_{max.}$ for H-shape Cross-section

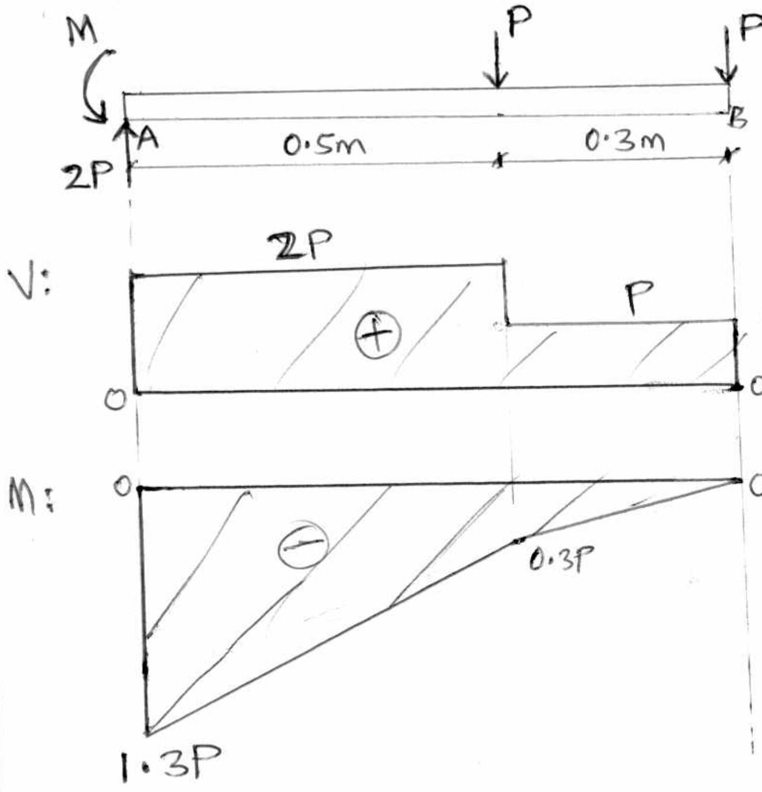
b) $P_{max.}$ for I-shape Cross-section

c) To Compare (a) and (b)



SOLUTION

$$\sigma_{allowable} = \frac{\sigma_{ultimate}}{F.S.} = \frac{180}{3} = 60\,MPa$$



$+\circlearrowleft \Sigma M = 0;$

$-P(0.5) - P(0.8) + M = 0$

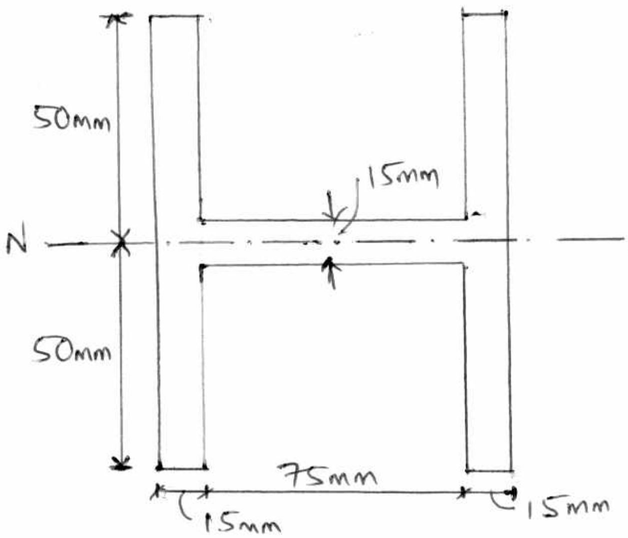
$\therefore M = 1.3P$

$+\uparrow \Sigma F_y = 0;\ A_y - P - P = 0$

$\therefore A_y = 2P$

Hence, $M_{max} = 1.3P$

a) For H-cross section



$$I_{N.A.} = 2\left[\frac{(0.015)(0.1)^3}{12}\right] + \left[\frac{0.075 \times 0.015^3}{12}\right]$$

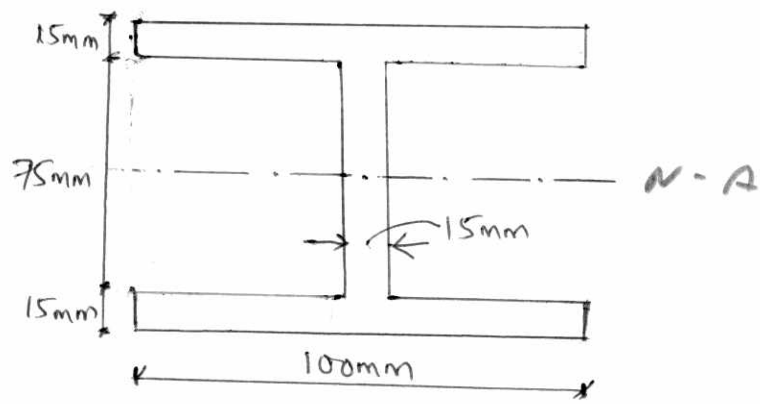
$$= 2.5 \times 10^{-6} + 2.109375 \times 10^{-8}$$

$$= 2.52109375 \times 10^{-6}\, m^4$$

$$\Rightarrow \sigma_{allow} = \sigma_{max} = 60 \times 10^6 = \frac{1.3P\,(0.05)}{2.52109375 \times 10^{-6}}$$

$$\Rightarrow P_{max} = 2.327\,KN$$

b) For I- cross section,



$$I_{N.A} = 2\left[\frac{(0.1)(0.015)^3}{12} + (0.1)(0.015)(0.045)^2\right] + \frac{(0.015)(0.075)^3}{12}$$

$$= 6.13125\times10^{-6} + 5.2734375\times10^{-7}$$

$$= 6.65859375\times10^{-6}\, m^4$$

$$\sigma_{allow} = \sigma_{max.} = 60\times10^6 = \frac{(1.3P)(0.0525)}{6.65859375\times10^{-6}}$$

$$\Rightarrow P_{max.} = 5.854\, KN$$

c) Since $P_{max(I)} > P_{max(H)}$, hence the I-shape Cross section is better.

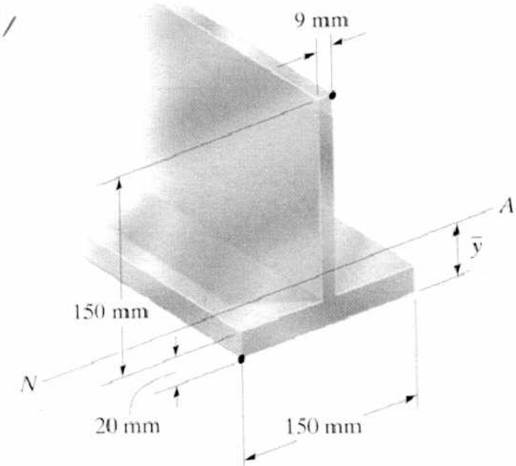
PROBLEM #4

Given:

The beam cross section shown,

$\sigma_{allow(tension)} = 40\,MPa$

$\sigma_{allow(Compr.)} = 70\,MPa$

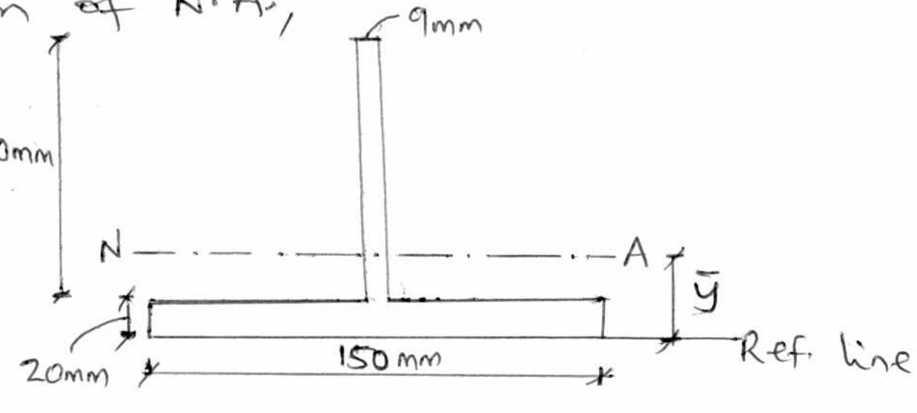


Required

- Positive M_{max}.
- Negative M_{max}.

SOLUTION

- Location of N.A.,



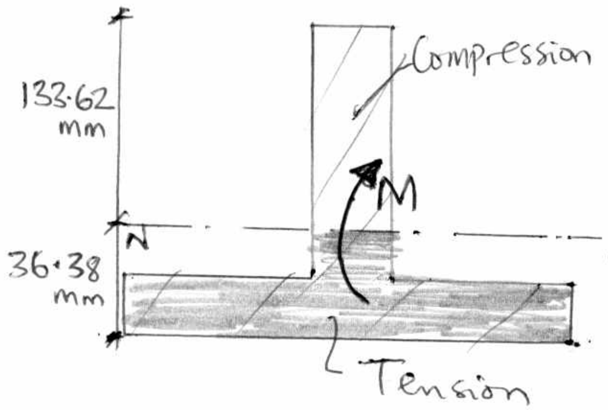
$$\bar{y} = \frac{(0.15)(0.02)(0.01) + (0.009)(0.15)(0.095)}{(0.15)(0.02) + (0.009)(0.15)}$$

$$= 0.03638\,m$$

$$I_{N.A.} = \left[\frac{(0.15)(0.02)^3}{12} + (0.15)(0.02)(0.02638)^2\right] + \left[\frac{(0.009)(0.15)^3}{12} + (0.009)(0.15)(0.05862)^2\right]$$

$$= 9.35797 \times 10^{-6}\,m^4$$

– Positive moment



$$\sigma_{max\,(tension)} = \sigma_{allow\,(tension)} = 40\times10^{6} = \frac{M\times0.03638}{9.35797\times10^{-6}}$$

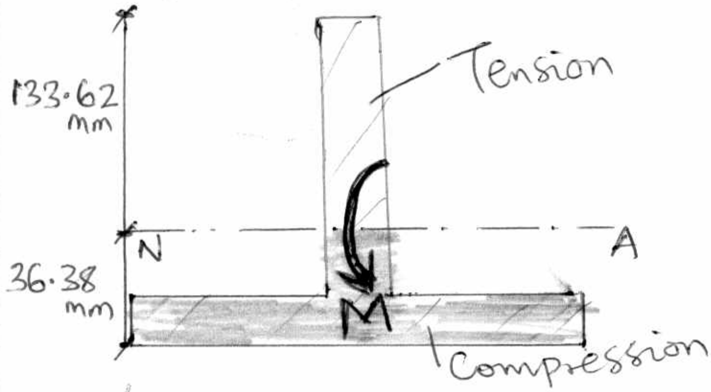
$$\Rightarrow M_1 = 10.289\,KN.m$$

$$\sigma_{allow\,(Compr.)} = 70\times10^{6} = \frac{M\times0.13362}{9.35797\times10^{-6}}$$

$$\Rightarrow M_2 = 4.902\,KNm$$

Hence, $\boxed{\text{Positive } M_{max} = 4.902\,KNm}$

– Negative moment



$$\sigma_{max.\,(tension)} = \sigma_{allow\,(tension)} = 40\times10^{6} = \frac{M\times0.13362}{9.35797\times10^{-6}}$$

$$\therefore M_1 = 2.801\,KNm$$

$$\sigma_{allow\,(Compr.)} = 70\times10^{6} = \frac{M\times0.03638}{9.35797\times10^{-6}}$$

$$\Rightarrow M_2 = 18.006\,KNm$$

Hence, $\boxed{\text{Negative } M_{max.} = 2.801\,KNm}$