Solution of HW # 12

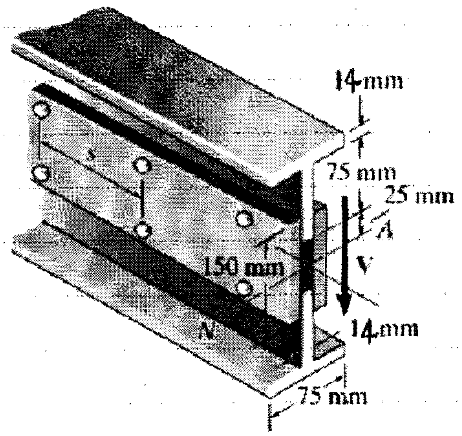
Prob # 1:

Given:

The beam shown

$V = 250\ kN$

$R_{bolt} = 75\ kN$



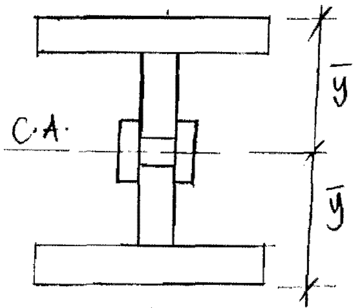
Required:

Max spacing of bolts (s)

Solution:

First, we need to locate $\bar{y}$ and calculate $\bar{I}$.
Due to the double symmetry, the Centroid is in the "middle".
Thus,

$$\bar{y} = 14 + 75 + 25 = 114\ mm$$



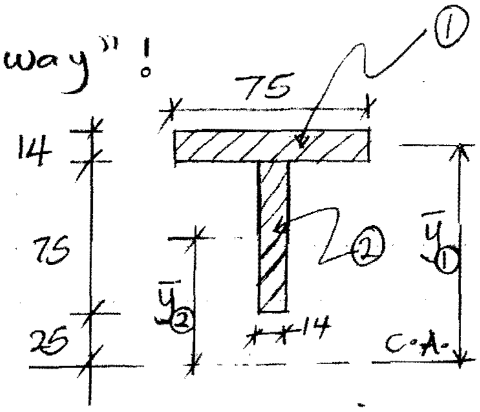
$$\bar{I} = \frac{1}{12}\left[75(114+114)^3 - (75-14)\left[2(75+25)\right]^3 - 14(25+25)^3 + 2(14)(150)^3\right]$$
$$= 4.11397 \times 10^7\ mm^4$$

Try to find $\bar{I}$ by "another way"!

$$q = \frac{VQ}{I}$$

For the shaded area,

$$Q = \Sigma Q_i = \Sigma (A\bar{y})_i$$
$$Q = Q_1 + Q_2$$
$$= 75(14)(114-7) + 14(75)\left(\frac{75}{2} + 25\right)$$
$$= 177{,}975\ mm^3$$



All dimensions in mm

$$\Rightarrow q = \frac{250(10^3)(177{,}975)}{4.11397(10^7)} = 1081.53\ N/mm$$

$$\frac{R_{bolt}}{S} = q$$

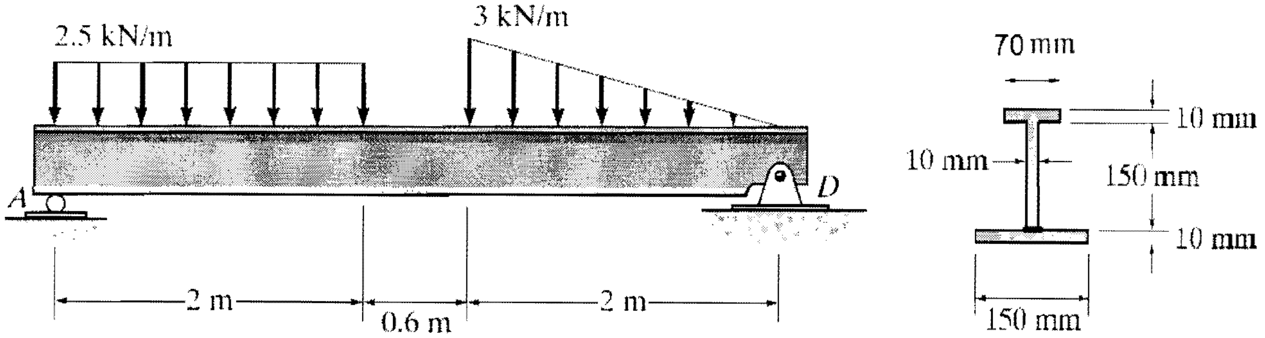
The bolts act in "double shear" $\Rightarrow$

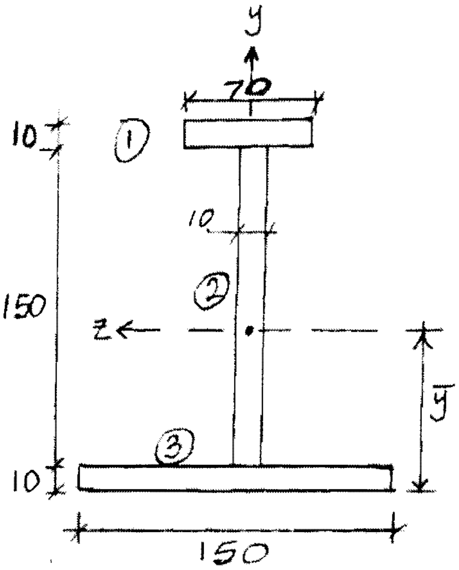
$$\frac{2(75)(10^3)}{S} = 1081.53 \Rightarrow$$

$$\boxed{S = 138.7 \text{ mm}}$$

Note that due to symmetry, the spacing of the bolts in the lower row is the same as the upper one.

Prob #2





Given:
The beam with the cross-section shown

Required:
The min. strength of the glue connecting the Lower flange to the web

Solution:
First, we need to find $\bar{y}$ and I.
We will use a table. We need to first locate $\bar{y}$, then calculate I (why?!)

Segment part	A_i (mm^2)	$\bar{y}_i$ (mm)	$A_i\bar{y}_i$ (mm^3)
①	700	10+150+5=165	115,500
②	1500	10+75 = 85	127,500
③	1500	5	7,500
Σ	3700		250,500

$$\bar{y} = \frac{\Sigma (A\bar{y})_i}{\Sigma A_i} = \frac{250,500}{3,700}$$

$$= 67.7027 \text{ mm}$$

(Reasonable?!)

* What is "d"?! Review statics!!!

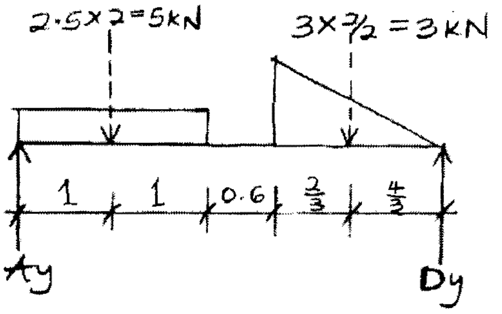
Segment part	A_i (mm^2)	d_i^{*} (mm)	$A_i d_i^2$ (mm^4)	$\bar{I}_i^{*}$ (mm^4)	$\bar{I}_i + A_i d_i^2$ (mm^4)
①	700	$10+150+5-\bar{y} = 97.2973$	6626735	5833.333	6632569
②	1500	$10+75-\bar{y} = 17.2973$	448795	2812500	3261295
③	1500	$\bar{y}-5 = 62.7027$	5897443	12500	5909943
Σ	3700				15.8038×10^6

* $I = \frac{1}{12} b h^3$

$$\tau_{max}^{glue} = \frac{V_{max}\, Q}{It}$$

To determine V_{max}, we need to draw the SFD. But first, we need to determine the reactions.

From the FBD



$\circlearrowleft + \Sigma M_D = 0 \Rightarrow$

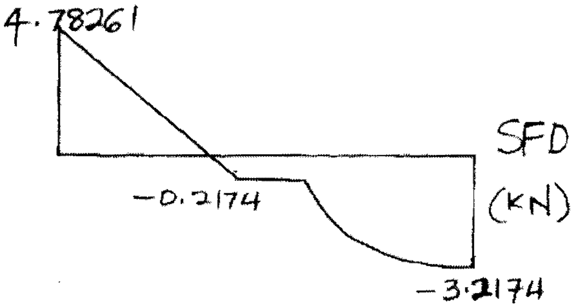
$-4.6 A_y + 5(3.6) + 3\left(\frac{4}{3}\right) = 0$

$\Rightarrow A_y = 4.78261$ kN

$+\uparrow \Sigma F_y = 0 \Rightarrow$

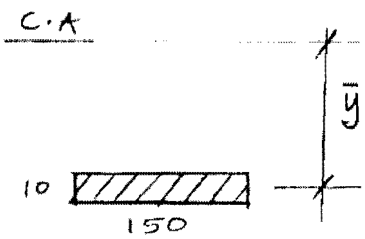
$4.78261 - 5 - 3 + D_y = 0$

$\Rightarrow D_y = 3.21739$ kN



Clearly, $V_{max} = 4.78261$ kN

$$\tau_{max}^{glue} = \frac{V_{max} Q}{It}$$



$\bar{y} = 67.7027 - 5 = 62.7027$

$Q = A\bar{y} = 10(150)(62.7027)$

$= 94\,054.1\ mm^3$

$$\tau_{max}^{glue} = \frac{4782.61(94054.1)}{(15.8038\times10^6)(10)} = 2.8463\ MPa$$

$\Rightarrow$ $\boxed{\text{Required glue strength} = 2.85\ MPa}$

Prob #3

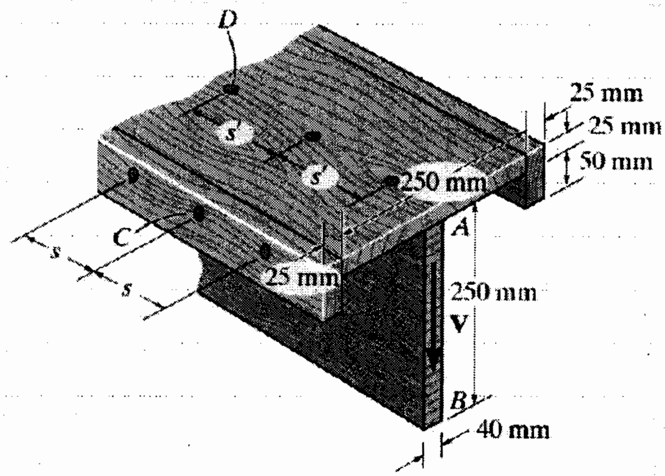
Given:

The cross-section of the beam shown

$\tau_{allow} = 2$ MPa

$R_{nail} = 600$ N

$S = 100$ mm, $S' = 120$ mm



Required:

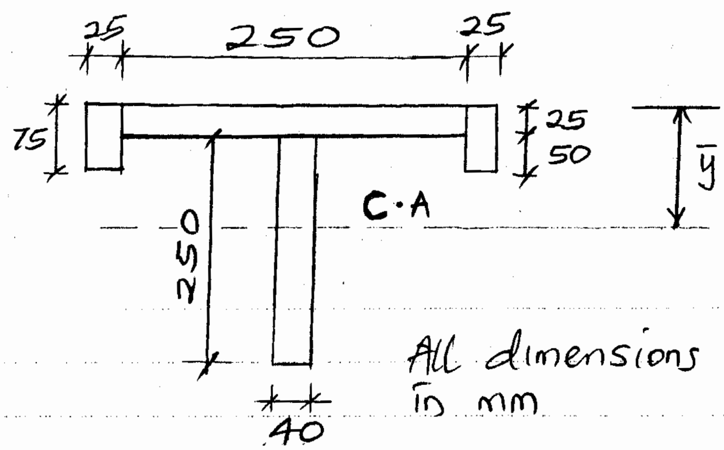
V_{max}

Solution:

$$\tau_{max} = \frac{V_{max} Q_{max}}{It}$$

First, we need to locate the centroid ($\bar{y}$), then calculate I.

$$\bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i}$$



All dimensions in mm

$$\bar{y} = \frac{40(250)(\frac{250}{2}+25)+25(250)(\frac{25}{2})+2(25)(75)(\frac{75}{2})}{40(250)+25(250)+2(25)(75)} = \underline{85.9375 \text{ mm}}$$

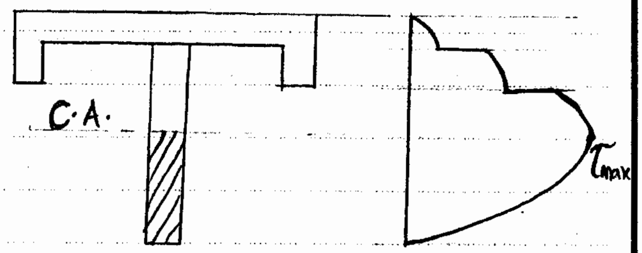
$$\bar{I}_{CA} = \left[\tfrac{1}{12}(40)(250^3)+40(250)(125+25-85.9375)^2\right]$$
$$+\left[\tfrac{1}{12}(250)(25^3)+250(25)(85.9375-12.5)^2\right]$$
$$+\left[\tfrac{1}{12}(2)(25)(75^3)+2(25)(75)(85.9375-\tfrac{75}{2})^2\right]$$

$$= 1.3771\times10^8 \text{ mm}^4$$

Solution of HW #12

Criterion ①: $\tau_{allow} = 2$ MPa

Clearly, τ_{max} @ C.A.
It is easier to take the lower (shaded) area to calculate $Q \rightarrow q \rightarrow \tau$
(Why?!)
Try the upper part!



τ-distribution

$$Q_{C.A.} = A\bar{y} = 40(250+25-85.9375)^2/2$$

$$= 714,893 \text{ mm}^3$$

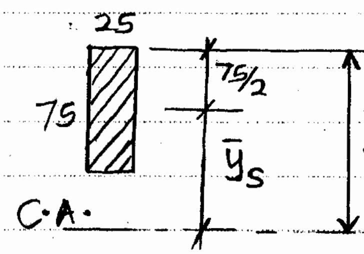
$$\tau_{max} = \frac{V_{max}^{①} Q_{C.A}}{I t_{C.A.}} = \frac{V_{max}^{①} \, 714,893}{1.3771\,(10^8)(40)} = 1.2978 \times 10^{-4} V_{max}^{①}$$

Set $\tau_{max} \equiv \tau_{allow} = 2$ MPa $\Rightarrow$

$$V_{max}^{①} = 2 / 1.2978 \times 10^{-4} = 1.5411 \text{ kN}$$

Criterion ②: $R_{nail} = 600$ N ; $S = 100$ mm (side nails)

$$R_n = qS = \frac{VQ}{I} S$$



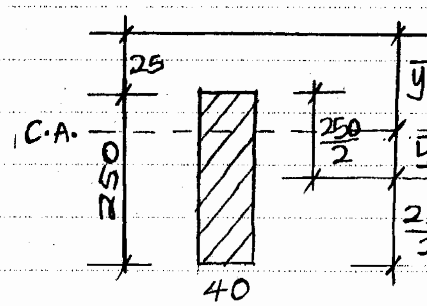
$$Q = A\bar{y}_s = 25(75)(85.9375 - \tfrac{75}{2})$$
$$= 90,820 \text{ mm}^3$$

$$600 = \frac{V_{max}^{②}(90,820)}{1.3771 \times 10^8} 100 \Rightarrow$$

$$V_{max}^{②} = 9.0978 \text{ kN}$$

Criterion ③

$R_{nail} = 600$ N ; $S' = 120$ mm (top nails)



$$R_n = qS' = \frac{VQ}{I} S'$$

$$Q = A\bar{y}_{s'} = 40(250)(\tfrac{250}{2} + 25 - 85.9375) = 640,625$$

Note that it is easier to take the <u>lower</u> area; you may choose the upper part. (Try!)

$$600 = \frac{V_{max}^{③}\,(640{,}625)}{1.3771 \times 10^{8}}\,(120) \Rightarrow$$

$$\underline{V_{max}^{③} = 1.0748\,kN}$$

For $V_{max\,allow}$, we choose $\min\left(V_{max}^{①},\ V_{max}^{②},\ V_{max}^{③}\right)$
(why?!)

$$\Rightarrow \boxed{V_{max\,allow} = 1.0748\ kN}$$

Solution of HW #12

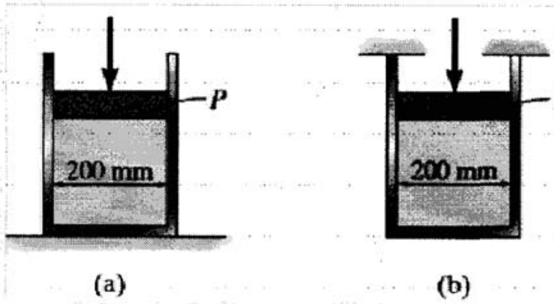
Prob#4

Given:

The two figures shown

$P = 0.5$ MPa, $t = 6$ mm

$D = 200$ mm



Required:

The state of stress in the cylinder wall in cases (a) and (b)

Solution:

Case (a)

Note that in case (a), there is a circumferential (hoop) stress, but no longitudinal (axial) stress, as it is "free to expand" in the axial (vertical) direction (up)

Thus,

$$\sigma_c = \sigma_1 = \frac{Pr}{t} = \frac{0.5\times10^{6}\times\frac{200}{2}\times10^{-3}}{6\times10^{-3}}$$

$$\Rightarrow \boxed{\sigma_c = \sigma_1 = 8.333 \text{ MPa}}$$

$$\boxed{\sigma_l = \sigma_2 = 0}$$

Case (b)

In case (b), there is an axial stress on the cylinder as we can see it from the boundary conditions. However, σ_c is as in case (a)

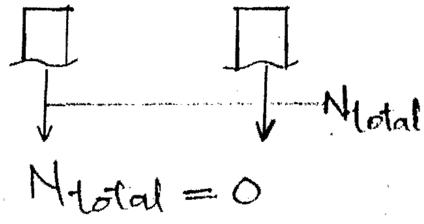
$$\Rightarrow \boxed{\sigma_c = \sigma_1 = 8.333 \text{ MPa}}$$

$$\sigma_l = \sigma_2 = \frac{Pr}{2t} = 0.5\times\frac{200}{2}\Big/2\times6$$

$$\boxed{\sigma_l = \sigma_2 = 4.167 \text{ MPa}}$$

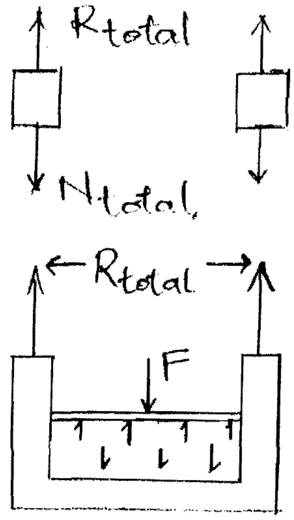
We can also find σ_L as we did in the "axially-loaded member" chapter.

For case (a), from the FBD,



$$\boxed{\sigma_L = 0}$$

For case (b), from the FBD,



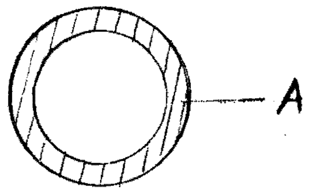
$$N_{total} = R_{total}$$

and $R_{total} = F$, and $F = PA_{pressure}$

Thus, $N = R = F = \text{Pressure (Area)}$

$$\Rightarrow N = 0.5\times10^6 \left[\pi \times \left(\frac{200}{2}\right)^2\right]\times10^{-6}$$
$$= 5000\pi \text{ (N)}$$

$$\sigma_{axial} = \sigma_L = \sigma_2 = \frac{N}{A}$$



A = Area of material

$$\approx \pi D(t) = 1200\pi \text{ mm}^2$$

$$\Rightarrow \sigma_L = \sigma_2 = \frac{5000\pi}{1200\pi\times10^{-6}} = 4.167 \text{ MPa (As above)}$$

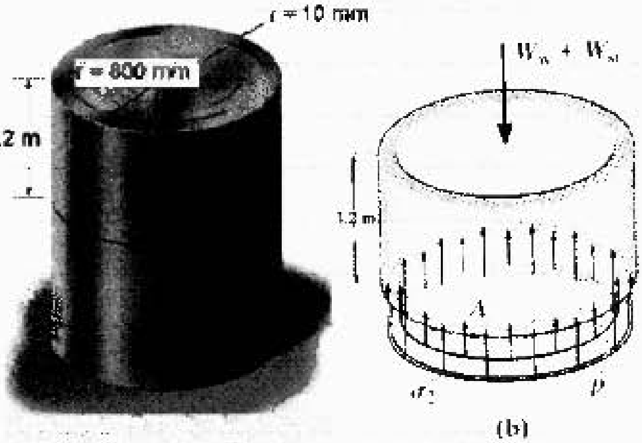
Prob #5

Given:

The figure shown, a steel tank filled with water

$\gamma_{water} = 10 kN/m^3$, and

$\gamma_{steel} = 70 kN/m^3$



Required

State of stress at A

Solution:

The FBD after making a section through A and taking the "upper" part (why?!) is shown

The stress at A is caused by the weight of the steel above A. "Steel carries itself" and "water carries itself" vertically.

In addition, the water causes "hydrostatic pressure" on the steel wall according to "Pascal's law", that is, $P = (\gamma h)_{water}$. (Review it if you took it; read about it or ask your instructor if you did not).

This P causes σ_h (or σ_c) as studied in the pressure vessels section

$$W_{steel} = \gamma_{steel} V_{steel}$$
$$= 70(810^2 - 800^2)\pi \times 10^{-6} \times 1.2$$
$$= 4.2487 \text{ kN}$$

$$\sigma_{vert} = \sigma_{long} = \sigma_{axial} = \frac{W_{steel}}{A_{steel}}$$ at that section

$$= \frac{-4.2487 \times 10^3}{(810^2 - 800^2)\pi \times 10^{-6}}$$

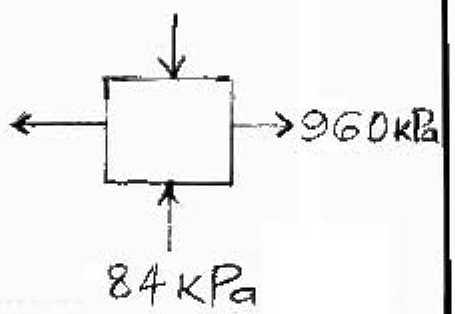
$$\boxed{\sigma_{vert} = -84.00\,kPa = 84\,kPa \text{ "C"}}$$

$$\sigma_{horiz} = \sigma_{hoop} = \sigma_{circumferential} = \frac{Pr}{t}$$

$$P = \gamma_{water} h_{water} = 10 \times 10^3 \times 1.2 = 12\,kPa$$

$$\Rightarrow \sigma_h = \frac{12 \times 800}{10} = \boxed{960\,kPa \text{ "T"} = \sigma_h}$$

The state of stress is as shown



Note that the internal pressure due to water does not cause longitudinal stress in the open tank (why?!)