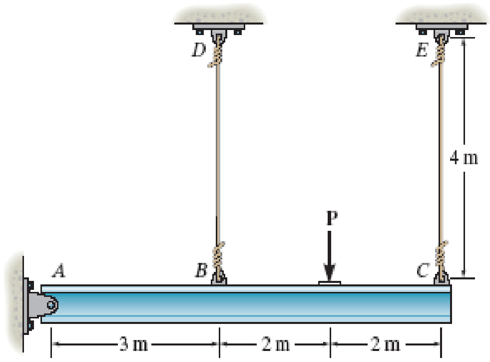
HW#3 Key Solutions

2–3. The rigid beam is supported by a pin at A and wires BD and CE. If the load **P** on the beam causes the end C to be displaced 10 mm downward, determine the normal strain developed in wires CE and BD.



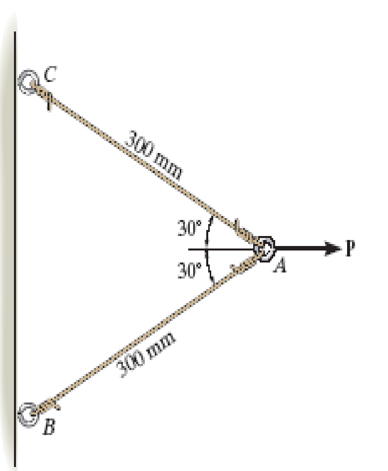
$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}$$

$$\Delta L_{BD} = \frac{3\,(10)}{7} = 4.286 \text{ mm}$$

$$\varepsilon_{CE} = \frac{\Delta L_{CE}}{L} = \frac{10}{4000} = 0.00250 \text{ mm/mm}$$ **Ans.**

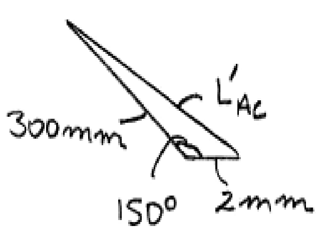
$$\varepsilon_{BD} = \frac{\Delta L_{BD}}{L} = \frac{4.286}{4000} = 0.00107 \text{ mm/mm}$$ **Ans.**

***2–4.** The two wires are connected together at A. If the force **P** causes point A to be displaced horizontally 2 mm, determine the normal strain developed in each wire.

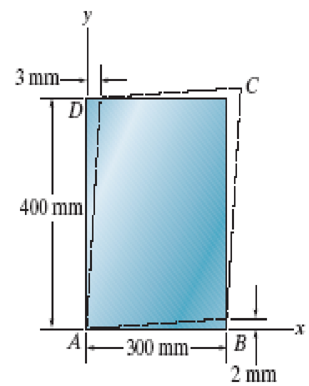


$$L'_{AC} = \sqrt{300^2 + 2^2 - 2(300)(2)\cos 150^\circ} = 301.734 \text{ mm}$$

$$\varepsilon_{AC} = \varepsilon_{AB} = \frac{L'_{AC} - L_{AC}}{L_{AC}} = \frac{301.734 - 300}{300} = 0.00578 \text{ mm/mm}$$ **Ans.**



*2–12. The piece of rubber is originally rectangular. Determine the average shear strain γ_{xy} at A if the corners B and D are subjected to the displacements that cause the rubber to distort as shown by the dashed lines.

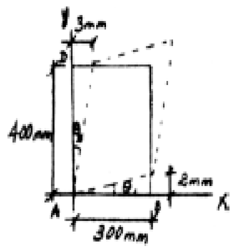


$$\theta_1 = \tan \theta_1 = \frac{2}{300} = 0.006667 \text{ rad}$$

$$\theta_2 = \tan \theta_2 = \frac{3}{400} = 0.0075 \text{ rad}$$

$$\gamma_{xy} = \theta_1 + \theta_2$$

$$= 0.006667 + 0.0075 = 0.0142 \text{ rad}$$ **Ans.**



•3–1. A concrete cylinder having a diameter of 6.00 in. and gauge length of 12 in. is tested in compression. The results of the test are reported in the table as load versus contraction. Draw the stress–strain diagram using scales of 1 in. = 0.5 ksi and 1 in. = $0.2(10^{-3})$ in./in. From the diagram, determine approximately the modulus of elasticity.

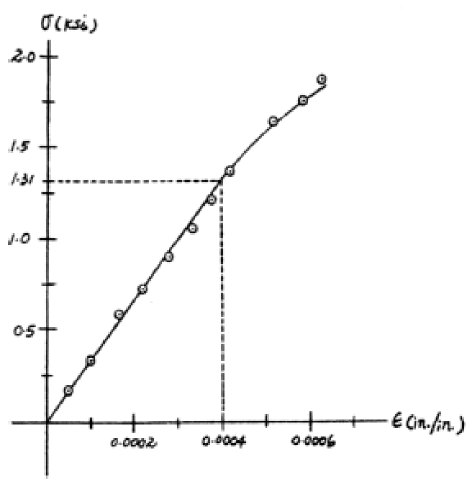
Load (kip)	Contraction (in.)
0	0
5.0	0.0006
9.5	0.0012
16.5	0.0020
20.5	0.0026
25.5	0.0034
30.0	0.0040
34.5	0.0045
38.5	0.0050
46.5	0.0062
50.0	0.0070
53.0	0.0075

Stress and Strain:

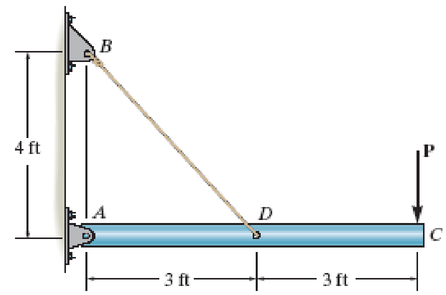
$\sigma = \dfrac{P}{A}$(ksi)	$\varepsilon = \dfrac{\delta L}{L}$(in./in.)
0	0
0.177	0.00005
0.336	0.00010
0.584	0.000167
0.725	0.000217
0.902	0.000283
1.061	0.000333
1.220	0.000375
1.362	0.000417
1.645	0.000517
1.768	0.000583
1.874	0.000625

Modulus of Elasticity: From the stress–strain diagram

$$E_{\text{approx}} = \frac{1.31 - 0}{0.0004 - 0} = 3.275\left(10^3\right) \text{ ksi} \qquad \textbf{Ans.}$$



3–15. The rigid pipe is supported by a pin at A and an A-36 guy wire BD. If the wire has a diameter of 0.25 in., determine the load P if the end C is displaced 0.075 in. downward.



Here, we are only interested in determining the force in wire BD. Referring to the FBD in Fig. a

$$\zeta + \Sigma M_A = 0; \qquad F_{BD}\left(\tfrac{4}{5}\right)(3) - P(6) = 0 \qquad F_{BD} = 2.50\,P$$

The unstretched length for wire BD is $L_{BD} = \sqrt{3^2 + 4^2} = 5\text{ ft} = 60\text{ in}$. From the geometry shown in Fig. b, the stretched length of wire BD is

$$L_{BD'} = \sqrt{60^2 + 0.075^2 - 2(60)(0.075)\cos 143.13°} = 60.060017$$

Thus, the normal strain is

$$\epsilon_{BD} = \frac{L_{BD'} - L_{BD}}{L_{BD}} = \frac{60.060017 - 60}{60} = 1.0003(10^{-3})\text{ in./in.}$$

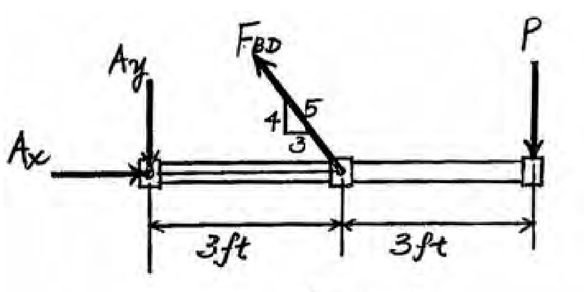
Then, the normal stress can be obtain by applying Hooke's Law.

$$\sigma_{BD} = E\epsilon_{BD} = 29(10^3)\left[1.0003(10^{-3})\right] = 29.01\text{ ksi}$$

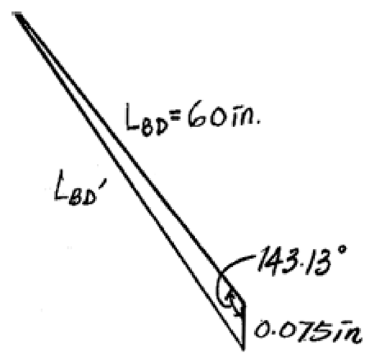
Since $\sigma_{BD} < \sigma_y = 36$ ksi, the result is valid.

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}}; \qquad 29.01(10^3) = \frac{2.50\,P}{\frac{\pi}{4}(0.25^2)}$$

$$P = 569.57\text{ lb} = 570\text{ lb} \qquad \textbf{Ans.}$$

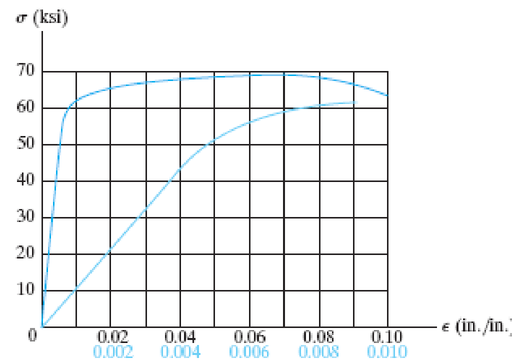


(a)



(b)

3–17. A tension test was performed on an aluminum 2014-T6 alloy specimen. The resulting stress–strain diagram is shown in the figure. Estimate (a) the proportional limit, (b) the modulus of elasticity, and (c) the yield strength based on a 0.2% strain offset method.



Proportional Limit and Yield Strength: From the stress–strain diagram, Fig. *a*,

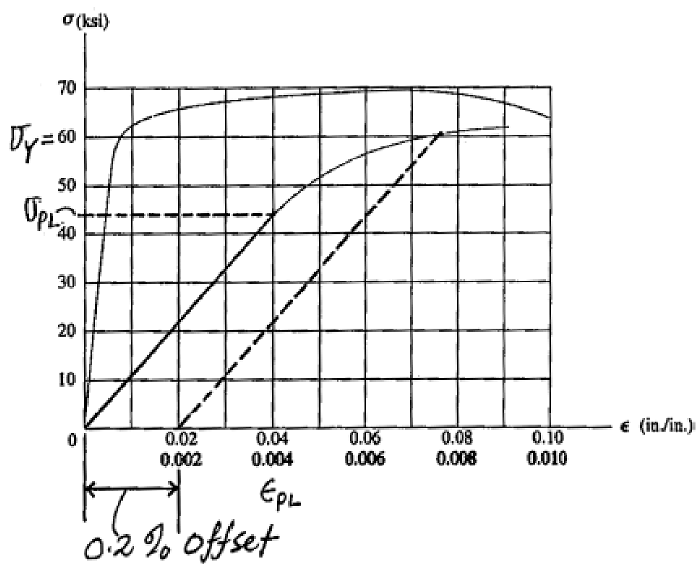
$$\sigma_{pl} = 44 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_Y = 60 \text{ ksi} \qquad \textbf{Ans.}$$

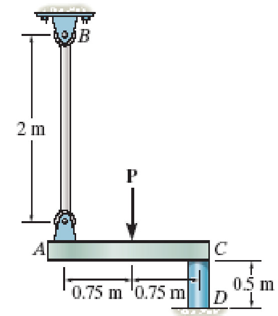
Modulus of Elasticity: From the stress–strain diagram, the corresponding strain for $\sigma_{PL} = 44$ ksi is $\varepsilon_{pl} = 0.004$ in./in. Thus,

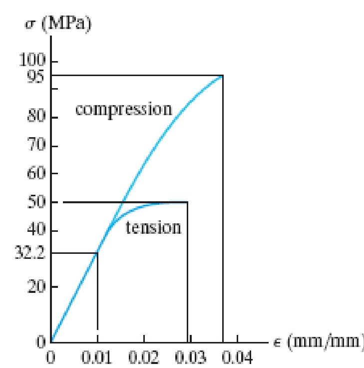
$$E = \frac{44 - 0}{0.004 - 0} = 11.0(10^3) \text{ ksi} \qquad \textbf{Ans.}$$

Modulus of Resilience: The modulus of resilience is equal to the area under the



3–22. The stress–strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut AB and post CD made from this material, determine the largest load P that can be applied to the beam before it ruptures. The diameter of the strut is 12 mm and the diameter of the post is 40 mm.





Rupture of strut AB:

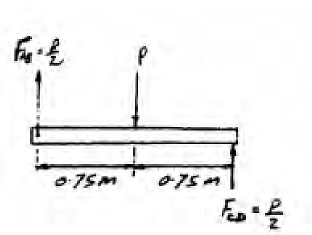
$$\sigma_R = \frac{F_{AB}}{A_{AB}}; \qquad 50(10^6) = \frac{P/2}{\frac{\pi}{4}(0.012)^2};$$

$$P = 11.3 \text{ kN (controls)}$$ **Ans.**

Rupture of post CD:

$$\sigma_R = \frac{F_{CD}}{A_{CD}}; \qquad 95(10^6) = \frac{P/2}{\frac{\pi}{4}(0.04)^2}$$

$$P = 239 \text{ kN}$$



3–38. A short cylindrical block of 6061-T6 aluminum, having an original diameter of 20 mm and a length of 75 mm, is placed in a compression machine and squeezed until the axial load applied is 5 kN. Determine (a) the decrease in its length and (b) its new diameter.

a) $$\sigma = \frac{P}{A} = \frac{-5(10^3)}{\frac{\pi}{4}(0.02)^2} = -15.915 \text{ MPa}$$

$$\sigma = E\,\varepsilon_{long}; \qquad -15.915(10^6) = 68.9(10^9)\,\varepsilon_{long}$$

$$\varepsilon_{long} = -0.0002310 \text{ mm/mm}$$

$$\delta = \varepsilon_{long} L = -0.0002310(75) = -0.0173 \text{ mm}$$ **Ans.**

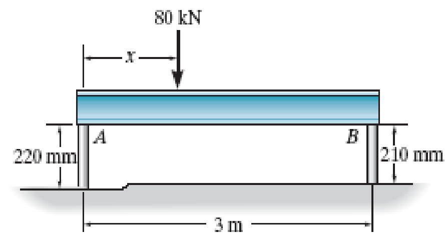
b) $$v = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}; \qquad 0.35 = -\frac{\varepsilon_{lat}}{-0.0002310}$$

$$\varepsilon_{lat} = 0.00008085 \text{ mm/mm}$$

$$\Delta d = \varepsilon_{lat}\, d = 0.00008085(20) = 0.0016 \text{ mm}$$

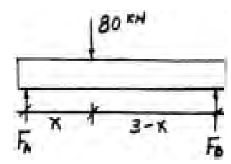
$$d' = d + \Delta d = 20 + 0.0016 = 20.0016 \text{ mm}$$ **Ans.**

3–39. The rigid beam rests in the horizontal position on two 2014-T6 aluminum cylinders having the *unloaded* lengths shown. If each cylinder has a diameter of 30 mm, determine the placement x of the applied 80-kN load so that the beam remains horizontal. What is the new diameter of cylinder A after the load is applied? $\nu_{al} = 0.35$.



$$\zeta + \Sigma M_A = 0; \quad F_B(3) - 80(x) = 0; \quad F_B = \frac{80x}{3} \qquad \textbf{(1)}$$

$$\zeta + \Sigma M_B = 0; \quad -F_A(3) + 80(3 - x) = 0; \quad F_A = \frac{80(3 - x)}{3} \qquad \textbf{(2)}$$



Since the beam is held horizontally, $\delta_A = \delta_B$

$$\sigma = \frac{P}{A}; \qquad \varepsilon = \frac{\sigma}{E} = \frac{\frac{P}{A}}{E}$$

$$\delta = \varepsilon L = \left(\frac{\frac{P}{A}}{E}\right) L = \frac{PL}{AE}$$

$$\delta_A = \delta_B; \qquad \frac{\frac{80(3 - x)}{3}(220)}{AE} = \frac{\frac{80x}{3}(210)}{AE}$$

$$80(3 - x)(220) = 80x(210)$$

$$x = 1.53 \text{ m} \qquad \textbf{Ans.}$$

From Eq. (2),

$$F_A = 39.07 \text{ kN}$$

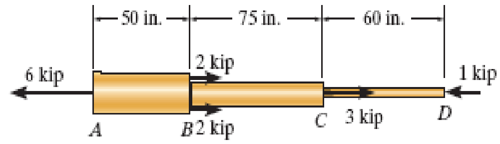
$$\sigma_A = \frac{F_A}{A} = \frac{39.07(10^3)}{\frac{\pi}{4}(0.03^2)} = 55.27 \text{ MPa}$$

$$\varepsilon_{long} = \frac{\sigma_A}{E} = -\frac{55.27(10^6)}{73.1(10^9)} = -0.000756$$

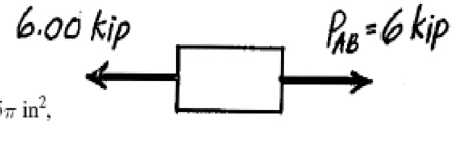
$$\varepsilon_{lat} = -\nu\varepsilon_{long} = -0.35(-0.000756) = 0.0002646$$

$$d'_A = d_A + d\,\varepsilon_{lat} = 30 + 30(0.0002646) = 30.008 \text{ mm} \qquad \textbf{Ans.}$$

4-2. The copper shaft is subjected to the axial loads shown. Determine the displacement of end A with respect to end D. The diameters of each segment are $d_{AB} = 3$ in., $d_{BC} = 2$ in., and $d_{CD} = 1$ in. Take $E_{cu} = 18(10^3)$ ksi.



The normal forces developed in segment AB, BC and CD are shown in the *FBDS* of each segment in Fig. *a*, *b* and *c* respectively.



(a)

The cross-sectional area of segment AB, BC and CD are $A_{AB} = \frac{\pi}{4}(3^2) = 2.25\pi$ in^2, $A_{BC} = \frac{\pi}{4}(2^2) = \pi$ in^2 and $A_{CD} = \frac{\pi}{4}(1^2) = 0.25\pi$ in^2.

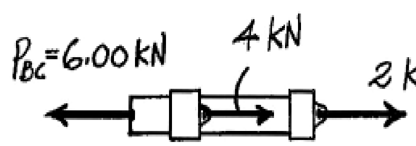
Thus,

$$\delta_{A/D} = \Sigma\frac{P_iL_i}{A_iE_i} = \frac{P_{AB}\,L_{AB}}{A_{AB}\,E_{Cu}} + \frac{P_{BC}\,L_{BC}}{A_{BC}\,E_{Cu}} + \frac{P_{CD}L_{CD}}{A_{CD}\,E_{Cu}}$$

$$= \frac{6.00\,(50)}{(2.25\pi)\left[18(10^3)\right]} + \frac{2.00\,(75)}{\pi\left[18(10^3)\right]} + \frac{-1.00\,(60)}{(0.25\pi)\left[18(10^3)\right]}$$

$$= 0.766(10^{-3})\text{ in.}$$ **Ans.**

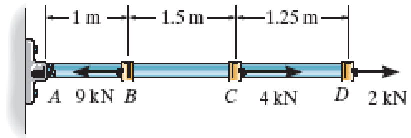
The positive sign indicates that end A moves away from D.



(b)

(c)

4–3. The A-36 steel rod is subjected to the loading shown. If the cross-sectional area of the rod is 50 mm^2, determine the displacement of its end D. Neglect the size of the couplings at B, C, and D.



The normal forces developed in segments AB, BC and CD are shown in the $FBDS$ of each segment in Fig. a, b and c, respectively.

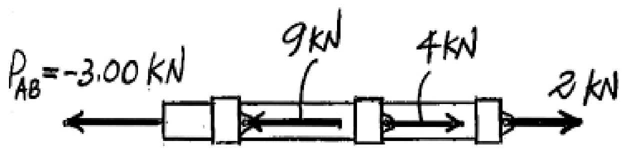
The cross-sectional areas of all the segments are $A = \left(50\text{ mm}^2\right)\left(\frac{1\text{ m}}{1000\text{ mm}}\right)^2 = 50.0(10^{-6})\text{ m}^2$.

$$\delta_D = \Sigma\frac{P_i L_i}{A_i E_i} = \frac{1}{A\,E_{SC}}\left(P_{AB}\,L_{AB} + P_{BC}\,L_{BC} + P_{CD}\,L_{CD}\right)$$

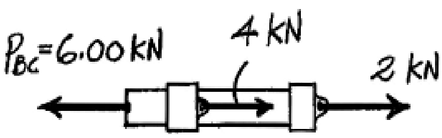
$$= \frac{1}{50.0(10^{-6})\left[200(10^9)\right]}\left[-3.00(10^3)(1) + 6.00(10^3)(1.5) + 2.00(10^3)(1.25)\right]$$

$$= 0.850(10^{-3})\text{ m} = 0.850\text{ mm}$$ **Ans.**

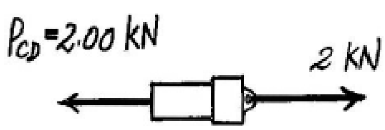
The positive sign indicates that end D moves away from the fixed support.



(a)

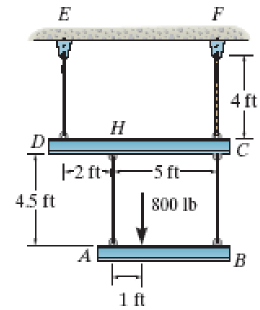


(b)



(c)

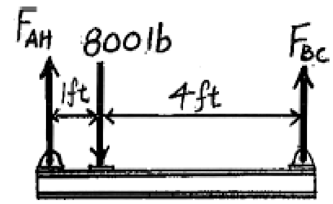
4–7. The load of 800 lb is supported by the four 304 stainless steel wires that are connected to the rigid members AB and DC. Determine the vertical displacement of the load if the members were horizontal before the load was applied. Each wire has a cross-sectional area of 0.05 in^2.



Referring to the *FBD* of member AB, Fig. *a*

$$\zeta + \Sigma M_A = 0; \qquad F_{BC}(5) - 800(1) = 0 \qquad F_{BC} = 160 \text{ lb}$$

$$\zeta + \Sigma M_B = 0; \qquad 800(4) - F_{AH}(5) = 0 \qquad F_{AH} = 640 \text{ lb}$$

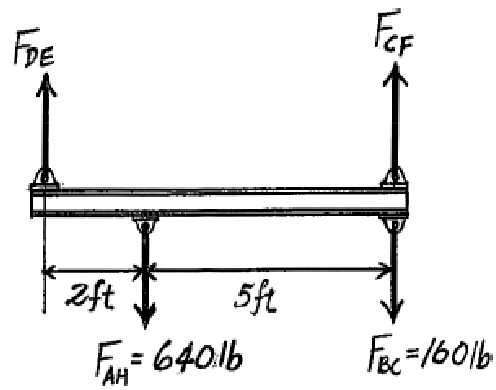


(a)

Using the results of F_{BC} and F_{AH}, and referring to the *FBD* of member DC, Fig. *b*

$$\zeta + \Sigma M_D = 0; \qquad F_{CF}(7) - 160(7) - 640(2) = 0 \qquad F_{CF} = 342.86 \text{ lb}$$

$$\zeta + \Sigma M_C = 0; \qquad 640(5) - F_{DE}(7) = 0 \qquad F_{DE} = 457.14 \text{ lb}$$



(b)

Since E and F are fixed,

$$\delta_D = \frac{F_{DE}\,L_{DE}}{A\,E_{st}} = \frac{457.14(4)(2)}{0.05\left[28.0\,(10^6)\right]} = 0.01567 \text{ in} \downarrow$$

$$\delta_C = \frac{F_{CF}\,L_{CF}}{A\,E_{st}} = \frac{342.86\,(4)(12)}{0.05\left[28.0\,(10^6)\right]} = 0.01176 \text{ in} \downarrow$$

From the geometry shown in Fig. *c*,

$$\delta_H = 0.01176 + \frac{5}{7}(0.01567 - 0.01176) = 0.01455 \text{ in} \downarrow$$

Subsequently,

$$\delta_{A/H} = \frac{F_{AH}\,L_{AH}}{A\,E_{st}} = \frac{640(4.5)(12)}{0.05\left[28.0(10^6)\right]} = 0.02469 \text{ in} \downarrow$$

$$\delta_{B/C} = \frac{F_{BC}\,L_{BC}}{A\,E_{st}} = \frac{160(4.5)(12)}{0.05\left[28.0(10^6)\right]} = 0.006171 \text{ in} \downarrow$$

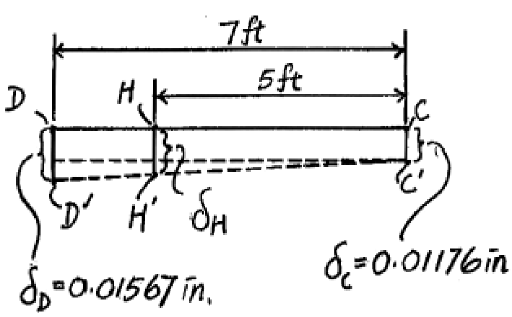
Thus,

$$(+\downarrow) \quad \delta_A = \delta_H + \delta_{A/H} = 0.01455 + 0.02469 = 0.03924 \text{ in} \downarrow$$

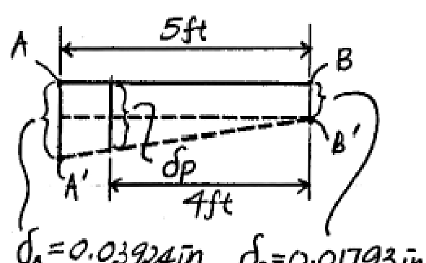
$$(+\downarrow) \quad \delta_B = \delta_C + \delta_{B/C} = 0.01176 + 0.006171 = 0.01793 \text{ in} \downarrow$$

From the geometry shown in Fig. *d*,

$$\delta_P = 0.01793 + \frac{4}{5}(0.03924 - 0.01793) = 0.0350 \text{ in} \downarrow \qquad \textbf{Ans.}$$

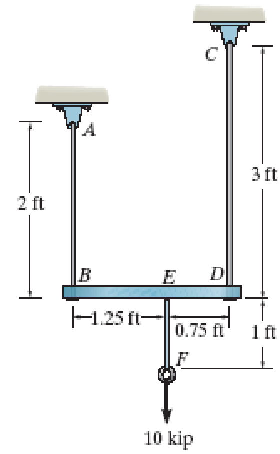


(c)



(d)

4–18. The assembly consists of two A-36 steel rods and a rigid bar BD. Each rod has a diameter of 0.75 in. If a force of 10 kip is applied to the bar as shown, determine the vertical displacement of the load.



Here, $F_{EF} = 10$ kip. Referring to the FBD shown in Fig. a,

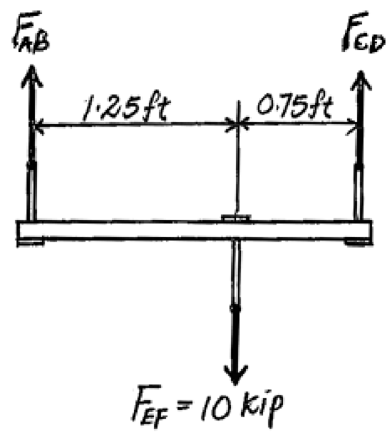
$$\zeta + \Sigma M_B = 0; \qquad F_{CD}(2) - 10(1.25) = 0 \qquad F_{CD} = 6.25 \text{ kip}$$

$$\zeta + \Sigma M_D = 0; \qquad 10(0.75) - F_{AB}(2) = 0 \qquad F_{AB} = 3.75 \text{ kip}$$

The cross-sectional area of the rods is $A = \frac{\pi}{4}(0.75^2) = 0.140625\pi \text{ in}^2$. Since points A and C are fixed,

$$\delta_B = \frac{F_{AB}\,L_{AB}}{A\,E_{st}} = \frac{3.75\,(2)(12)}{0.140625\pi\left[29.0(10^3)\right]} = 0.007025 \text{ in.} \downarrow$$

$$\delta_D = \frac{F_{CD}\,L_{CD}}{A\,E_{st}} = \frac{6.25(3)(12)}{0.140625\pi\left[29.0(10^3)\right]} = 0.01756 \text{ in} \downarrow$$



(a)

From the geometry shown in Fig. b

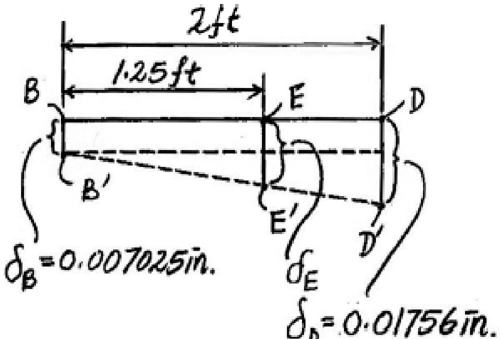
$$\delta_E = 0.007025 + \frac{1.25}{2}(0.01756 - 0.00725) = 0.01361 \text{ in.} \downarrow$$

Here,

$$\delta_{F/E} = \frac{F_{EF}\,L_{EF}}{A\,E_{st}} = \frac{10\,(1)\,(12)}{0.140625\pi\left[29.0(10^3)\right]} = 0.009366 \text{ in} \downarrow$$

Thus,

$$(+\downarrow) \quad \delta_F = \delta_E + \delta_{F/E} = 0.01361 + 0.009366 = 0.0230 \text{ in} \downarrow \qquad \textbf{Ans.}$$



(b)