HW #4 - Solution

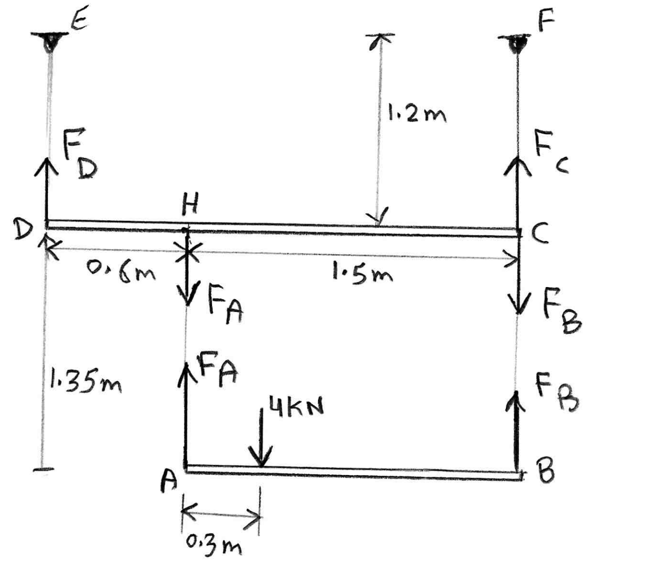
Pr #1:

Given: $A_{AH} = A_{BC} = 40\times10^{-6}\,m^2$.

$E_{AH} = E_{BC} = 100\times10^{9}\,Pa$.

and $A_{DE} = A_{CF} = 50\times10^{-6}\,m^2$.

$E_{DE} = E_{CF} = 150\times10^{9}\,Pa$.



F.B.D.

- From the F.B.D.:

For the rigid member AB;

$\circlearrowleft + \Sigma M_B = 0;\ +(4KN)(1.2m) - F_A(1.5m) = 0$

$\Rightarrow F_A = 3.2\,KN.$

$+\uparrow \Sigma F_y = 0;\ F_A + F_B - 4KN = 0 \Rightarrow F_B = 0.8\,KN.$

Similarly, for rigid member DC;

$\circlearrowleft + \Sigma M_c = 0;\ +(3.2KN)\cdot(1.5m) - F_D\cdot(2.1m) = 0$

$\Rightarrow F_D = 2.286\,KN.$

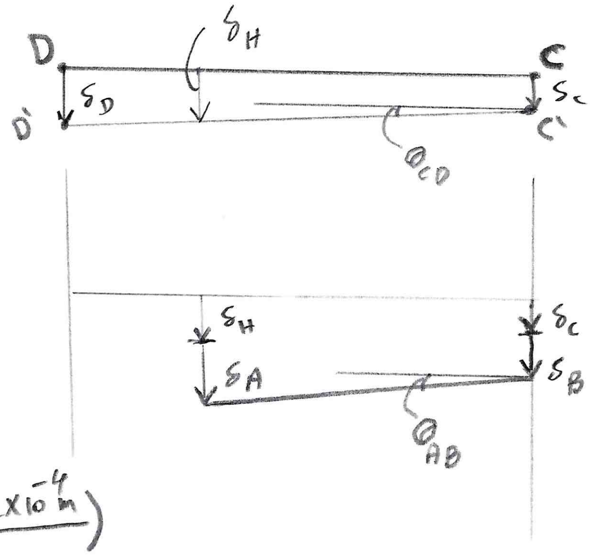
$+\uparrow \Sigma F_y = 0;\ F_D + F_c - 3.2KN - 0.8KN = 0$

$\Rightarrow F_c = 1.714\,KN.$

- The displacement of member DC:

$$\delta_c = \frac{F_c\cdot(1.2m)}{A_{CF}\cdot E_{CF}} = \frac{(1.714\times10^{3}N)\cdot(1.2m)}{(50\times10^{-6}m^2)\cdot(150\times10^{9}Pa)} = 2.742\times10^{-4}\,m.$$

$$\delta_D = \frac{F_D\cdot(1.2m)}{A_{DE}\cdot E_{DE}} = \frac{(2.286\times10^{3}N)(1.2m)}{(50\times10^{-6}m^2)\cdot(150\times10^{9}Pa)} = 3.658\times10^{-4}\,m.$$



The angle of tilt of member CD:

$$\theta_{CD} = \tan^{-1}\left(\frac{\delta_D - \delta_c}{2.1m}\right) = \tan^{-1}\left(\frac{3.658\times10^{-4}m - 2.742\times10^{-4}m}{2.1\,m}\right)$$

$$= \tan^{-1}(4.3619\times10^{-5}) = \underline{\underline{2.5\times10^{-3}\,Deg.}}$$

The displacement of member AB:

$$\delta_H = \frac{1.5m}{2.1m}(\delta_D) = 2.613 \times 10^{-4} m \quad \text{(in member DC)}.$$

$$\delta_A = \frac{F_A \cdot (1.35m)}{A_{AH} \cdot E_{AH}} = \frac{(3.2 \times 10^3 N) \cdot (1.35m)}{(40 \times 10^{-6} m^2) \cdot (100 \times 10^9 Pa)} = 1.08 \times 10^{-3} m.$$

$$\delta_B = \frac{F_B \cdot (1.35m)}{A_{BC} \cdot E_{BC}} = \frac{(0.8 \times 10^3 N)(1.35m)}{(40 \times 10^{-6} m^2) \cdot (100 \times 10^9 Pa)} = 2.7 \times 10^{-4} m.$$

The displacement of A (down) $= \delta_H + \delta_A = 2.613 \times 10^{-4} m + 1.08 \times 10^{-3} m$
$= 1.341 \times 10^{-3} m.$

The displacement of B (down) $= \delta_C + \delta_B$
$= 2.742 \times 10^{-4} m + 2.7 \times 10^{-4} m$
$= 5.442 \times 10^{-4} m.$

The angle of tilt of member AB:

$$\theta_{AB} = \tan^{-1}\left(\frac{\delta_A - \delta_B}{1.5m}\right) = \tan^{-1}\left(\frac{1.341 \times 10^{-3} m - 5.442 \times 10^{-4} m}{1.5m}\right)$$

$$= \tan^{-1}(5.312 \times 10^{-4})$$

$$= \underline{\underline{0.0304^\circ}}.$$

Pr # 2 :

Given: $(\sigma_{all})_{BC} = 80 \text{ MPa}$

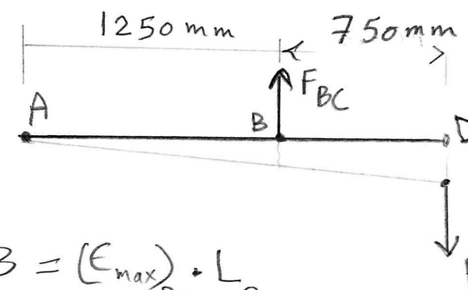
$r_{BC} = 3 \text{ mm}$

$E = 100 \text{ GPa}$.

Since the maximum movement (down) of point D is 4mm

$\Rightarrow$ the corresponding displacement (down) of point B $= \dfrac{4\text{ mm} \times 1250\text{ mm}}{2000\text{ mm}} = 2.5\text{ mm}$

$\because$ σ_{all} for cable BC = 80 MPa



$\Rightarrow (\epsilon_{max})_{BC} = \dfrac{80 \times 10^6 \text{ Pa}}{100 \times 10^9 \text{ Pa}} = 0.8 \times 10^{-3} \dfrac{\text{mm}}{\text{mm}}$

$\Rightarrow$ The max. allow displacement of B $= (\epsilon_{max})_{BC} \cdot L_{BC}$

$= \left(0.8 \times 10^{-3} \dfrac{\text{mm}}{\text{mm}}\right) \cdot (1000 \text{ mm})$

$= 0.8 \text{ mm} < 2.5 \text{ mm}$

$\Rightarrow \therefore$ The max Force in Cable BC $= (\sigma_{all})_{BC} \cdot A_{BC}$

$= (80 \times 10^6 \text{ Pa}) \cdot \left[\pi \cdot (3.0 \times 10^{-3} \text{ m})^2\right]$

$= \underline{\underline{2{,}261.95 \text{ N}}}$

By applying equilibrium equ:

$\circlearrowleft \Sigma M_A = 0$; $+P \cdot (2000\text{m}) - (2{,}261.95 \text{ N}) \cdot (1250 \text{ mm}) = 0$

$\Rightarrow \boxed{P = 1{,}413.72 \text{ N}}$

Pr #3

$$\delta = \frac{P \cdot L}{A E}$$

From the table in the inside cover of the textbook:

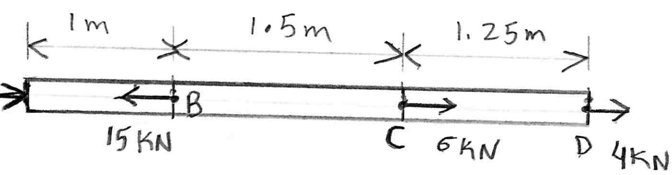
For A36 steel $\Rightarrow E_{st} = 200\,GPa$

For C 8340 Red Brass $\Rightarrow E_{br} = 101\,GPa$

The cross-section area of the rod, $A = \frac{\pi}{4}(0.01m)^2$

Internal Forces:

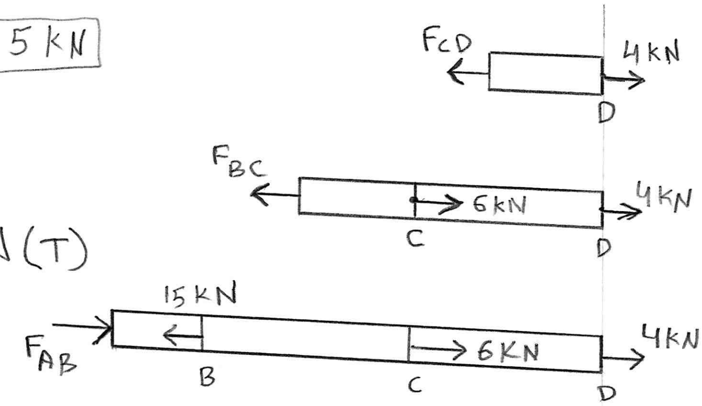
By applying the section method and equilibrium equations, the internal forces and in the three segments can be obtained as shown below:



F.B.D

For the entire rod; $\xrightarrow{+} \Sigma F_x = 0$; $+F_A - 15KN + 6KN + 4KN = 0$

$\Rightarrow \boxed{F_A = 5\,KN}$



In segment CD: $F_{CD} = +4KN\ (T)$

In segment BC: $F_{BC} = 6KN + 4KN = +10KN\ (T)$

In Segment AB: $F_{AB} = 15KN - 6KN - 4KN$
$= 5\,KN\ (C)$

a) Total change in length $= \delta_{B/A} + \delta_{C/B} + \delta_{D/C} = \frac{F_{AB} \cdot L_{AB}}{A \cdot E_{st}} + \frac{F_{BC} \cdot L_{BC}}{A \cdot E_{br}} + \frac{F_{CD} \cdot L_{CD}}{A \cdot E_{st}}$

$$= \frac{(-5 \times 10^3 N) \cdot (1m)}{\frac{\pi}{4}(0.01m)^2 \cdot 200 \times 10^9 Pa} + \frac{(10 \times 10^3 N) \cdot (1.5m)}{\frac{\pi}{4}(0.01m)^2 \cdot 101 \times 10^9 Pa} + \frac{(4 \times 10^3 N) \cdot (1.25m)}{\frac{\pi}{4}(0.01m)^2 \cdot (200 \times 10^9 Pa)}$$

$= -318.3 \times 10^{-6} m + 1{,}891 \times 10^{-6} m + 318.3 \times 10^{-6} m$ $= \underline{+1{,}891 \times 10^{-6} m}$

b) $\delta_{D/B} = \delta_{D/C} + \delta_{C/B} = 1{,}891 \times 10^{-6} m + 318.3 \times 10^{-6} m = \underline{+2{,}209.3 \times 10^{-6} m}$

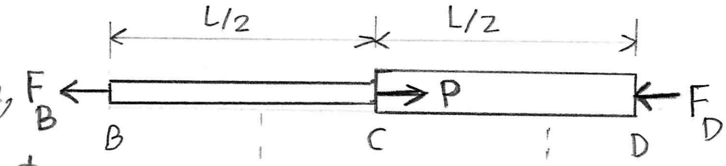
c) $\delta_{D/C} = \underline{+318.3 \times 10^{-6} m.}$

$\delta_{D/A}$ = total change in length $= \underline{+1{,}891 \times 10^{-6} m.}$

Pr #4 :

Given : $A_{BC} = 100\,mm^2$, $A_{CD} = 250\,mm^2$.

$P = 5\,KN$, $E = 100\,GPa$.



This problem is statically indeterminate, the equlibrium equation is not sufficient to determine the two reactions at B and D.

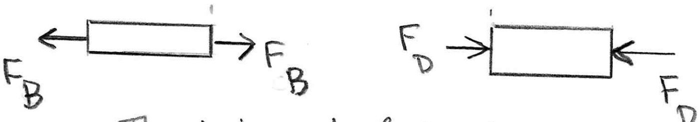
F.B.D.

From equilibrium equation:

$\overset{+}{\rightarrow} \sum F_x = 0; \quad P - F_D - F_B = 0$

$\Rightarrow \boxed{5\,KN = F_D + F_B}$ — 1

Additional equation is needed to solve this problem.



The internal forces in both rods

Since both rods are fixed at both ends, then the compatibility condition becomes:

$\delta_{B/D} = 0$ — 2

$$\delta_{B/D} = \delta_{B/C} + \delta_{C/D} = \frac{(F_B)\cdot(\cancel{\tfrac{L}{2}})}{(A_{BC})\cdot(\cancel{E})} - \frac{(F_D)\cdot(\cancel{\tfrac{L}{2}})}{(A_{CD})\cdot(\cancel{E})} = 0$$

$$\Rightarrow F_B = \frac{A_{BC}}{A_{CD}} \cdot F_D = \left(\frac{100\,mm^2}{250\,mm^2}\right)\cdot F_D$$

$$\Rightarrow \boxed{F_B = 0.4\,F_D}$$

substitute into equ (1) $\Rightarrow 5\,KN = F_D + 0.4 F_D \quad \Rightarrow F_D = 3.57\,KN$

$\Rightarrow F_B = 1.43\,KN$

$$\therefore \sigma_{BC} = \frac{+1.43\times10^3\,N}{100\times10^{-6}\,m^2} = 14.3\times10^6\,Pa = \underline{14.3\,MPa}\ (T).$$

$$\sigma_{CD} = \frac{-3.57\times10^3\,N}{250\times10^{-6}\,m^2} = -14.28\times10^6\,Pa = -14.28\,MPa\ (C).$$

The displacement of point C $= \dfrac{F_B\cdot(\frac{L}{2})}{A_{CB}\cdot E} = \dfrac{1.43\times10^3 N\,(\frac{L}{2})}{(100\times10^{-6} m^2)(100\times10^9\,Pa} = \underline{\underline{7.15\times10^{-5}\,L}}\ (\rightarrow)$