King Fahd University of Petroleum & Minerals

CIVIL ENGINEERING DEPARTMENT

CE 203 STRUCTURAL MECHANICS I

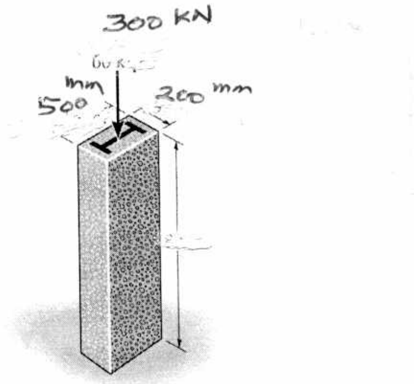
First Semester 2012 / 2013 (**121**)

HOMEWORK NO. 5 Solution

#1 (4.39 modified)

Req:

Steel area so that $F_{st} = F_{con}$

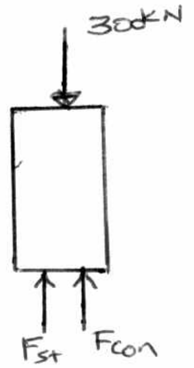


Compatibility requirement

$$\Delta_{st} = \Delta_{con}$$

in terms of forces

$$\frac{F_{st}\,L_{st}}{A_{st}\,E_{st}} = \frac{F_{con}\,L_{con}}{A_{con}\,E_{con}}$$



Since forces & lengths are equal

$$\frac{A_{con}}{A_{st}} = \frac{E_{st}}{E_{con}}$$

substituting known values

$$\frac{500 \times 200 - A_{st}}{A_{st}} = \frac{200}{29} \quad \text{— (1)}$$

Solving for steel area we get:

$$\underline{\underline{A_{st} = 12{,}664 \ mm^2}}$$

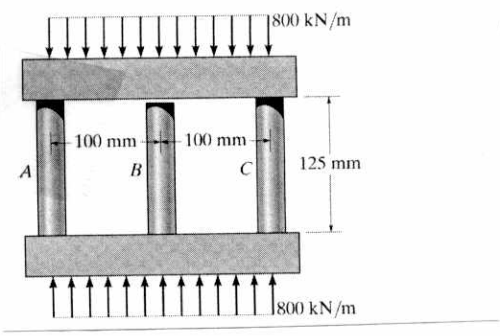
∵ The column will shorten by

$$\Delta_{st} = \frac{F_{st}\,L_{st}}{A_{st}\,E_{st}} = \frac{150\,(\quad)}{12664\,(200)} = 0.142^{mm}$$

2 (4.64 text modified)

Req:

Find stresses in steel & aluminum.

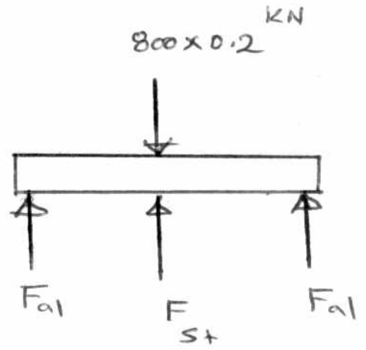


Assume gab closes and look at FBD of rigid beam

$\uparrow \Sigma F_y = 0$

$$2F_{al} + F_{st} = 160 \quad — (1)$$

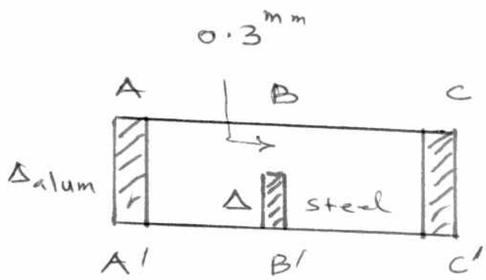
(note colum A & C carry the same force !)



Statically Indeterminate Problem

an extra equation can be obtained from deformation geometry

$$\Delta_{al} = 0.3 + \Delta_{st}$$



use force def. relation

$$\frac{F_{al}L_{al}}{A_{al}E_{al}} = 0.3 + \frac{F_{st}L_{st}}{A_{st}E_{st}}$$

$$\frac{F_{al}(125)}{400(70)} = 0.3 + \frac{F_{st}(125)}{400(200)}$$

which yields

$$F_{st} = \frac{2500}{875}F_{al} - 192 \quad — (2)$$

Solving (1) & (2) we get

$$F_{st} = 15^{\,kN} \; ; \; F_{al} = 72.5^{\,KN}$$

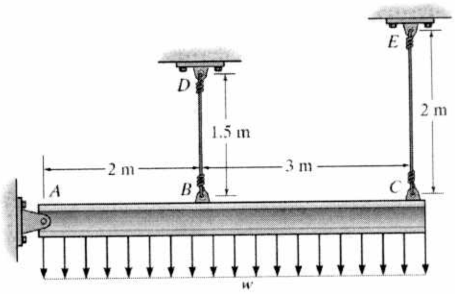
The stresses will be

$$\sigma_{st} = \frac{F_{st}}{A_{st}} = 37.5 \text{ MPa} \; ; \; \sigma_{al} = 181 \text{ MPa}$$

Note that stresses are compressive

3 Req:

Vertical displacement of point C.



From shown FBD

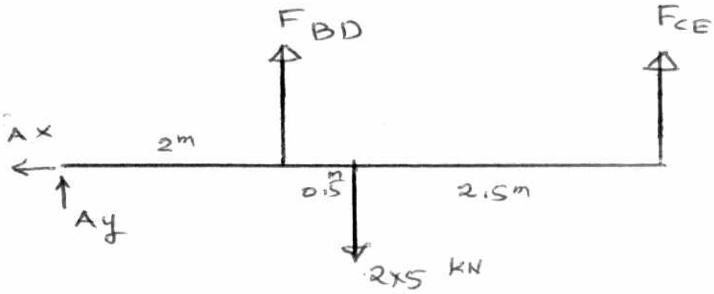
$\overset{+}{\circlearrowleft} \Sigma M_A = 0$

$5 F_{CE} + 2 F_{BD} - 10 \times 2.5 = 0$

$F_{CE} + 0.4 F_{BD} = 5 \quad —(1)$

Statically Indeterminate, need a compatibility equation.

use the given deformation geometry (similar triangles)

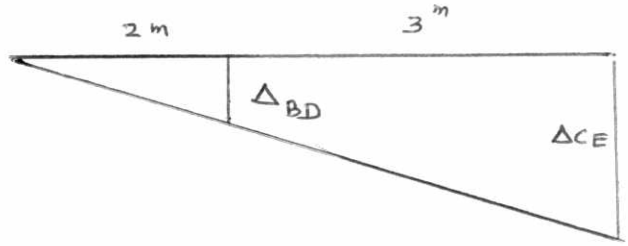


FBD

$$\frac{\Delta_{BD}}{2} = \frac{\Delta_{CE}}{5}$$

$\therefore \quad \Delta_{CE} = 2.5 \, \Delta_{BD}$

use force - def. relation



$$\frac{F_{CE} L_{CE}}{A_{CE} E_{CE}} = 2.5 \frac{F_{BD} L_{BD}}{A_{BD} E_{BD}}$$

$\therefore \quad F_{CE} = 1.875 F_{BD} \quad —(2)$

solving (1) & (2) we get

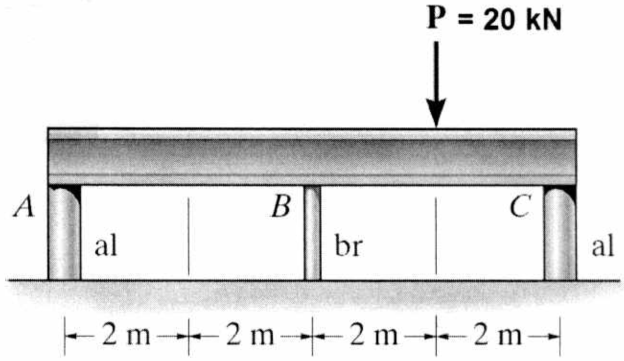
$F_{BD} = 2.2^{KN} \qquad F_{CE} = 4.12^{KN}$

Now calculate the vertical movement of Point C which is equal to Δ_{CE}

$$= \frac{F_{CE} L_{CE}}{A E} = \frac{4.12 (2000)}{\frac{\pi}{4}(15)^2 (200)} = 0.233^{mm}$$

#4. Required: stresses in rods

using Equlibrium equation applied to shown FBD



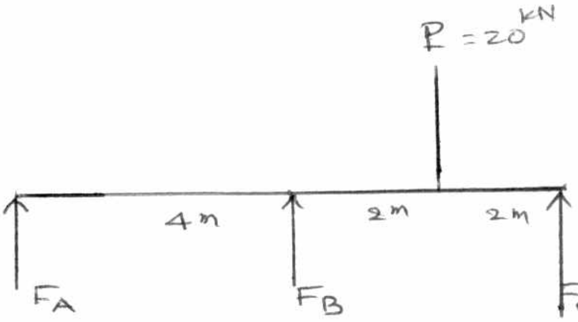
$\uparrow \Sigma F_y = 0$

$$F_A + F_B + F_C = 20 \quad —①$$

$\circlearrowleft \Sigma M_C = 0$

$$+4F_B + 8F_A - 20 \times 2 = 0$$

$$\Rightarrow F_B + 2F_A = 10 \quad —②$$



FBD

Statically indeterminate, get an extra equation from deformation geometry, which is

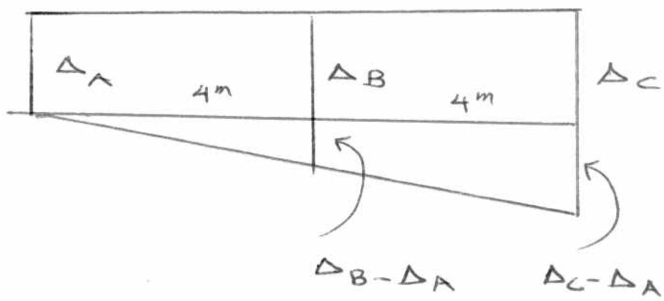
$$\frac{\Delta_C - \Delta_A}{8} = \frac{\Delta_B - \Delta_A}{4}$$

which simplifies to

$$\Delta_B = \frac{1}{2}(\Delta_A + \Delta_C)$$

using force-def. relation

$$\frac{F_B L_B}{\frac{\pi}{4}(d_B)^2 E_{Br}} = \frac{1}{2}\left[\frac{F_A L_A}{\frac{\pi}{4}(d_A)^2 E_{al}} + \frac{F_C L_C}{\frac{\pi}{4}(d_C)^2 E_{al}}\right]$$



Deformation geometry

substituting

$$5.333 F_B = F_A + F_C \quad —③$$

solving ①, ② and ③ we get

$F_A = 3.421$ kN ; $F_B = 3.158$ kN ; $F_C = 13.421$ kN

and

$$\sigma_A = \frac{F_A}{A_A} = 1.2 \text{ MPa}$$

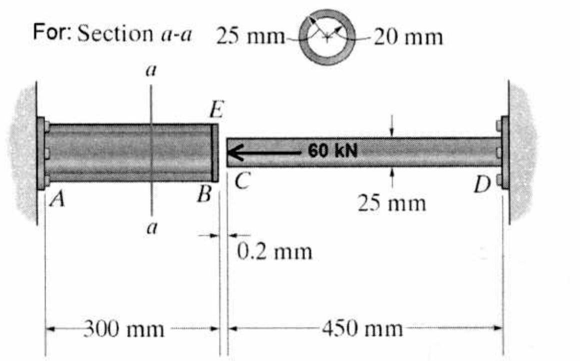
$$\sigma_B = \frac{F_B}{A_B} = 4.5 \text{ MPa}$$

$$\sigma_C = \frac{F_C}{A_C} = 4.7 \text{ MPa}$$

#5 Required

a) stresses in both materials due to load

b) due to load plus temp change.



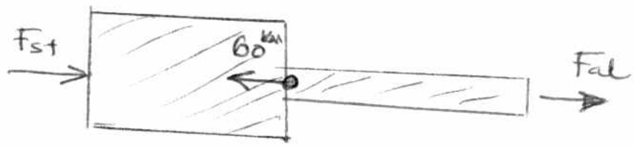
Material Properties from tables

$\alpha_{st} = 12\times10^{-6}/^\circ C$; $\alpha_{al} = 24\times10^{-6}/^\circ C$

$E_{st} = 200\,GPa$, $E_{al} = 68.9\,GPa$

Solution Assuming gab closes

(a) From shown FBD



$\rightarrow \sum F_x = 0$

$$F_{st} + F_{al} = 60 \quad -①$$

SI, compatibility requrement is

$$\Delta_{st} + \Delta_{al} = 0.2$$

in terms of forces

$$-\frac{F_{st}L_{st}}{A_{st}E_{st}} + \frac{(60-F_{st})L_{al}}{A_{al}E_{al}} = 0.2$$

$$\frac{-F_{st}(300)}{\pi(25^2-20^2)(200)} + \frac{(60-F_{st})(450)}{\frac{\pi}{4}(25)^2(68.9)} = 0.2$$

which reduces to

$$F_{st} = 38.78\,kN$$

and using ① we get $F_{al} = 21.22\,kN$

and the resulting stresses are

$$\sigma_{st} = \frac{F_{st}}{A_{st}} = 54.9\,MPa \quad \text{(compression)}$$

$$\sigma_{al} = \frac{F_{al}}{A_{al}} = 43.2\,MPa$$

b) If the temperature is increased then the compatibility equation will be

$$\Delta_{st}^{total} + \Delta_{al}^{total} = 0.2$$

substituting

$$-\frac{F_{st} L_{st}}{A_{st} E_{st}} + \alpha_{ST} \Delta T L_{st} + \frac{(60 - F_{st}) L_{al}}{A_{al} E_{al}}$$

$$+ \alpha_{al} \Delta T L_{al} = 0.2$$

Substituting the values and solving we get

$$F_{st} = 48.1 \ kN$$

and from ①

$$F_{al} = 11.9 \ kN$$

and the stresses become

$$\sigma_{st} = 68 \ MPa$$

$$\sigma_{al} = 24.2 \ MPa$$