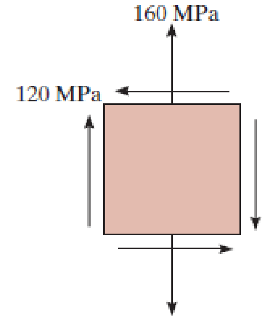
9–19. The state of stress at a point is shown on the element. Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case. Sketch the results on each element.



In accordance to the established sign Convention,

$$\sigma_x = 0 \qquad \sigma_y = 160\text{ MPa} \qquad \tau_{xy} = -120\text{ MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + 160}{2} \pm \sqrt{\left(\frac{0 - 160}{2}\right)^2 + (-120)^2}$$

$$= 80 \pm \sqrt{20800}$$

$$\sigma_1 = 224\text{ MPa} \qquad \sigma_2 = -64.2\text{ MPa}$$ **Ans.**

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-120}{(0 - 160)/2} = 1.5$$

$$\theta_p = 28.15^\circ \quad \text{and } -61.85^\circ$$

Substitute $\theta = 28.15^\circ$ into Eq. 9-1,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{0 + 160}{2} + \frac{0 - 160}{2}\cos 56.31^\circ + (-120)\sin 56.31^\circ$$

$$= -64.22 = \sigma_2$$

Thus,

$$(\theta_p)_1 = -61.8^\circ \qquad (\theta_p)_2 = 28.2^\circ$$ **Ans.**

The element that represents the state of principal stress is shown in Fig. *a*

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{0 - 160}{2}\right)^2 + (-120)^2} = 144\text{ MPa}$$ **Ans.**

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(0 - 160)/2}{-120} = -0.6667$$

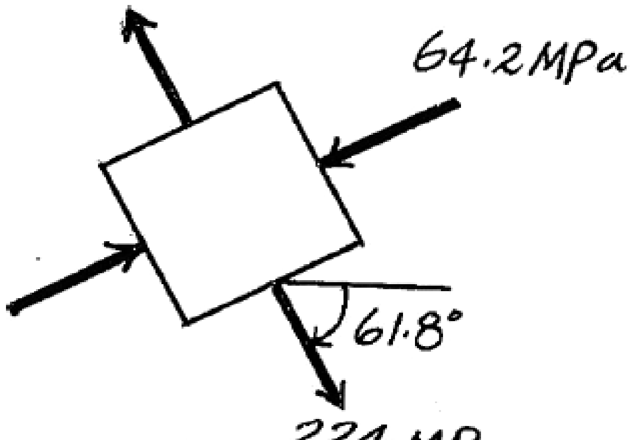
$$\theta_s = -16.8^\circ \quad \text{and} \quad 73.2^\circ$$ **Ans.**

By inspection, $\tau_{\substack{\max \\ \text{in-plane}}}$ has to act in the sense shown in Fig. *b* to maintain equilibrium.

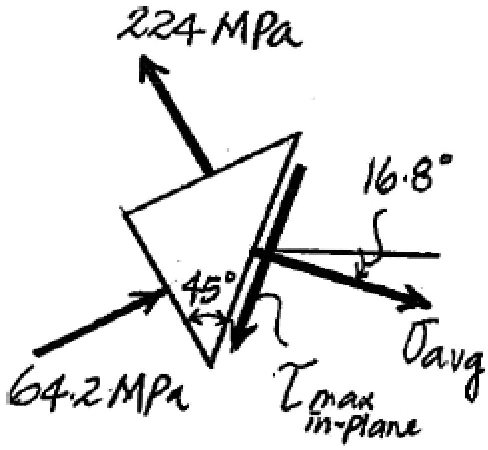
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + 160}{2} = 80\text{ MPa}$$ **Ans.**

The element that represents the state of Maximum in - plane shear stress is shown in Fig. (c)

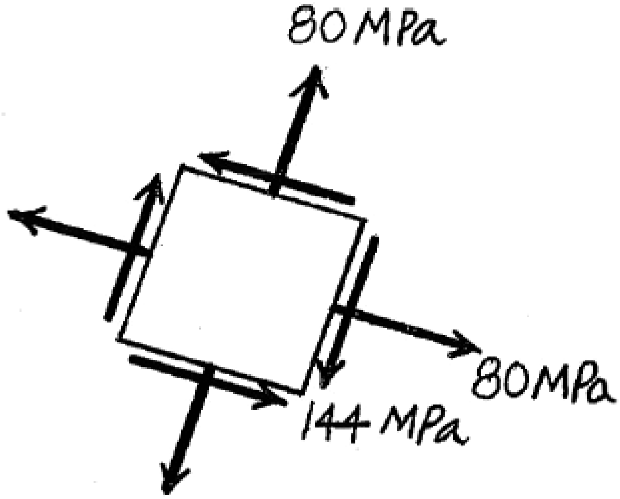
9–19. Continued



(a)

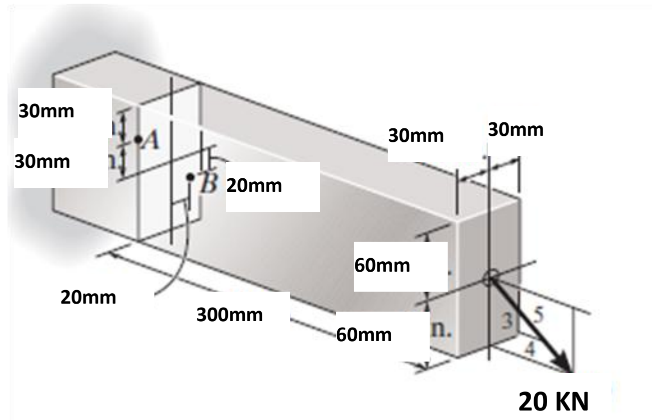


(b)

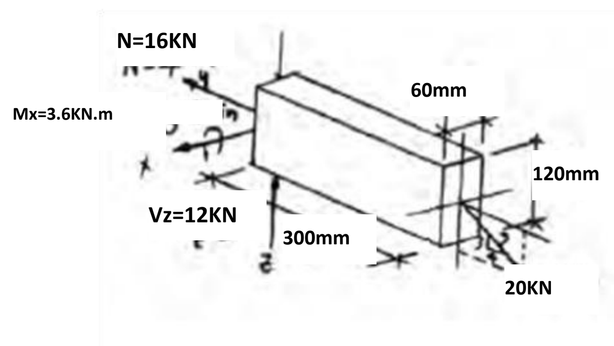


(c)

9–30. The cantilevered rectangular bar is subjected to the force of 20 KN. Determine the principal stress at points *A* and *B*.



$$I = \frac{1}{12} * 60 * 120^3 = 8640000\, mm^4,$$
$$A = 120 * 60 = 7200\, mm^2 \quad ;$$
$$Q_A = 45 * 30 * 60 = 81000 mm^3;\; Q_B = 40 * 40 * 60 = 96000 mm^3$$



Point A.

$$\sigma_A = \frac{P}{A} + \frac{M_x * z}{I} = \frac{16 * 1000}{7200} + \frac{3.6 * 10^6 * 30}{8640000}.$$
$$\sigma_A = 14.72\, MPa.$$

$$\tau_A = \frac{V_z Q_A}{It} = \frac{12 * 1000 * 81000}{8640000 * 60} = 1.88\, MPa.$$
$$\sigma_x = 14.72\, MPa;\;\; \sigma_y = 0\;\; ; \tau_{xy} = 1.88\, MPa$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{14.72 + 0}{2} \pm \sqrt{\left(\frac{14.72 - 0}{2}\right)^2 + 1.88^2} = 7.36 \pm 7.60$$

$$\boldsymbol{\sigma_1 = 14.96\, MPa\;\; Ans.;\; \sigma_2 = -0.24\, MPa\;\; Ans}$$

Point B.

$$\sigma_B = \frac{P}{A} - \frac{M_x * z}{I} = \frac{16 * 1000}{7200} + \frac{3.6 * 10^6 * 20}{8640000}.$$
$$\sigma_A = -6.11\, MPa.$$

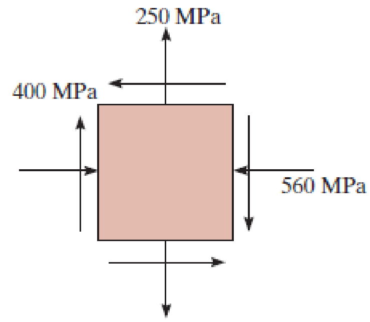
$$\tau_A = \frac{V_z Q_A}{It} = \frac{12 * 1000 * 96000}{8640000 * 60} = 2.22\, MPa.$$
$$\sigma_x = -6.11\, MPa;\;\; \sigma_y = 0\;\; ; \tau_{xy} = 2.22\, MPa$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{-6.11 + 0}{2} \pm \sqrt{\left(\frac{-6.11 - 0}{2}\right)^2 + 2.22^2}$$

$$\boldsymbol{\sigma_1 = 0.72\, MPa\;\; Ans.;\; \sigma_2 = -6.83\, MPa\;\; Ans}$$

•9–61. Determine the equivalent state of stress for an element oriented 60° counterclockwise from the element shown. Show the result on the element.



In accordance to the established sign convention, $\sigma_x = -560$ MPa, $\sigma_y = 250$ MPa and $\tau_{xy} = -400$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-560 + 250}{2} = -155 \text{ MPa}$$

Then, the coordinate of reference points A and C are

$$A(-560, -400) \qquad C(-155, 0)$$

The radius of the circle is

$$R = CA = \sqrt{[-560 - (-155)]^2 + (-400)^2} = 569.23 \text{ MPa}$$

Using these results, the circle shown in Fig. *a* can be constructed.

Referring to the geometry of the circle, Fig. *a*

$$\alpha = \tan^{-1}\left(\frac{400}{560 - 155}\right) = 44.64^\circ \qquad \beta = 120^\circ - 44.64^\circ = 75.36^\circ$$

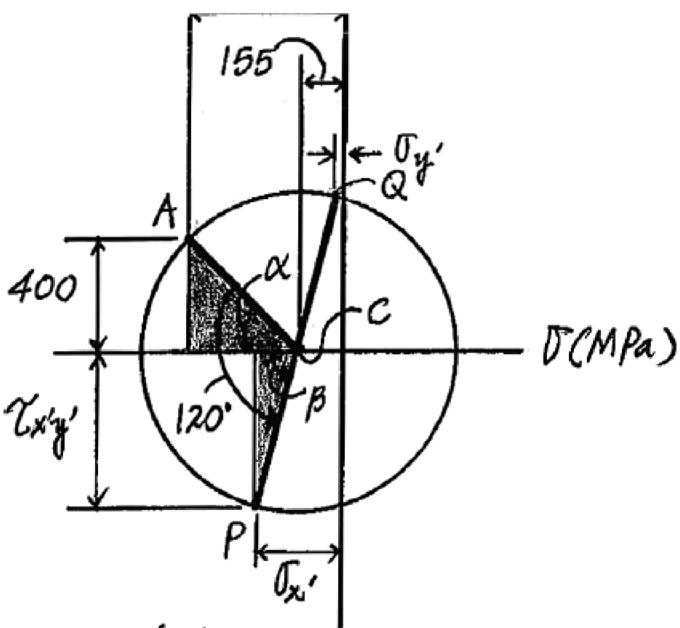
Then,

$$\sigma_{x'} = -155 - 569.23 \cos 75.36^\circ = -299 \text{ MPa}$$ **Ans.**

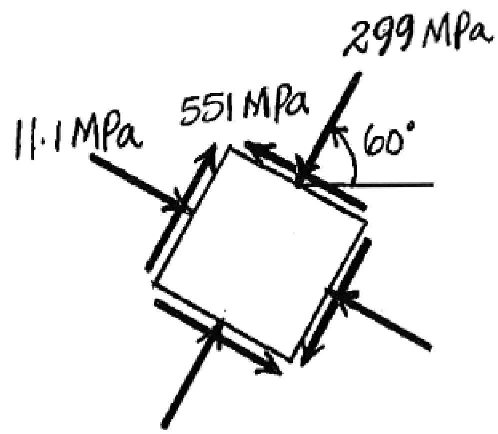
$$\tau_{x'y'} = 569.23 \sin 75.36^\circ = 551 \text{ MPa}$$ **Ans.**

$$\sigma_{y'} = -155 + 569.23 \cos 75.36^\circ = -11.1 \text{ MPa}$$ **Ans.**

The results are shown in Fig. *b*.

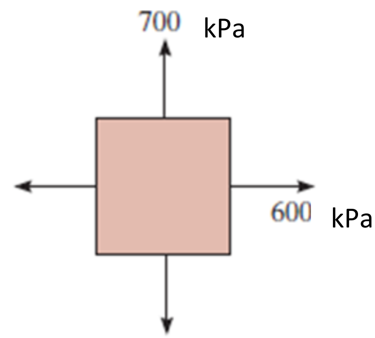


(a)

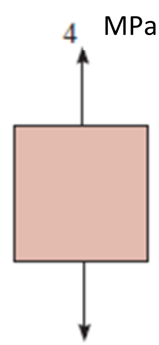


(b)

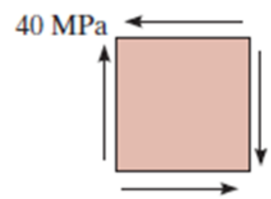
9–68. Draw Mohr's circle that describes each of the following states of stress.



(a)



(b)



(c)

a) Here, $\sigma_x = 600$ kPa, $\sigma_y = 700$ kPa and $\tau_{xy} = 0$. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{600 + 700}{2} = 650 \text{ kPa}$$

Thus, the coordinate of reference point A and center of circle are

$$A(600, 0) \qquad C(650, 0)$$

Then the radius of the circle is

$$R = CA = 650 - 600 = 50 \text{ kPa}$$

The Mohr's circle represents this state of stress is shown in Fig. *a*.

b) Here, $\sigma_x = 0$, $\sigma_y = 4$ MPa and $\tau_{xy} = 0$. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + 4}{2} = 2 \text{ MPa}$$

Thus, the coordinate of reference point A and center of circle are

$$A(0, 0) \qquad C(2, 0)$$

Then the radius of the circle is

$$R = CA = 2 - 0 = 2 \text{ MPa}$$

c) Here, $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = -40$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = 0$$

Thus, the coordinate of reference point A and the center of circle aı

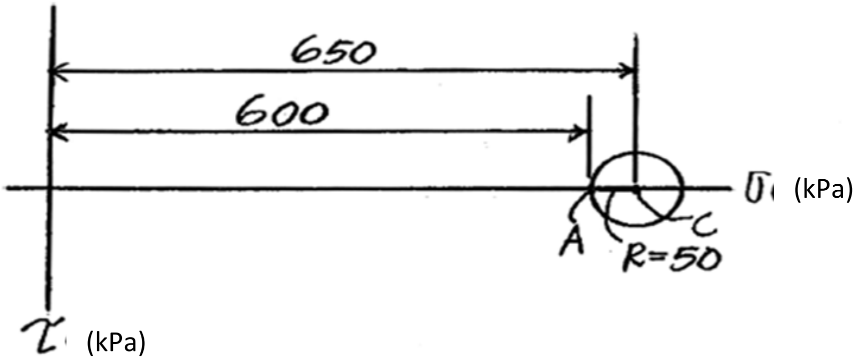
$$A(0, -40) \qquad C(0, 0)$$

Then, the radius of the circle is

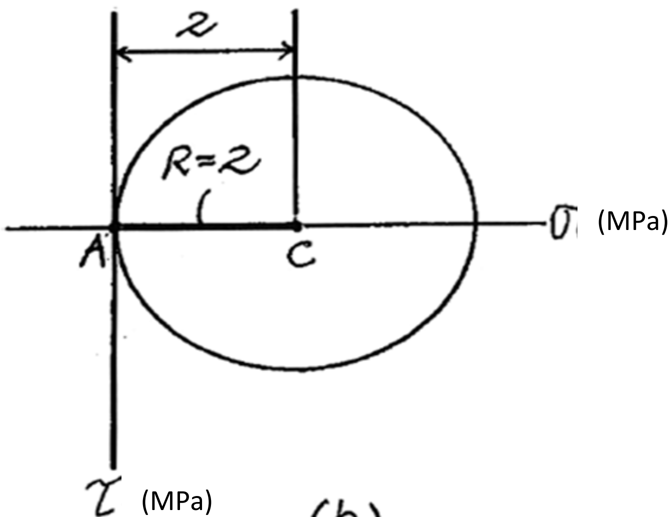
$$R = CA = 40 \text{ MPa}$$

The Mohr's circle represents this state of stress shown in Fig. *c*

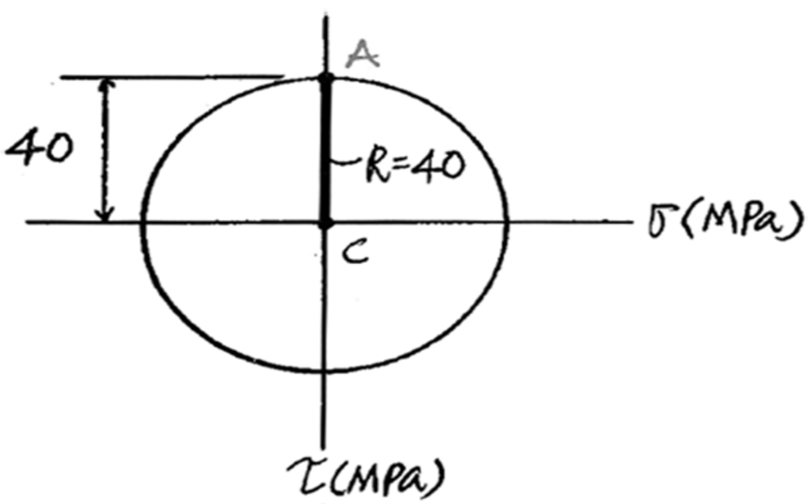
9–68. Continued



(a)



(b)



(c)

9–75. The 50 mm.-diameter drive shaft *AB* on the helicopter is subjected to an axial tension of 50 KN and a torque of 0.45 KN.m Determine the principal stress and the maximum in-plane shear stress that act at a point on the surface of the shaft.

$$\sigma = \frac{P}{A} = 50 * \frac{1000}{\pi * 25^2} = 25.47\ MPa.$$

$$\tau = \frac{Tc}{J} = \frac{0.45 * 10^6 * 25}{\frac{\pi}{2} * 25^4} = 18.34\ MPa.$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{25.47 + 0}{2} \pm \sqrt{\left(\frac{25.47 - 0}{2}\right)^2 + 18.34^2} = 12.74 \pm 22.33$$

$$\boldsymbol{\sigma_1 = 35.07\ MPa \quad Ans.\ ;\ \sigma_2 = -9.59\ MPa \quad Ans.}$$

$$\tau_{\max\ in-plane} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{25.47 - 0}{2}\right)^2 + 18.34^2}$$

$$= \boldsymbol{22.33\ MPa. \quad Ans.}$$