Key Solution HW # 14

1 (9-70)

$\sigma_x = 350$ MPa

$\sigma_y = -200$ MPa

$\tau_{xy} = 300$ MPa

center = (75, 0)

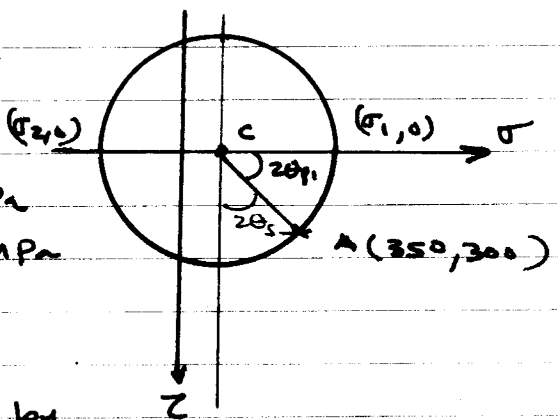
radius = 407 MPa

Principal stresses are

$\sigma_1 = 407 + 75 = 482$ MPa

$\sigma_2 = 75 - 407 = -332$ MPa

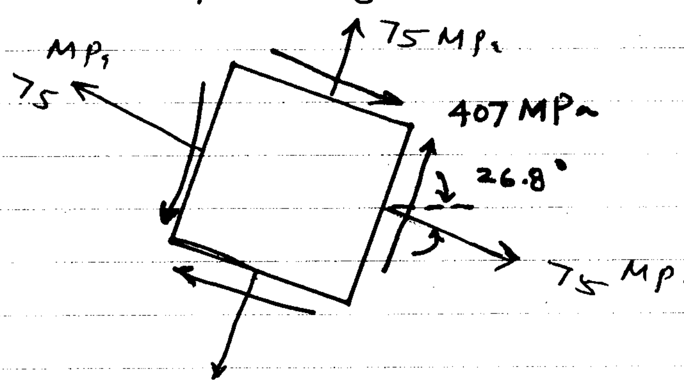
$\tau_{max} = 407$ MPa



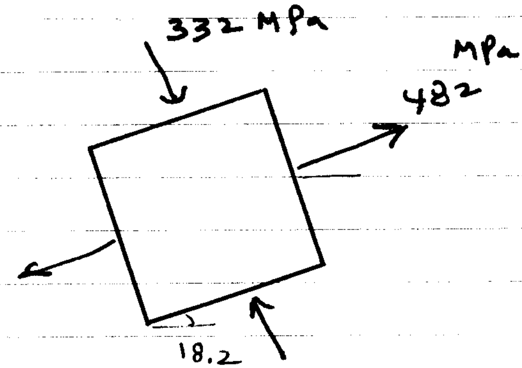
orientations are given by

$$\tan 2\theta_{p_1} = \frac{300}{R} \Rightarrow \theta_{p_1} = 18.2^\circ \text{ CCW}$$

$$\theta_s = 26.8 \text{ CW}$$



max shear

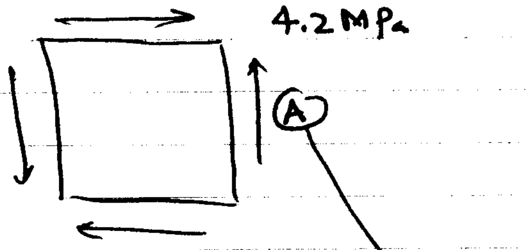


Principal stress

2 (9.75)

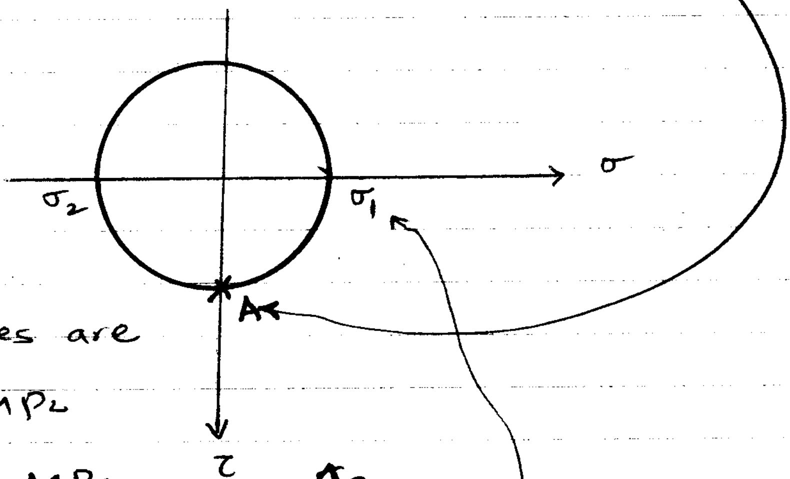
$$\tau_{xy} = \frac{5\ \mathrm{kN/m}}{12\ \mathrm{mm}} = \frac{50}{12} = 4.2\ \mathrm{MPa}$$

$$\sigma_x = \sigma_y = 0$$



Construct Mohr circle

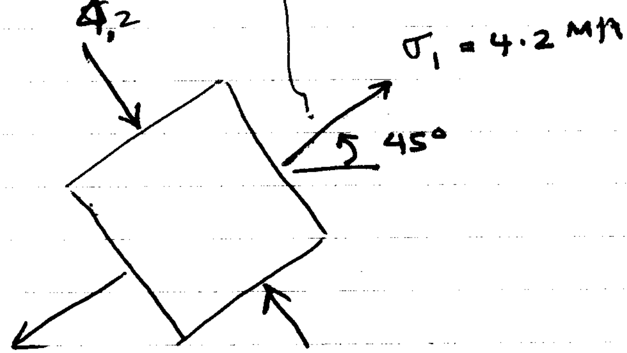
center $(0,0)$; $R = 4.2$



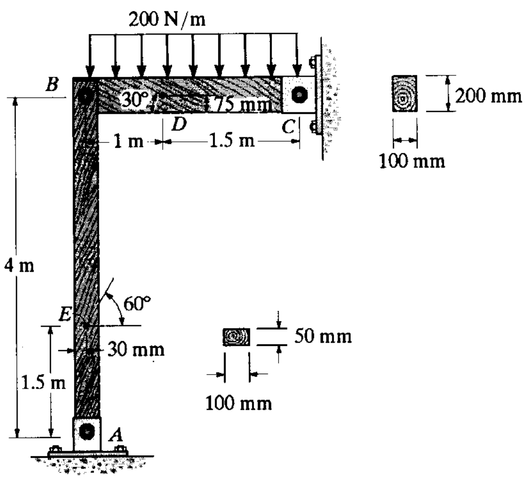
Principal stresses are

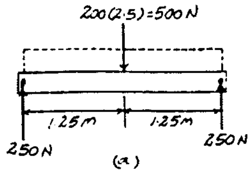
$\sigma_1 = 4.2\ \mathrm{MPa}$

$\sigma_2 = -4.2\ \mathrm{MPa}$



9–85. The frame supports the distributed loading of 200 N/m. Determine the normal and shear stresses at point D that act perpendicular and parallel, respectively, to the grains. The grains at this point make an angle of 30° with the horizontal as shown.

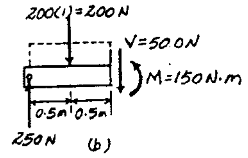




Support Reactions: As shown on FBD(a).

Internal Forces and Moment: As shown on FBD(b).

Section Properties:

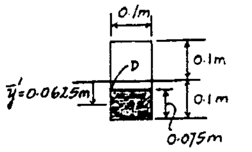


$$I = \frac{1}{12}(0.1)(0.2^3) = 66.667(10^{-6})\ \text{m}^4$$

$$Q_D = \bar{y}'A' = 0.0625(0.075)(0.1) = 0.46875(10^{-3})\ \text{m}^3$$

Normal Stress: Applying the flexure formula,

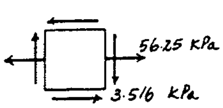
$$\sigma_D = -\frac{My}{I} = -\frac{150(-0.025)}{66.667(10^{-6})} = 56.25\ \text{kPa}$$



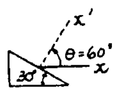
Shear Stress: Applying the shear formula,

$$\tau_D = \frac{VQ_D}{It} = \frac{50.0\left[0.46875(10^{-3})\right]}{66.667(10^{-6})(0.1)} = 3.516\ \text{kPa}$$

Construction of the Circle: In accordance to the established sign convention, $\sigma_x = 56.25$ kPa, $\sigma_y = 0$ and $\tau_{xy} = -3.516$ kPa. Hence,



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{56.25 + 0}{2} = 28.125\ \text{kPa}$$

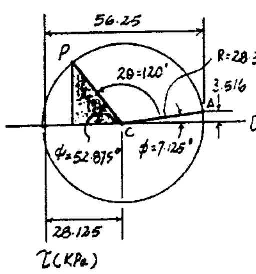


The coordinates for reference point A and C are

$$A(56.25,\ -3.516) \qquad C(28.125,\ 0)$$

The radius of the circle is

$$R = \sqrt{(56.25 - 28.125)^2 + 3.516^2} = 28.3439\ \text{kPa}$$



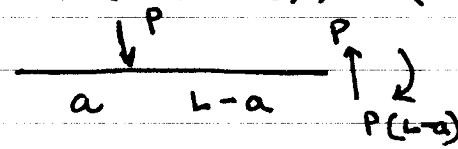
Stresses on The Rotated Element: The normal and shear stress components ($\sigma_{x'}$ and $\tau_{x'y'}$) are represented by the coordinates of point P on the circle. Here, $\theta = 60°$.

$$\sigma_{x'} = 28.125 - 28.3439\cos 52.875° = 11.0\ \text{kPa} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = -28.3439\sin 52.875° = -22.6\ \text{kPa} \qquad \textbf{Ans}$$

12.5

Find Rxn at fixed End



In region I

$$EI \frac{d^2 v_1}{dx_1^2} = M_1(x_1) = 0$$

integrate to get

$$EI\ v_1 = c_1 x_1 + c_2 \qquad — ①$$

In region II

$$EI \frac{d^2 v_2}{dx_2^2} = M_2(x) = P x_2 - P(L-a)$$

integrate to get

$$EI\ v_2 = \frac{P}{6} x_2^3 - \frac{P}{2}(L-a)\, x^2 + c_3 x + c_4 \qquad ②$$

In equation ① & ② we have 4 Konstants to be evaluated from 2 BC + 2 CE

$$v_2(0) = 0 \ ; \ \frac{dv_2}{dx_2}(0) = 0 \ ; \ v_1(a) = v_2(L-a)$$

$$\text{and} \quad v_1'(a) = -v_2'(L-a)$$

solve to get

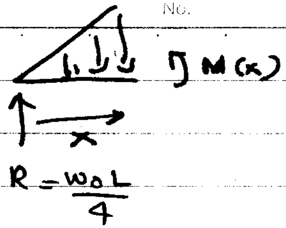
$$v_1(x_1) = \frac{P}{6EI}\left[3(L-a)^2 x_1 - 3a(L-a)^2 - 2(L-a)^3\right]$$

and

$$v_2(x_2) = \frac{P}{6EI}\left[x_2^3 - 3(L-a)\, x_2^2\right]$$

max def obviously at free end $= -\dfrac{P(L-a)^3}{3EI}$

5 (12.27)



$$M(x) = \frac{w_0 L}{4} x - \frac{w_0}{3L} x^3$$

used 2nd order GDE

$$EI \frac{d^2 v}{dx^2} = M(x) = \frac{w_0 L}{4} x - \frac{w_0}{3L} x^3$$

Integrate and make use of symmetry i.e. max def at middle

$$EI \frac{dv}{dx} = \frac{w_0 L}{8} x^2 - \frac{w_0}{12L} x^4 + C_1 \quad \text{— (1)}$$

BC #1 $\frac{dv}{dx}\left(\frac{L}{2}\right) = 0 \Rightarrow C_1 = \frac{-5 w_0 L^3}{192}$

substitute and integrate (1) then evaluate

BC #2 $v(0) = 0 \Rightarrow C_2 = 0$

$$\therefore \; v(x) = \frac{w_0 x}{960 EIL}\left(40 L^2 x^2 - 16 x^4 - 25 L^4\right)$$

Max def at $x = \frac{L}{2}$

$$v_{max} = -\frac{w_0 L^4}{120 EI}$$