King Fahd University of Petroleum & Minerals

CIVIL ENGINEERING DEPARTMENT

CE 203 STRUCTURAL MECHANICS I

First Semester 2012 / 2013 (**121**)

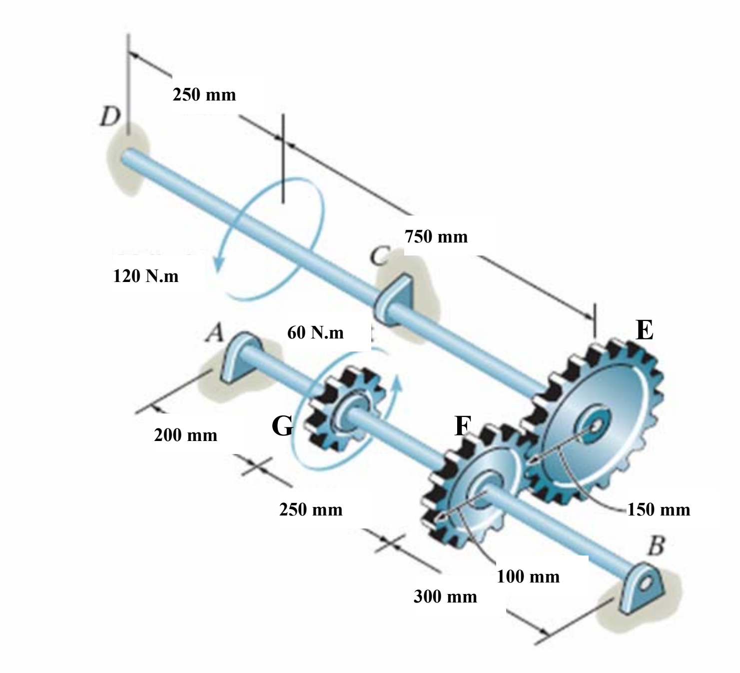
HOMEWORK NO. 8 (Key Solution)

- **Textbook Sections Covered: 5.4 - 5.7 ,** Torsion : Statically-Indeterminate shafts & Non-circular Sections

Problem # 1 (Problem 5-62 Page 211 in the text book with the revised data):-

Given Data:

- The two-Gear system given.
- Material of A-36.
- Diameter = 30 mm.
- Bearing supports at A, B, and C (allow free rotation).
- Support at D is fixed.

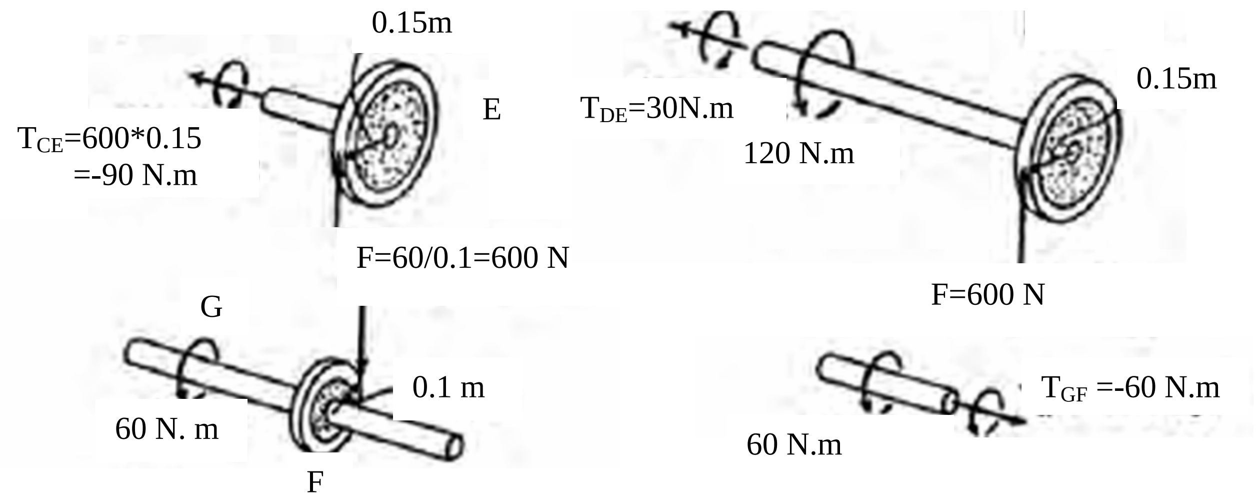


Required:

- ❖ **The angle of twist of end A.**
- ❖ **The magnitude and the location of the maximum stress in the whole shaft.**

Solution:

From the following shown free body diagrams:-



$$\emptyset_{E/D} = \emptyset_E = \sum \frac{TL}{JG}$$

$$\therefore \emptyset_E = \frac{1}{\frac{\pi}{2}(0.015)^4 * 75 * 10^9}[-90 * 0.75 + 30 * 0.25] = -0.010\ Rad = 0.010\ Rad.$$

$$\emptyset_E * 100 = \emptyset_F * 150 \quad \Rightarrow \quad \emptyset_F = \frac{150}{100}\emptyset_E = \frac{150}{100} * 0.010 = 0.015\ Rad.$$

$$\emptyset_{A/F} = \sum \frac{TL}{JG} = \frac{T_{GF} * L_{GF}}{JG} = \frac{-60 * 0.25}{\frac{\pi}{2}(0.015)^4 * 75 * 10^9} = -0.00252 = 0.00252\ Rad.$$

$$\emptyset_A = \emptyset_F + \emptyset_{A/F} = 0.015 + 0.00252 = 0.01752\ Rad = 1.002^\circ \qquad Ans.$$

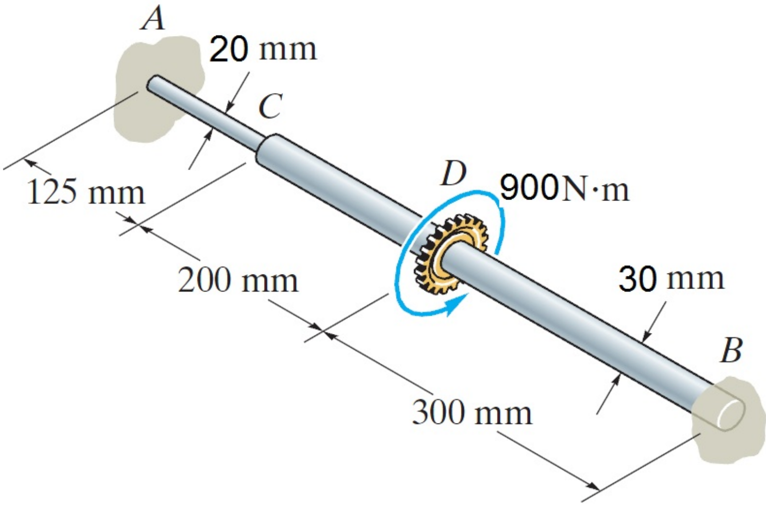
The magnitude of the maximum stress in the whole shaft is as follows:-

$$(\tau_{max})_{at\ E} = \frac{90 * 0.015}{\frac{\pi}{2}(0.015^4)} = 16.98\ \text{MPa.} \qquad \textbf{Ans}$$

Problem # 2:-

Given Data:

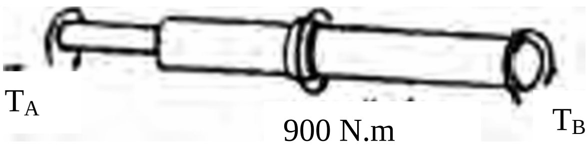
- The shown figure.
- Steel shaft (AC d=20mm and CB d =30mm)
- Fixed at ends A and B.
- G =100 GPa.



Required:

- **The support reactions at A and B.**
- **The angle of twist of point D.**

Solution:



Equilibrium : $\Rightarrow T_A + T_B - 900 = 0 \ldots\ldots\ldots\ldots \quad (1)$

Compatibility condition: $- \quad \phi_{D/A} = \phi_{D/B}$

$$\frac{T_A * 0.125}{\frac{\pi}{2} * (0.01)^4 * G} + \frac{T_A * 0.2}{\frac{\pi}{2} * (0.015)^4 * G} = \frac{T_B * 0.3}{\frac{\pi}{2} * (0.015)^4 * G}$$

□ $2.776 T_A = T_B \ldots\ldots\ldots (2)$

From the Equations (1) and (2)

□ $T_A = 238.35\ N.m$ *and* $T_B = 661.65\ N.m$ ***Ans***

The angle of twist of point D is as follows:-

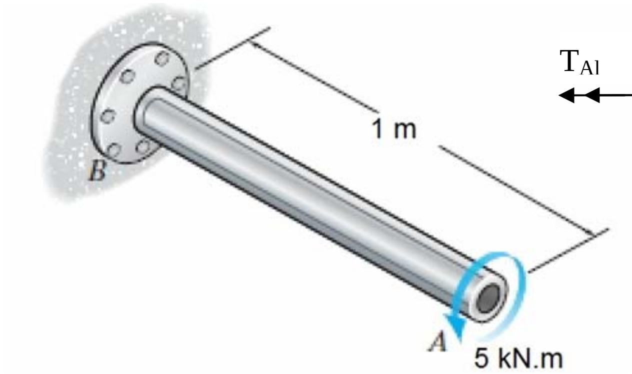
$$\phi_D = \sum \frac{TL}{JG} = \frac{T_A * 0.125}{\frac{\pi}{2} * (0.01)^4 * G} + \frac{T_A * 0.2}{\frac{\pi}{2} * (0.015)^4 * G} = \frac{16450617.3 T_A}{\frac{\pi}{2} * G} = \frac{3.921 * 10^{\wedge}9}{\frac{\pi}{2} * 100 * 10^{\wedge}9}$$

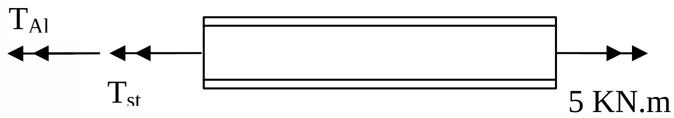
$\phi_D = 0.02496 \quad Rad.$ ***Ans***

Problem # 3:-

Given Data:

- The shown figure (composite shaft).
- Fixed at B.
- Solid circular steel part outer radius = 40mm and G = 100 GPa.
- Aluminum tube (inner radius =40mm and outer radius =70 mm and G=30GPa).





Required:

- **The values of the shear stress along the radius of the shaft.**
- **Plotting of the distribution of the shear stress along the radius of the shaft.**
- **The angle of the twist of point A.**

Solution:

$$\text{Equilibrium:} \quad -T_{Al} - T_{St} + 5000 = 0 \quad \ldots\ldots (1)$$

$$\text{Compatibility:} \; \emptyset = \emptyset_{Al} = \emptyset_{St} \quad \Rightarrow$$

$$\frac{T_{Al} * L}{J_{Al} * G_{Al}} = \frac{T_{St} * L}{J_{St} * G_{St}} \quad \Rightarrow \frac{T_{Al} * L}{\frac{\pi}{2}(0.07^4 - 0.04^4) * 30 * 10^9} = \frac{T_{St} * L}{\frac{\pi}{2}(0.04^4) * 100 * 10^9}$$

$$\rightarrow \; T_{Al} = 2.5137 T_{St} \quad \ldots\ldots\ldots (2) \qquad \rightarrow \; T_{Al} = 3577 \text{ N.m} \;\; \text{and} \;\; T_{St} = 1423 \text{ N.m}$$

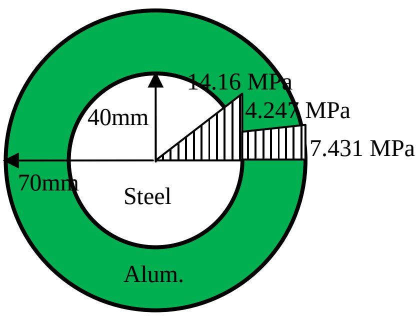
$$(\tau_{St})_{max} = \frac{1423 * 0.04}{\frac{\pi}{2}(0.04^4)} = 14.155 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{Al})_{min} = \frac{3577 * 0.04}{\frac{\pi}{2}(0.07^4 - 0.04^4)} = 4.247 \text{ MPa} \qquad \textbf{Ans}$$

$$(\tau_{Al})_{max} = \frac{3577 * 0.07}{\frac{\pi}{2}(0.07^4 - 0.04^4)} = 7.431 \text{ MPa} \qquad \textbf{Ans}$$

$$\emptyset_A = \frac{3577 * 1}{\frac{\pi}{2}(0.07^4 - 0.04^4) * 30 * 10^9} = 0.00354 \text{ Rad.} \qquad \textbf{Ans}$$

The plot of the distribution of the shear stress along the radius of the shaft as follows:-

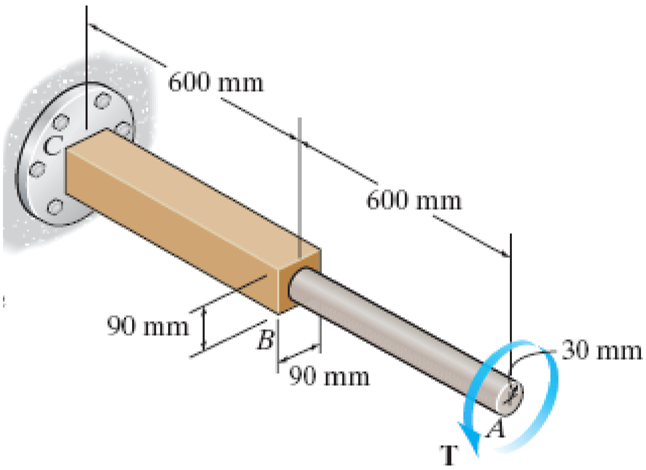


Shear stress distribution along the
radius of the shaft.

Problem # 4 (Problem 5-101 Page 230 in the text book with the revised data):-

Given Data:

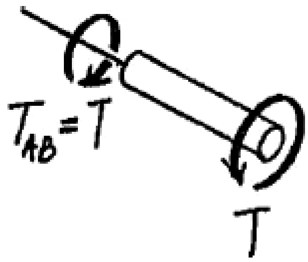
- The shown figure.
- Material of A-36.
- Allowable shear stress = 50 MPa.
- Allowable angle of twist at end A = 0.03 rad.
- The shaft is fixed at end C.

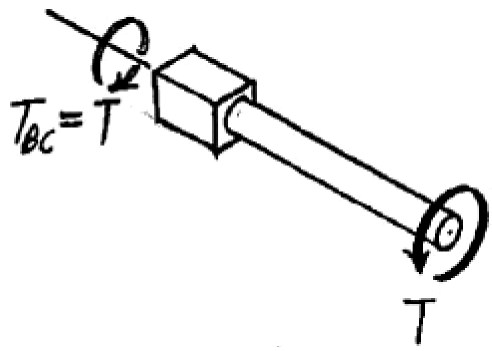


Required:

- **The maximum allowable torque T that can be applied at end A.**

Solution:





Allowable Shear Stress: For segment AB,

$$\tau_{allow} = \frac{T_{AB}c}{J}; \qquad 50*10^6 = \frac{T(0.03)}{\frac{\pi}{2}\left(0.03^4\right)}$$

T = 2120.57 N.m ….. (1)

For segment BC,

$$\tau_{allow} = \frac{4.81T_{BC}}{a^3}; \qquad 50*10^6 = \frac{4.81T}{(0.09)^3}$$

T= 7577.96 N.m …….(2)

Angle of Twist:

$$\phi_A = \frac{T_{AB}L_{AB}}{JG} + \frac{7.10T_{BC}L_{BC}}{a^4G}$$

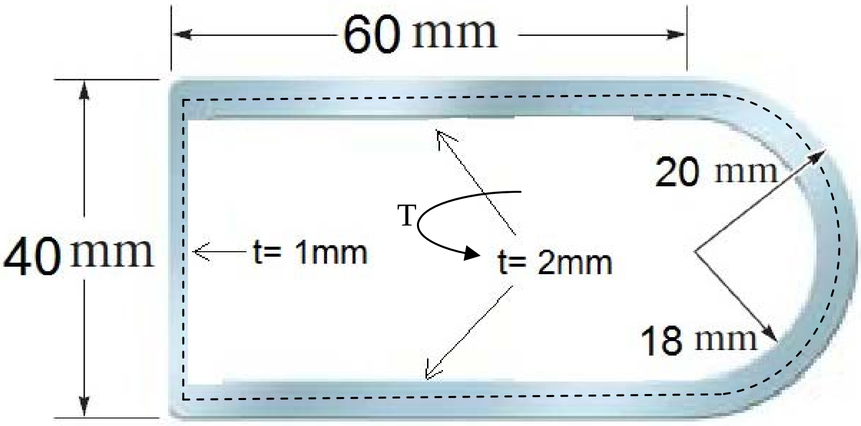
$$0.03 = \frac{T(0.6)}{\frac{\pi}{2}\left(0.03^4\right)(75)(10^9)} + \frac{7.10T(0.6)}{(0.09)^4(75)(10^9)}$$

T =4193.86 N.m ……(3), ***from the three values*** □ **the minimum value is the Allowable T = 2120.57 N.m** **Ans.**

Problem # 5:-

Given Data:

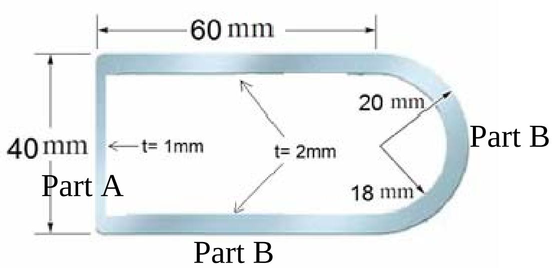
- The shown figure.
- Subjected torque T is 10 N.m
- G=100 GPa.
- Shaft Length =800 mm.



Required:

- **The magnitude and location of the maximum shear stress in the whole shaft.**
- **The angle of twist of the shaft end.**

Solution:



For semicircle part, $r = \frac{20+18}{2} = 19\ mm = 0.019 m.$

$\Rightarrow A_m = \frac{\pi}{2} * 0.019^2 + (0.038 * 0.0595) = 0.00283\ m^2$

$\left(\oint ds\right)_A = 0.038\ m$

$\left(\oint ds\right)_B = \pi * 0.019 + 2 * 0.0595 = 0.1787\ m$

$\Rightarrow \tau_{max} = \tau_A = \frac{T}{2t_{min}A_m} = \frac{10}{2 * 0.001 * 0.00283} = 1.767\ MPa$ **Ans**

$\Rightarrow \tau_B = \frac{T}{2tA_m} = \frac{10}{2 * 0.002 * 0.00283} = 0.8834\ MPa$ **Ans**

□ **The maximum stress at point A in the shown figure and equal to 1.767 MPa.** **Ans**

Angle of twist:—

$\emptyset = \frac{TL}{4A_m{}^2 G}\oint\frac{ds}{t} = \frac{10 * 0.8}{4 * 0.00283^2 * 100 * 10^9}\left(\frac{0.038}{0.001} + \frac{0.1787}{0.002}\right)$

$= 0.00318\ Rad$ **Ans**