Key Solution

HOME WORK # 9

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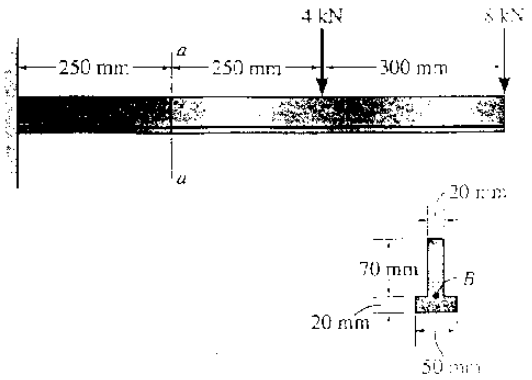
KEY TO HOMEWORK #9.

PROBLEM #① (7-7):-

7–7. Determine the shear stress at point B on the web of the cantilevered strut at section a–a.

***7–8.** Determine the maximum shear stress acting at section a–a of the cantilevered strut.



SOLUTION:-

REQUIRED:-

$\tau_B = ?$

$\because \tau = \frac{VQ}{It}$

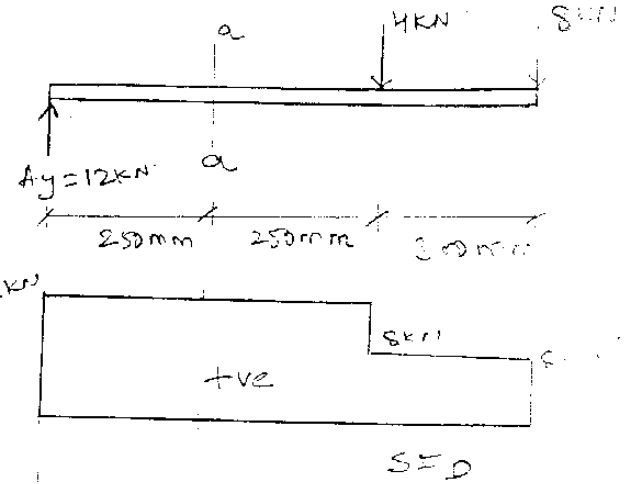
$V = 12$ kN at section a-a.

$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A}$

$\Rightarrow \bar{y}_{bot} = \frac{50\times20\times10 + 70\times20\times55}{50\times20 + 70\times20}$

$\Rightarrow \bar{y}_{bot} = 36.25$ mm.

$\bar{y}_{top} = 53.75$ mm.



$I = \frac{50\times20^3}{12} + 50\times20\times(36.25-10)^2 + \frac{20\times70^3}{12} + 70\times20(53.75-35)^2$

$\Rightarrow I = 1.786\times10^6$ mm^4.

$Q_{at\,B} = A'\bar{y}' = 20\times50\times\frac{(36.25-10)}{2}$.

$\Rightarrow Q = 13.125\times10^3$ mm^3.

& $t = 20$ mm.

$\therefore \tau_B = \frac{12\times10^3\times13.125\times10^3}{1.786\times10^6\times20}$

$= 4.41$ MPa

$\therefore \tau_B = 4.41$ MPa.

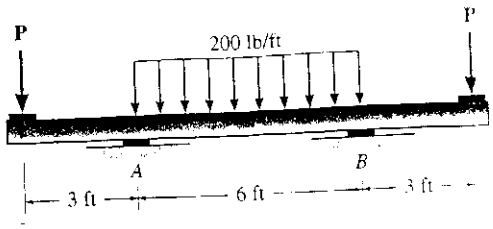
PROBLEM #② (7-17):-

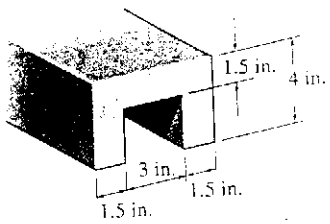
SOLUTION:-

REQUIRED:- P = ?

7–17. Determine the largest end forces P that the member can support if the allowable shear stress is $\tau_{allow} = 10$ ksi. The supports at A and B only exert vertical reactions on the beam.

7–18. If the force $P = 800$ lb, determine the maximum shear stress in the beam at the critical section. The supports at A and B only exert vertical reactions on the beam.





Probs. 7–17/7–18

$$\bar{y}_{top} = \frac{\Sigma A\bar{y}}{\Sigma A}$$

$$= \frac{6 \times 1.5 \times \frac{1.5}{2} + 2 \times 2.5 \times 1.5 \times \left(\frac{2.5}{2} + 1.5\right)}{6 \times 1.5 + 2 \times 2.5 \times 1.5}$$

$$\Rightarrow \bar{y}_{top} = 1.659 \text{ in}$$

$$\bar{y}_{bott} = 2.341 \text{ in}$$

$$I = \frac{6 \times 1.5^3}{12} + 6 \times 1.5 \times \left(1.659 - \frac{1.5}{2}\right)^2 + 2\left[\frac{1.5 \times 2.5^3}{12} + 1.5 \times 2.5 \times \left(2.341 - \frac{2.5}{2}\right)^2\right]$$

$$\Rightarrow I = 21.96 \text{ in}^4$$

$$Q = A'\bar{y}' = 2 \times 2.341 \times 1.5 \times \frac{2.341}{2}$$

$$\Rightarrow Q = 8.22 \text{ in}^3$$

$$\therefore \tau_{all} = \frac{VQ}{It}$$

taking $V = P$, $t = 3$ in

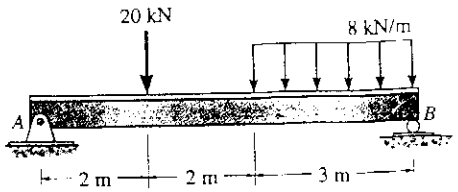
$$\therefore \tau_{all} = \frac{P \times 8.22}{21.96 \times 3} = 10 \text{ ksi}$$

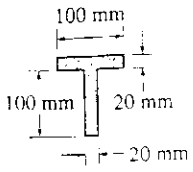
$$\Rightarrow P = 80.1 \text{ kip}$$

$$\therefore P = 80.1 \text{ kip}$$

PROBLEM NO:- (3) (7-19).-

7–19. The T-beam is subjected to the loading shown. Determine the maximum transverse shear stress in the beam at the critical section.





Prob. 7–19

SOLUTION:-

REQUIRED:- $\tau_{max} = ?$

$\because \tau_{max} = \dfrac{VQ}{It}$

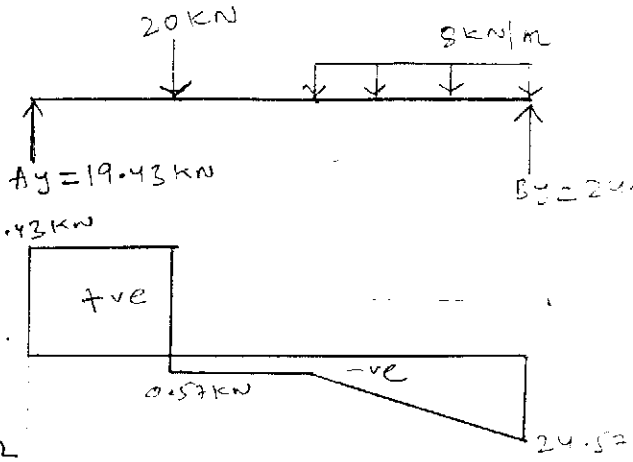
$V = V_{max} = 24.57\ kN$.

$\bar{y}_{top} = \dfrac{\Sigma A\bar{y}}{\Sigma A}$

$= \dfrac{100\times20\times10 + 20\times100\times70}{100\times20 + 100\times20}$

$= 40\ mm$

$\therefore \bar{y}_{top} = 40\ mm$ & $\bar{y}_{bot} = 80\ mm$.



S.F.D.

$\therefore I = \dfrac{100\times20^3}{12} + 100\times20\times(40-10)^2 + \dfrac{20\times100^3}{12} + 100\times20\times(80-50)^2$

$= 5.33\times10^6\ mm^4$.

$Q = A'\bar{y}' = 20\times80\times\dfrac{80}{2} = 64\times10^3\ mm^3$.

$t = 20\ mm$.

$\therefore \tau_{max} = \dfrac{VQ}{It} = \dfrac{24.57\times10^3\times64\times10^3}{5.33\times10^6\times20}$

$\therefore \tau_{max} = 14.75\ MPa$.

PROBLEM # 4 (7-23):-

SOLUTION:-

REQUIRED:-

τ at joints = ?

$$I = 2\left[\frac{8\times2^3}{12} + 8\times2\times(6-1)^2\right] + \frac{2\times8^3}{12}$$

$$= 896.0 \text{ in}^4.$$

$t = 2$ in.

Q at joint A.

$$Q_A = \bar{y}'A'$$

$$= 8\times2\times5 = 80 \text{ in}^3.$$

Shear stress $\tau = \frac{VQ}{It}$

$V = 800$ lb. (Maximum)

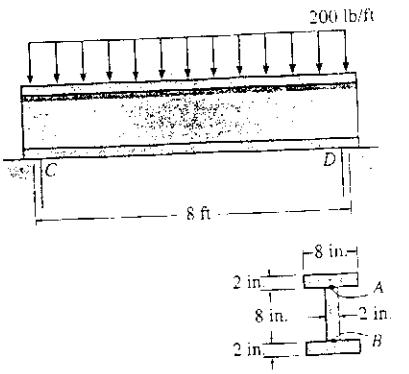
$$\therefore \tau_A = \frac{800\times80}{896\times2}$$

$$\Rightarrow \tau_A = 35.71 \text{ psi}.$$

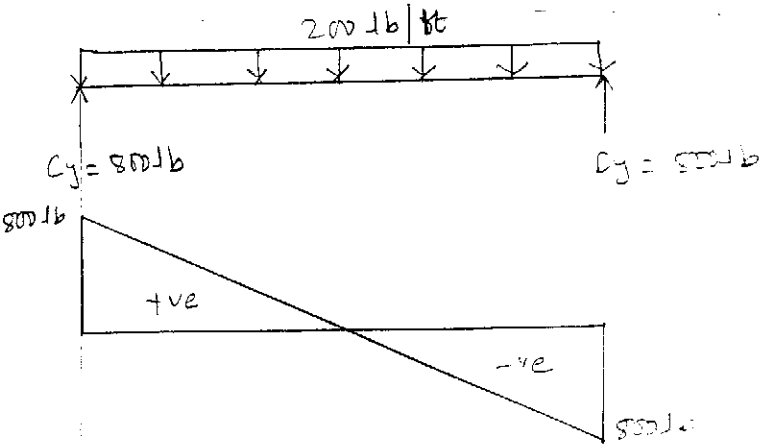
∴ Shear stress at joints is 35.71 psi.

7–23. The beam is made from three plastic pieces glued together at the seams A and B. If it is subjected to the loading shown, determine the shear stress developed in the glued joints at the critical section. The supports at C and D exert only vertical reactions on the beam.

***7–24.** The beam is made from three plastic pieces glued together at the seams A and B. If it is subjected to the loading shown, determine the vertical shear force resisted by the top flange of the beam at the critical section. The supports at C and D exert only vertical reactions on the beam.



Probs. 7–23/7–24



PROBLEM # 5 (7-33):-

SOLUTION:-

REQUIRED:-

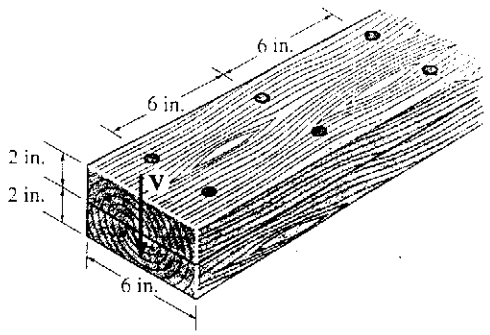
Shear force resisted by each nail = f = ?

GIVEN:-

V = 600 lb

Spacing = S = 6 in.

7–33. The beam is constructed from two boards fastened together at the top and bottom with two rows of nails spaced every 6 in. If an internal shear force of $V = 600$ is applied to the boards, determine the shear force resisted by each nail.



Prob. 7–33

$\therefore q = \frac{VQ}{I}$

$I = \frac{6 \times 4^3}{12} = 32 \text{ in}^4$

$Q = A' \cdot \bar{y}' = 2 \times 6 \times \frac{2}{2} = 12 \text{ in}^3$

$\therefore q = \frac{VQ}{I} = \frac{600 \times 12}{32} = 225 \text{ lb/in}$

Shear flow = $q = 225$ lb/in.

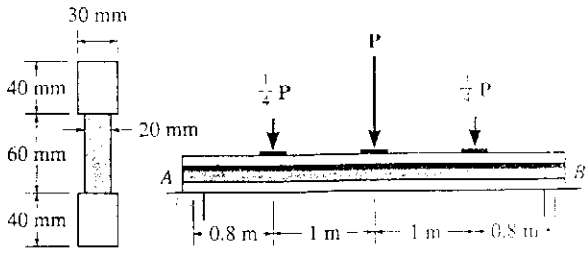
∴ Shear force resisted by each nail = f

$\Rightarrow f = \frac{qs}{2} = \frac{225 \times 6}{2} = 675 \text{ lb}$

$\therefore f = 675$ lb.

PROBLEM # 6 (7-39):-

7–39. The beam is made from three polystyrene strips that are glued together as shown. If the glue has a shear strength of 80 kPa, determine the maximum load P that can be applied without causing the glue to lose its bond.



Prob. 7–39

SOLUTION:-

REQUIRED:- P = ?

Glue shear strength = 80 kPa.

$V_{max} = \frac{3}{4}P$.

$$I = 2\times\left[\frac{30\times40^3}{12} + 30\times40\times(70-20)^2\right] + \frac{20\times60^3}{12}$$

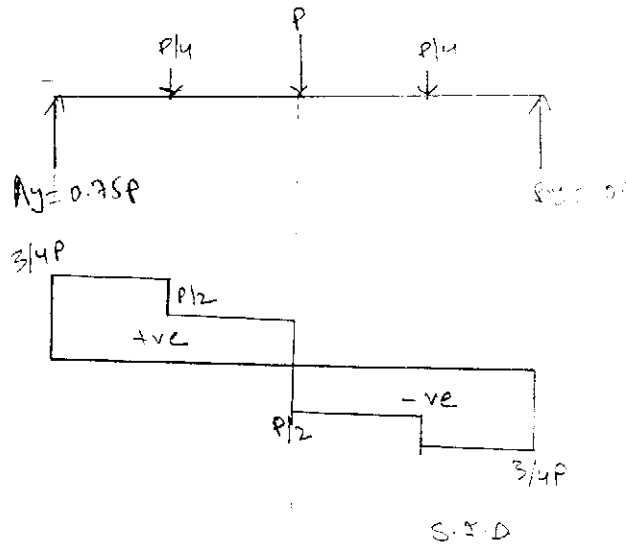
$$\Rightarrow I = 6.68\times10^6\ mm^4.$$

$t = 20\ mm$.

$Q = 5'A'$ (at glue level)

$= 30\times40\times50$

$= 60\times10^3\ mm^3$.



S.F.D

$\therefore$ Shear Stress $= \tau_{max} = \frac{VQ}{It}$

$$\Rightarrow \tau_{max} = \frac{3/4\ P\times69\times10^3}{6.68\times10^6\times20}$$

$\therefore \tau_{max} \leq 80\ kPa$.

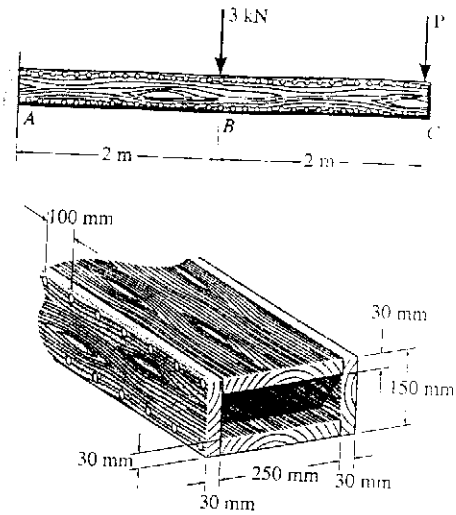
$$\therefore \frac{\frac{3}{4}P\times60\times10^3}{6.68\times10^6\times20} = 80\times10^3$$

$$\Rightarrow P = 237.6\ N.$$

$\therefore P = 238\ N$.

PROBLEM #7 (7-43):-

7–43. The beam is subjected to the loading shown, when $P = 7$ kN. Determine the average shear stress developed in the nails within region AB of the beam. The nails are located on each side of the beam and are spaced 100 mm apart. Each nail has a diameter of 5 mm.



Prob. 7–43

SOLUTION:-

REQUIRED:-

Average Shear Stress developed in the nails = ?

$\tau_{nail} = ?$

$V = V_{max} = 10\,kN$

$$I = 2\left[\frac{250 \times 30^3}{12} + 250 \times 30 \times (75-15)^2 + \frac{1}{12} \times 30 \times 150^3\right]$$

$$\Rightarrow I = 72 \times 10^6\ mm^4.$$

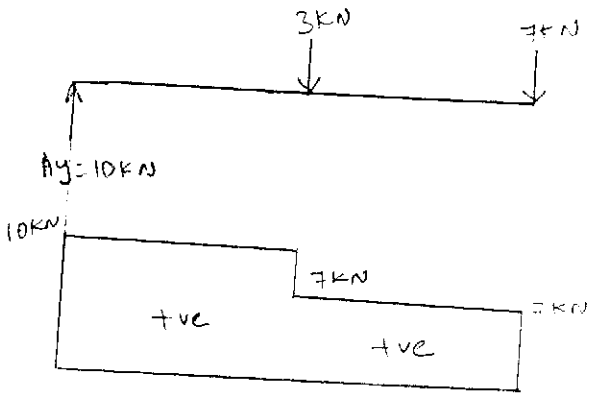
$$Q = A'\bar{y}' = 250 \times 30 \times (75-15)$$

$$= 45 \times 10^4\ mm^4.$$

$$\therefore Q = 45 \times 10^4\ mm^4.$$

$$q = \frac{VQ}{I} = \frac{10 \times 10^3 \times 45 \times 10^4}{72 \times 10^6}$$

$$\Rightarrow q = 62.5\ N/mm.$$



SFD

for each row shear flow is

$$\frac{q}{2} = \frac{62.5}{2} = 31.25\ N/mm.$$

Shear force in each nail $= 31.25 \times 100 = 3125\ N.$

$$\therefore \tau_{nail} = \frac{3125}{\frac{\pi}{4}(5)^2} = 159\ N/mm^2$$

$$\Rightarrow \tau_{nail} = 159\ MPa.$$