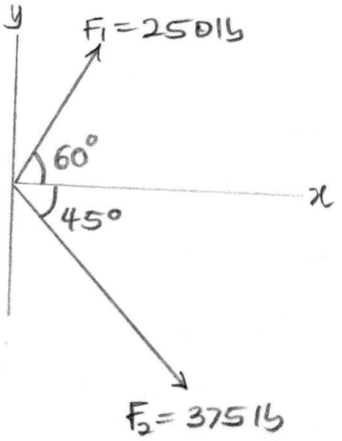
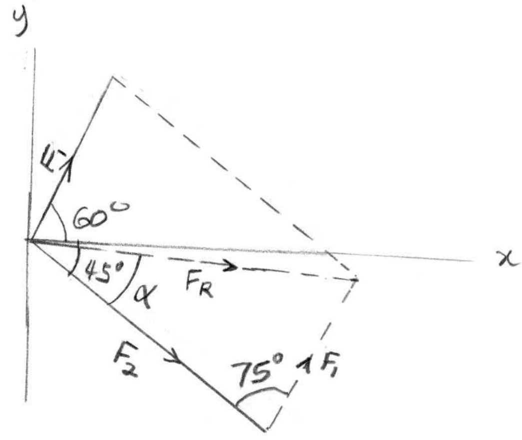
CE 202 — Statics and Strength of Materials

Homework #1 Solution

1.



1(a)



1(b)

From fig 1(b), F_R is the resultant of F_1 & F_2 both in magnitude and direction as given by triangle/parallelogram law of vector addition.

Thus

$$|\bar{F}_R|^2 = |\bar{F}_1|^2 + |\bar{F}_2|^2 - 2|\bar{F}_1||\bar{F}_2| \cos 75° \quad \text{(Cosine rule)}$$

$$= 250^2 + 375^2 - 2(250)(375)\cos 75°$$

$$= 154596.429$$

$$|F_R| = \sqrt{154596.429} = 393.1875 \text{ lb} \quad \text{(Magnitude)}$$

Direction

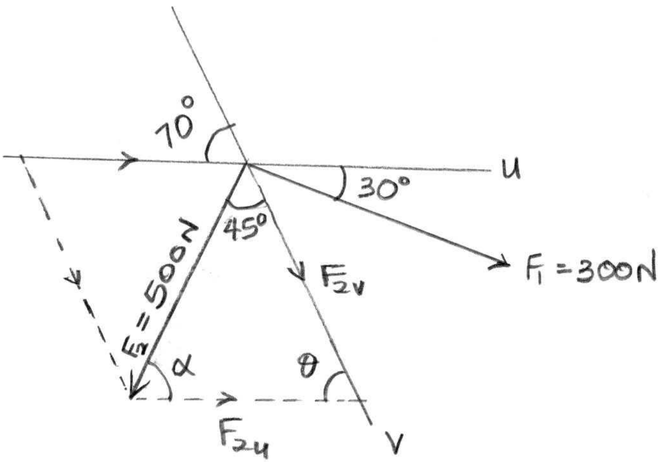
$$\frac{|F_1|}{\sin\alpha} = \frac{|F_R|}{\sin 75°} \quad \text{(Sine rule)}$$

$$\sin\alpha = \frac{|F_1| \sin 75°}{|F_R|} = \frac{250 \sin 75°}{393.1875} = 0.6142$$

$$\alpha = \sin^{-1} 0.6142 = 37.89°$$

Direction $= 270° + 45° + \alpha = 270° + 45 + 37.89° = 352.89°$ counterclockwise from +ve x-axis

2.



From the figure above,

$\theta = 70°$ (alternate angles)

$45° + 70° + \alpha = 180$ (sum of angles in a Δ)

$\alpha = 65°$

Using sine rule

$$\frac{F_{2u}}{\sin 45°} = \frac{F_2}{\sin \theta} \Rightarrow F_{2u} = \frac{F_2 \sin 45}{\sin \theta}$$

$$F_{2u} = \frac{500 \sin 45°}{\sin 70°}$$

$= 376.2437\,N$ — In the positive u direction

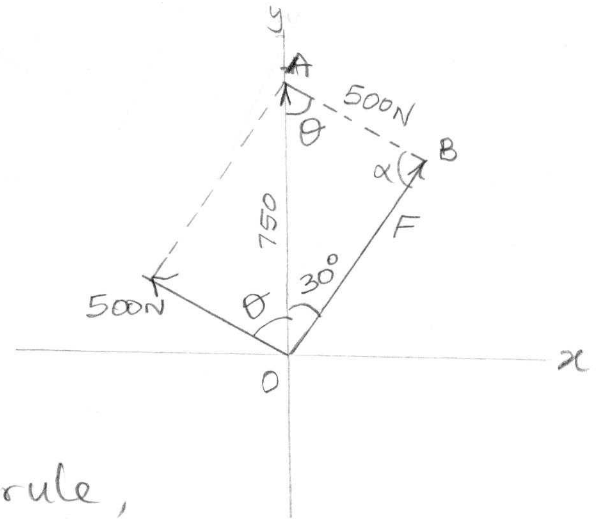
In the same way,

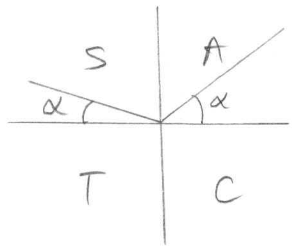
$$\frac{F_{2v}}{\sin 65°} = \frac{500}{\sin 70°} \Rightarrow F_{2v} = \frac{500 \sin 65°}{\sin 70°}$$

$F_{2v} = 482.2363\,N$ — in the negative v direction

Hence the component of F_2 acting along the u and v axes have the magnitudes 376.2437N and 482.2363N respectively.

3.





Using sine rule,

$$\frac{500}{\sin 30^\circ} = \frac{750}{\sin \alpha} \Rightarrow \sin \alpha = \frac{750 \sin 30^\circ}{500}$$

$$\sin \alpha = 0.75 \Rightarrow \alpha = 48.59^\circ \text{ or } 131.41^\circ$$

Summing angles in ΔOAB,

$$\theta + \alpha + 30 = 180^\circ, \text{ or } \theta = 150^\circ - \alpha$$

$$\Rightarrow \theta = 150^\circ - 48.59^\circ \text{ or } \theta = 150^\circ - 131.41^\circ$$

$$\theta = 101.41^\circ \text{ or } \theta = 18.59^\circ$$

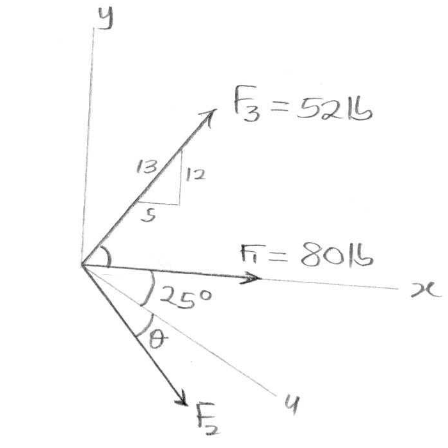
Since $0^\circ \leq \theta \leq 90^\circ$, we choose $\theta = \underline{18.59^\circ}$

Using sine rule again,

$$\frac{F}{\sin\theta} = \frac{500}{\sin 30^\circ} \Rightarrow F = \frac{500 \sin 18.59}{\sin 30^\circ}$$

$$\therefore \underline{F = 318.79\,N}$$

4.



Resolve the forces into their x and y components

Given $\theta = 55°$

$F_2 = 150$ lb

x-Components:

$F_{1x} = +80$ lb

$F_{2x} = +150\cos(25+55) = 26.0472$ lb

$F_{3x} = +52\left(\frac{5}{13}\right) = 20$ lb

$\Sigma F_x = 80 + 26.0472 + 20 = +126.0472$ lb

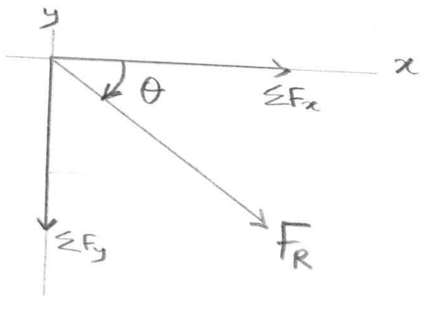
y-Components

$F_{1y} = 0$

$F_{2y} = -150\sin(25+55) = -147.7212$ lb

$F_{3y} = +52\left(\frac{12}{13}\right) = 48$ lb

$\Sigma F_y = -147.7212 + 48 = -99.7212$ lb



Resultant of the forces, F_R is given by

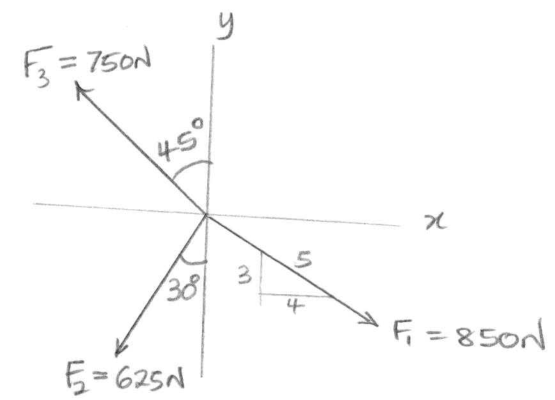
$F_R = \sqrt{[(\Sigma F_x)^2 + (\Sigma F_y)^2]} = \sqrt{126.0472^2 + (-99.7212)^2} = 160.724$ lb

Direction

$\tan\theta = \frac{|\Sigma F_y|}{|\Sigma F_x|} = \frac{99.7212}{126.0472} = 0.7911$

$\theta = 38.35°$, measured clockwise from the x-axis

5.



We resolve the three forces into their x- and y-components

x-components

$F_{1x} = +850(\frac{4}{5}) = +680N$

$F_{2x} = -625 \sin 30° = -312.5N$

$F_{3x} = -750 \sin 45° = -530.33N$

$\Sigma F_x = 680 - 312.5 - 530.33$

$= -162.83 N$

y-components

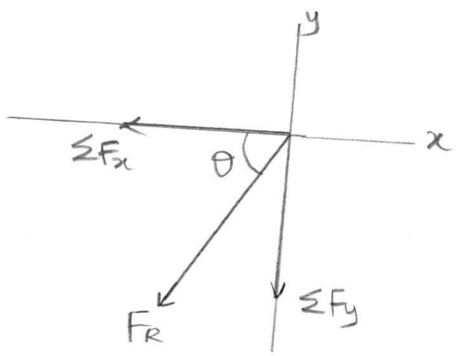
$F_{1y} = -850(\frac{3}{5}) = -510N$

$F_{2y} = -625 \cos 30° = -541.2659N$

$F_{3y} = +750 \cos 45° = 530.33N$

$\Sigma F_y = -510 - 541.2659 + 530.33$

$= -520.9359N$



$F_R = \sqrt{[(\Sigma F_x)^2 + (\Sigma F_y)^2]}$

$= [(-162.83)^2 + (-520.9359)^2]^{1/2}$

$= 545.791N$

Direction

$\tan \theta = \frac{|\Sigma F_y|}{|\Sigma F_x|} = \frac{520.9359}{162.83} = 3.1993$

$\theta = 72.64°$

∴ The resultant force direction measured counter-clockwise from +ve x-axis $= 180° + 72.64°$

$= \underline{252.64°}$