Modeling of jointed rock

(1), (2)

o Strength as a function of joint orientation:

From before:
$$\begin{split} \tau_{\text{max}} &= S_{\text{J}} + \sigma_{\text{d}} \ \text{tan} \ \phi_{\text{J}} \\ \sigma_{\text{d}} &= \frac{1}{2} \left(\sigma_{\text{1}} \! + \! \sigma_{\text{3}} \right) + \frac{1}{2} \left(\sigma_{\text{1}} \! - \! \sigma_{\text{3}} \right) \cos 2 \ \beta \\ \tau_{\text{d}} &= \frac{1}{2} \left(\sigma_{\text{1}} \! - \! \sigma_{\text{3}} \right) \sin 2 \ \beta \end{split}$$

Combining $\Rightarrow \frac{1}{2} (\sigma_1 - \sigma_3) \sin 2 \beta = S_J + \left[\frac{1}{2} (\sigma_1 + \sigma_3) + \frac{1}{2} (\sigma_1 - \sigma_3) \cos 2 \beta \right] \tan \phi_J$ after Algebraic

$$\frac{\sigma_{_{I}}}{S_{_{J}}} = \frac{2 + 2\left(\frac{\sigma_{_{3}}}{S_{_{J}}}\right) \tan \, \phi_{_{J}}}{(1 - \cos \beta \, \tan \, \tau_{_{J}}) \sin \, 2 \, \beta} + \frac{\sigma_{_{3}}}{S_{_{J}}}$$

 $\sigma_1 \& \sigma_3$: stresses at failure

- For what range of β , failure will be by sliding along joint??
- * Conditions for sliding Along a Discontinuity:

Failure criteria for intact rock: $\tau_{max} = S_R + \sigma \tan \phi_R$

Failure criteria for discontinuous rock: $\tau_{max} = S_J + \sigma \tan \phi_J$ if $\sigma > \sigma_T$

Patton's law: $\tau_{max} = \sigma \tan (\phi_u + i)$ if $\sigma < \sigma_T$

 $\, \Downarrow \,$

If
$$\beta > \frac{2\beta_1}{2}$$
 $< \frac{2\beta_2}{2}$ if sliding

if not \Rightarrow new crack

Failure will occur along the discontinuity of $2\beta_1<2\beta<2\beta_2$ or $~\beta_1<\beta<\beta_2.$

alternatively $\psi_1 + \psi > \psi_2$

Note: The value of β_1 & β_2 are function of

$$\beta_1 \;\&\; \beta_2 = f\left(\sigma_1,\,\sigma_3,\,\varphi_R,\,S_{R,}\,S_{J},\,\varphi_u,\,i,\,\sigma_T\right.$$

Failure is due to sliding along discontinuity

Failure through intact rock.

H.W. #7

Water Pressure affects in Discontinuous Rock:

$$\sigma' = \sigma_T - p_w$$

Pore water pressure required to cause failure must be considered separately for each of the three modes of failure.

For a particular ψ

a) Failure by riding over asperities

$$P_{w} = \sigma_{3} + (\sigma_{1} - \sigma_{3}) \left\{ \sin^{2} \psi - \frac{\sin \psi \cos \psi}{\tan (\phi_{u} + 2)} \right\}$$

b) Failure by shearing through asperities:

$$P_{w} = \frac{S_{J}}{\tan \phi_{J}} + \sigma_{3} + (\sigma_{1} - \sigma_{3}) \left\{ \sin^{2} \psi - \frac{\sin \psi \cos \psi}{\tan \phi_{J}} \right\}$$

c) Failure at intact rock:

$$P_{w} = \sigma_{3} - \frac{\left[(\sigma_{1} - \sigma_{3}) - 2 S_{R} \tan (45 + \frac{\phi R}{2}) \right]}{\tan^{2} (45 + \frac{\phi R}{2}) - 1}$$

Compute all three to find Pw minimum required to cause failure.

- 11. Deformability of rock masses (rather than intact rock) see Ch. 6 text.
- 11.1 <u>Def.</u>:

Modulus of permanent deformation.

M

- 11.2 In-situ Tests for determining rock mass modulus:
- 11.2.1 Plate Bearing test:
 - i) Reaction Against Anchor
 - ii) Reaction Against an opposite wall

Theory:
$$\varpi = \frac{G p(1-v^2)a}{E}$$

where $\overline{\omega}$ = displacement measured (average of three readings)

p = plate pressure (applied)

a = plate radius

 $G = \pi/2$ if plate is rigid or 1.7 if plate is flexible.

Typical Results:-

$$E = C a (1 - v^2) \frac{P}{\overline{\omega}_{elastic component}}$$

mass modulus

$$M = C a (1 - v^2) \frac{P}{\varpi - \varpi_{\text{elastic}}}$$

If extensometers are used:

$$E = \frac{(1+\nu) P}{\varpi} \left[\frac{-Z^2}{\sqrt{Z^2 + a^2}} + 2 (1-\nu) \sqrt{Z^2 + a^2} - (1-2\nu) \right]$$

 ϖ = displacement at depth Z.

See fig. pp.58

11.2.2 Borehole Dilatometer: pressure a borehole $\rightarrow \Delta u$

See fig. pp. 59

$$E = (1 - v)\Delta P \frac{a}{\Delta u}$$

easy test

11.3 Modulus of Fractured Rock (Rock mass) from intact Rock & Joint properties

A: Joint Testing

B: Intact Rock stress-strain test

elastic compression of intact rock

$$= \frac{\sigma_n}{E_{rock}} \cdot S \qquad \qquad \text{(assuming S much larger than joint width)}$$

compression of joint
$$\delta = \frac{\sigma_n}{K_n}$$

Total Deformation =
$$\frac{\sigma_n}{E_{rock}} S + \frac{\sigma_n}{K_n}$$

Strain in the rock mass =
$$\epsilon_{rock \; mass} = \frac{Total \; Deformatio \; n}{S}$$

$$= \frac{\sigma_{n}}{E_{rock}} + \frac{\sigma_{n}}{K_{n}S}$$

$$E_{\text{rock mass}} = \frac{\sigma_{\text{n}}}{\epsilon_{\text{rock mass}}} = \frac{\sigma_{\text{n}}}{\frac{\sigma_{\text{n}}}{E_{\text{rock}}} + \frac{\sigma_{\text{n}}}{K_{\text{n}}S}} = \frac{E_{\text{rock}} \cdot K_{\text{n}}S}{E_{\text{rock}} + K_{\text{n}}S}$$

Derivation for $G_{rock \ mass}$ follows in the same manner.

$$G_{\text{rock mass}} = \frac{G_{\text{rock}} \cdot K_s \cdot S}{G_{\text{rock}} + K_s \cdot S}$$

- \rightarrow Do both tests, to to field to measure S
- \rightarrow Do test on rock, go to field & do tests with small \Rightarrow Extrapolate for larger S.
- 11.4 Rock Indices for estimating E_{rock mass}: (approximate)
- a. <u>Bieniawski (1978)</u>: E(GPa) = 2(RMR) 100 for RMR 55
- b. using Modulus Reduction Factor = $\frac{E_{rock mass}}{E_{rock}} = f (RQD)$

see Fig. 7.2 from US army corp of engg.

H.W. # 8 Ch. #4 & 9

12. Slope Stability in Rocks

More complicated than soil _____ it can fail in tension

12.1 Types of Failure

- i. plane sliding (one joint of orientation is significant)
- ii. wedge-type failure (Two joints sets (or more) are significant

iii) Toppling failure

12.2 <u>Plane Sliding</u>:

a) <u>Kinematics</u>

 $\begin{array}{ll} \text{if } \delta > a & \quad \text{no failure} \\ \text{if } \delta < a & \quad \text{failure} \end{array}$

```
For failure to be kinematically feasible \delta < \alpha
(the discontinuity must (daylight))
o stereographic plot presentation for (a)
         represents dip vector \delta_1
if \delta = \delta_1 failure is kinematically possible
if \delta = \delta_2 failure is not possible
it is enough for \delta to be >\alpha failure to failure, but \delta have to be >\delta_{min}.
         strength: (\phi):
b)
         since \phi > \alpha: block will <u>not</u> slide.
         ∴ \phi < \alpha block will slike.
         o stereonet presentation for (b)
         If dip rector plots in this area, failure
         can't occur.
```

∴ joint is not steep enough

stereonet for (a) & (b).

Combining the 2 criteria:

failure can only <u>occur</u> if dip vector plots in shaded area.

joint must both daylight and its angle must exceed $\varphi_{J}.$

Note: This only due to self weight of block.

- still wt. of adjacent blocks.
- still wt. of adjacent water pressure.

Example of using stereonet for vector addition:

Ex.1 given two forces: $200 = acting in the direction N80^{\circ}W plunging 50^{\circ}$ $600 = acting in the direction S40^{\circ}W plunging 20^{\circ}$

Find the resultant force and its direction.

Solution: (Procedure)

1. Find the common plane of the two forces.

rotate until they plot on same circle. \rightarrow angle = 55°

2. Determine the resultant force by parallelogram theorem. ($\underline{\text{draw}}$ to scale). Find resultant. Find angle = 13° .

3. Plot direction of the resultant on stereonet.

S50°W, 290⁰

Forces acting on rock blocks:-

- 1. Block self weight.
- 2. Forces transmitted from adjacent blocks.
- 3. Forces due to water pressure.
- 4. Dynamic loads.
- 5. Reinforcement (rock bolts).

if $\phi < \phi_J$ stable

How to plot φ-circle?

To plot the friction circle on stereonet:

- 1. Locate the normal to the failure plane, $\hat{\mathbf{n}}$
- 2. Plot two points on the diameter of the circle each being ϕ_J^o from the normal (measured along the diameter).
- 3. Construct φ-circle having the line between the two points (in step # 2) as the diameter.

If resultant vector, î is inside the circle, failure doesn't occur.

factor of safety against sliding

$$F.S. = \frac{tan \ \phi_{available}}{tan \ \phi_{required}}$$

Ex. 1. A joint strikes S30°E and dips 60° NE

$$\phi_J\,25^o$$

resultant of all forces \hat{r} acts in the direction S50°W and plunges 20°,

Find the factor of safety.

<u>Procedure</u> 1. Plot the friction circle for the joint.

- 2. Plot resultant force $\hat{\mathbf{r}}$.
- 3. Find angle between \hat{r} and \hat{n} . (put them in same greater circle)

$$\Rightarrow$$
 $\phi_{regd.} = 15^{\circ}$

4.
$$\therefore \phi_{\text{available}} = \phi_{\text{J}} = 25^{\circ}$$

$$\therefore$$
 F.S. = $\frac{\tan 25}{\tan 15} = 1.74$

Ex. 2 Assume that the only force acting on the block is its <u>self-weight</u>. Determine the minimum bolt force and direction required to raise the F.S. to 1.0 for a block weighing 20 tons.

i.e. dipping
$$90^{\circ} \Rightarrow \text{plot } @ \text{ the center of streonet.}$$

 $\hat{\mathbf{w}} = \mathbf{weight} \ \mathbf{vector}$

$$\rightarrow$$
 \hat{w} not inside friction circle \Rightarrow F.S. $<< 1.0$.

<u>Procedure</u>: 1) \hat{w} will plot @ center of streonet \Rightarrow F.S. << 1.0.

2) Find the point on the ϕ -circle at which \hat{r} will make the smallest angle with \hat{w} .

if bolt driven S60°W

3) Construct force diagram

Determine a line of minimum length from tip of \hat{w} to the \hat{r} line. 4.

|B| = 11 Tons 35° up from horizontal.

Ex. 3. What bolt force would be required to achieve F.S. = 2.5.

$$\underline{Sol.} \hspace{0.5cm} F.S. = \frac{tan \hspace{0.1cm} \varphi_{available}}{tan \hspace{0.1cm} \varphi_{reqd.}} = \frac{tan \hspace{0.1cm} 25^{\circ}}{tan \hspace{0.1cm} \varphi_{reqd.}} = 2.5$$

$$\tan \, \phi_{reqd.} = \frac{\tan \, 25^{\circ}}{2.5} =$$
 $\Rightarrow \, \phi_{reed.} = 10.5^{\circ}$

min. angle between \hat{w} and $\phi_{circle} = 49.5^{\circ}$

$$|B| = 15 \text{ Tons}$$

Ex. 4. What both force would be required if the bolt are to be driven due west.

Analysis of Plane Slides by traditional Block Sliding Analysis

Ref. Book (Hock & Bray 1977)

#1.

Crack intercepts crest of slope

$$W = \frac{1}{2} \gamma H^{2} \left[\left(1 - \left(\frac{Z}{H} \right)^{2} \right) \cot \delta - \cot \alpha \right]$$

#2.

Crack intercepts face of the slope.

$$W = \frac{1}{2} \ \gamma \ H^2 \left[(1 - \frac{Z}{H})^2 \ \cot \ \delta \ (\cot \ \delta \ \tan \ \alpha - 1) \right]$$

Area of sliding surface:
$$A = \frac{H - Z}{\sin \delta} * 1$$

Resultant of water pressure along the vertical crack.

$$V = \int_{\rm o}^{Z_{\rm w}} Z \, dZ = \frac{1}{2} \, \gamma \, Z_{\rm w}^2$$

 $\begin{aligned} & \text{Resultant of water pressure acting on the sliding} \\ & \text{surface} = \ U = \gamma_w \ \frac{Z_w}{2} \cdot A \quad = \frac{1}{2} \ \gamma_w \ Z_w \ A. \end{aligned}$

Shear Forces Along Sliding Surface:

Forces resisting shear:

$$(S_J + {\sigma_n}' \tan \phi_J) \; A$$

$$\sigma_n' = \sigma - u = \frac{W \cos \delta - V \sin \delta - U}{A}$$

Sliding occurs when the driving shear forces = forces resisting shear.

F.S. (W sin
$$\delta$$
 + V cos δ) = S_J A + (W cos δ - V sin δ - U) tan ϕ _J

For Case #1:

$$\begin{aligned} \text{F.S.} \left[\frac{1}{2} \, \gamma \text{H}^2 \, \left[\left(1 - \left(\frac{Z}{\text{H}} \right)^2 \right) \cot \delta - \cot \alpha \right] \sin \, \delta + \frac{1}{2} \, \gamma_w \, Z_w^2 \, \cos \, \delta \right] \\ &= S_J \, \frac{\text{H-Z}}{\sin \delta} + \left[\frac{1}{2} \delta \, \text{H}^2 \, \left[\left(1 - \left(\frac{Z}{\text{H}} \right)^2 \right) \cot \, \delta - \cot \alpha \right] \cos \, \delta \\ &- \frac{1}{2} \, \gamma_w \, Z_w^2 \, \sin \, \delta - \frac{1}{2} \, \gamma_w \, Z_w \, A \right] \tan \, \phi_J \end{aligned}$$

Since we have some control over α_1 , we can solve for it.

$$\cot \alpha = \frac{\left[a\left(F.S. \sin \delta - \cos \delta \tan \phi\right) + U \tan \phi + V \left(\sin \delta \tan \phi + FS.\cos \delta\right)\right] - S_{J}A}{b\left(F.S. \sin \delta - \cos \delta \tan \phi\right)}$$

where
$$a = \frac{1}{2} \gamma H^2 \left(1 - \left(\frac{Z}{H} \right)^2 \right) Cot \delta$$

$$b = \frac{1}{2} \gamma H^2$$

 ΔS_J is more important for steep slopes (α is large)

 $\Delta \phi_{\rm J}$ is more important for high slopes (H is large)

Drainage of water can be very effective in stabilizing rock slopes.

- * Wedge Failure Analysis
- a) Kinematics: for sliding of a wedge, use \hat{I} (the interaction of 2 planes in place of the dip vector \hat{D} .
- \therefore if the plunge of \hat{I} is less than α , the wedge sliding is kinematically possible.
- Find normals to two planes $\hat{\mathbf{n}}_1$, $\hat{\mathbf{n}}_2$
- Rotate until $\,\hat{n}_{_1}\,\&\,\,\hat{n}_{_2}\,$ on one greater circle.

Failure can occur in one of three ways.

- 1. if \hat{D}_1 daylights
- 2. if \hat{D}_2 daylights plane sliding
- 3. if \hat{I}_{12} daylights \rightarrow sliding

If we have 3 sets of joints; Failure can occur on:

- 1. \hat{D}_1 daylight
- 2. \hat{D}_2 daylight
- 3. \hat{D}_3 daylight
- 4. \hat{I}_{12}
- 5. \hat{I}_{13}
- 6. \hat{I}_{23}
- b) Strength: If ϕ_J > plunge of \hat{I} failure cannot occur. Furthermore, for very acute (steep) wedges, considerable strength is obtained from roughness along the discontinuities, so \hat{I} can often be steeper than ϕ_J without failure.

Conventional Wedge Analysis

Ref. Hock & Bray "Rock Slope Engineering" Institution of Mining & Metallurgy 1981

Geometry:

 $\psi_{\rm fi}$ = inclination of slope, slope measured in the view at right angles to the line of intersection.

 $\phi_{\rm J}$ = joint friction angle.

actual slope is
$$> \psi_{f2}$$

 ψ_I = plunge of line of intersection.

Sliding will occur if $\psi_{fi} > \psi_I > \phi$

Forces:

Assume sliding is resisted by friction only.

$$F.S. = \frac{\left(R_A + R_B\right) \tan \phi_J}{W \sin \psi_i}$$

$$\Sigma F_H \implies R_A \sin \left(\beta - \frac{1}{2} \xi\right) = R_B \sin \left(\beta + \frac{1}{2} \xi\right)$$

 $\Sigma \; F_r \; along \; line \; of \; intersection \; \; \Rightarrow \; R_A \; \; Cos \; \left(\beta - \frac{1}{2} \xi\right) - \; R_B \; \; Cos \; \left(\beta + \frac{1}{2} \xi\right) = W \; Cos \; \psi_i$

Solving for RA & RB and adding

$$R_A + R_B = \frac{W \cos \psi_i \sin \beta}{\sin \frac{1}{2} \xi}$$

$$\therefore \quad F.S. = \frac{\sin \beta}{\sin \frac{1}{2} \xi} \cdot \frac{\tan \phi_J}{\tan \psi_i}$$
f.s. for plane sliding

$$(F.S.)_{wedge} = K * (F.S.)_{plane}$$

Fig. 96:

 ξ : angle between joints

 β : angle of tilt of line of intersection

Note: This is limited to joints having their normals coplan (same plan) 180° in phase.

$$F.S. = A \ tan \ \phi_A + B \ tan \ \phi_B$$

<u>Ex</u> .		Dip	Dip. direction	φ_{J}
	Plane A Plane B	40° 70°	165 285	35° 20
	Difference	30°	120	

$$A = 1.5$$
, $B = .7$

F.S. = $1.5 \tan 35 + 0.7 \tan 20 = 1.3$

This method is good better : $\xi \& \beta$ is difficult to find in field.

If F.S. ≥ 2 : Wedge failure is almost impossible.

o Wedge Analysis on stereographic projection.

Text pp.--- Fig. 8.16

- 1. Find $\hat{\mathbf{n}}_1$ and, $\hat{\mathbf{n}}_2$ then $\hat{\mathbf{I}}_{12}$
- 2. Draw greats circle through $\,\hat{n}_{_1}\,,\,I_{12}$ and $\,\hat{n}_{_2}\,,\,I_{12}$
- 3. Construct ϕ circles for n_l and $n_{\underline{p}}$. This gives the four points of intersection: p, q, s & t.
- 4. Construct great circles connecting p.s. and q.t.

if resultant of all forces.

Toppling Failure

a) flexural toppling

- b) block toppling
- c) combination of flexural and toppling
- Static Analysis of Block Toppling
- a. Single block:

if W is to left of \Rightarrow Failure

condition if impending failure

$$\cot \alpha = \frac{\Delta y}{\Delta x}$$

Cot
$$\alpha < \frac{\Delta y}{\Delta x}$$
 \Rightarrow failure will occur.

Long narrow blocks on steep slopes are more susceptible to failure.

b. Multiple blocks:

Resisting Moments:

$$P_{n-1}(L_n) + \mu P_n \Delta x + W \cos \alpha \frac{\Delta x}{2}$$

Overturning Moments

$$P_n(M_n) + \sin \alpha \frac{\Delta y}{2}$$

Setting R.M. = O.M. , toppling will occur if

$$P_{_{n-1}} < \frac{P_{_{n}}\left(M_{_{n}} - \mu \Delta x\right) + \frac{_{W}}{^{2}}\left(\Delta y \ sin \ \alpha - \Delta x \ cos \ \alpha\right)}{L_{_{n}}}$$

Many equations

Ref. Zanbak, Caner "Design Charts for Rock Slopes Susceptible to Toppling" J. of Geot. Engg., ASCE, Vol. 109, No. 8.

Stereographic Analysis for Flexural Toppling:

major principle direction∴ no stress normal to slope

Topping will occur only if there is relative motion between dipping layer.

slippage must occur

Criterion for toppling failure: resultant should be outside ϕ -circle.

If we lay off an angle ϕ_J from the normal to the dipping beds, and it falls outside of the slope, then failure will occur because the direction of applied compression is outside of the ϕ -circle.

Toppling will occur if $(90-\delta) + \phi_J < \alpha$

Or
$$\delta > 90 + \phi_J - \alpha$$

Toppling No Toppling

If \hat{n} in this region, flexural toppling will occur if strike is

if $> 30^{\circ}$: toppling

is not towards excavation.

H.W. #8

Ch. 8 Prob. # 1: Use: Set 1 Strike N40°E

Prob. # 2: Use $\phi_{J} = 30^{\circ}$

Prob. # 3: Use: dip 55°NE

P = 600 Tons

 $\phi = 35^{\circ}$

	 Excessive settlement → Bearing capacity 	compressibility of joints
Fig. 9.1	1	
	Karstic: Ca O ₂ , dissolvable	sinkable in Florida
a)	Shallow Foundations:	
		spread footings
b)	Deep Foundations H. piles Precast conc. Piles Pipe piles	

Foundation on Rock

13.0

Pier Socketed into rock

13.1 Shallow Foundations:

For intact rock in its elastic range:

Settlement may be predicted by:

$$u = \frac{C.P (1 - v^2)a}{E}$$

: plate bearing

where

$$C = -\pi/2$$
 if rigid
- 1.7 if flexible

P: applied stress

E: Young's Modulus

v : Poisson's ratio

a: radius of footing (or equivalent)

o If rock is not homogeneous se same as soil

Stress distribution beneath footings.

- a) for homogeneous isotropic rock, elasticity solution are generally available. \rightarrow point load integrate to get it for any shape.
- b) for heterogeneous, anisotropic rock, finite element methods may be required.

c) Fig. 9.7

Line load

Vertical

<u>Horizontal</u>

if there is a joint then tension will in opening crack \Rightarrow stress will not be transmitted if total stress is tension.

Inclined

Fig. 9.8 not intact rock

resultant can't go outside ϕ_J

 \Rightarrow slippage \Rightarrow realignment of stresses

Close Form Solution

1. Resolve Q & P

// & \perp to pidding planes

1977 Bray

$$\sigma_{r} = \frac{h}{\pi r} \left[\frac{X \cos \beta + yg \sin \beta}{(\cot \beta - g \sin^{2} \beta)^{2} + h^{2} \sin^{2} \beta \cos^{2} \beta} \right]$$

where
$$g = \left(1 + \frac{E}{(1 - v^2)k_n S}\right)^{1/2}$$

$$h = \left\{\left(\frac{E}{1 - v^2}\right)\left[\frac{2(1 + v)}{E} + \frac{1}{k_s \cdot S}\right] + 2\left(g - \frac{v}{1 - v}\right)\right\}^{1/2}$$

 E, ν : are intact rock properties

 k_n , k_s : are joint stiffnesses

S = Spacing

Fig. 9.9

* Strains and settlements beneath loaded rock masses.

Procedure

- 1. determine the stress distribution.
- 2. determine the equivalent elastic properties

$$G_{ns} = G_{nt} = \frac{G_{Rock} K_s S}{G_{Rock} + K_s S}$$

Similarly

$$E_n = \frac{E_{Rock} K_n S}{E_{Rock} + K_n S}$$

$$E_s = E_t = E_{rock}$$

$$v_{sn} = v_{tn} = \frac{\varepsilon_n}{\varepsilon_s} = v_{rock}$$

by symmetry,

$$\begin{split} \frac{\nu_{sn}}{E_s} &= \frac{\nu_{ns}}{E_n} \\ & \therefore \quad \nu_{ns} = \nu_{nt} = \frac{E_n}{E_s} \nu_{sn} \\ & = \frac{E_{rock} K_n S}{E_{rock} + K_n S} \cdot \frac{\nu_{rock}}{E_s = E_{rock}} \\ & = \frac{K_n S \cdot \nu_{rock}}{E_{rock} + K_n S} \end{split}$$

3. Employ the constitutive relationship for transversely isotropic media:

Bearing Capacity of Shallow Foundations on Rock.

Fig. 9.5

 \therefore The largest horizontal stress that can be developed to support the rock beneath the footing is the unconfined compressive strength, q_u .

since
$$\sigma_{1f} = \sigma_{3f} \tan^2 (45 + \phi/2) + q_u$$

:.
$$q_f = q_u \tan^2 (45 + \phi/2) + q_u$$

$$q_f = q_u \: (N_{\varphi} + 1)$$
 bearing capacity factor $\Rightarrow N_{\varphi} = tan^2 \: \left(45 \, + \, \frac{\varphi}{2}\right)$

$$q_{all} = \frac{q_f}{F.S.} = \frac{q_f}{3}$$
 especially F.S. = 3.0

o Table 9.2: most rock of the region is very conservative

New York Detroit \rightarrow largest values \rightarrow highest buildings.

Drop Foundations on Rock:

a) bearing capacity increases with depth due to increase in confinement.

$$\sigma_v \neq 0 \implies$$

b) bearing capacity doesn't increase with depth.

<u>Settlement of Deep Foundations:</u> - rock

- pier: pile or corrosion

a) due to rock deflection for

elastic material :
$$w_{base} = \frac{\frac{\pi}{2} P_{end} (1 - v^2) a}{E \cdot n}$$

n = f (embedment depth, radius and v)

√/a v	0	2	4	6	8	14
0	1.0	1.4	2.1	2.2	2.3	2.4
.3	1.0	1.6	1.8	1.8	1.9	2.0
.5	1.0	1.4	1.6	1.6	1.7	1.8

b) due to concrete deformation:

$$\omega_{\text{concrete}} = \frac{P \ell_c + \ell}{E_c}$$

But P is not constant along the pile.

c) Correction for load transfer due to side friction:-Neglect side friction through the soil.

derived in Appendix: same as one for consolidation

$$\sigma_{Z} = P_{\text{max}} \cdot e - \left[\frac{2 v_{c} \tan \phi_{rc}}{1 - v_{c} + (1 + v_{r}) \frac{E_{c}}{E_{r}}} \cdot \frac{Z}{a} \right]$$

 ν_c , $\nu_r\,$: Poisson's ratio of concrete & rock

 $\varphi_{cv}: \ friction \ angle \ for \ rock/concrete \ \& \ interface$

 E_c , E_v : Young's moduli for concrete & rock.

$$\therefore \Delta w = \frac{1}{E_c} \int_o^\ell (P_{max} - \sigma_z) dZ$$

d) : Total Deformation (settlement) = $w_{base} + w_{concrete} - \Delta w$

Determining depth of "socketing" into rock to insure against bearing capacity failure.

(1) <u>Ref</u>: Osterberg & Gill F.E.M. study (1971)

found that even small embedment (socketing) into rock greatly reduces P_{end} by taking load in side friction.

Therefore shaft diameters can be reduced.

Fig. 9.18

- (2) Ladanyi (1977) proposed a procedure for determining socketing depth.
- o Assumptions:
 - a) no load transfer along concrete/soil interface.
 - b) bond between concrete and rock broken⇒ use residual strength.

$$\sigma_{max} = \frac{P}{\pi a^2}$$

$$\sigma_z = \sigma_{max} e$$

For soft rock:

$$\tau_{residual} = \alpha S_u$$

S_u: undrained shear strength

 α : reduction factor, $0.3 < \alpha < 0.9$, typically $\alpha = 0.5$

$$\tau_{\rm res.} \simeq \frac{q_{\rm u}}{20}$$

where $q_u = unconfined comp.$ strength.

Procedure:

1. Determine shaft diameter "a" based on concrete strength.

2. Assume that
$$Q_p=0$$
 , then $\ell=\ell_1=\frac{P}{2\,\pi a\,\,\tau_{all}}$

- 3. Choose $\ell = \ell_2$ such that $\ell_2 < \ell_1$.
- 4. Compute σ_Z @ $Z = \ell_2$.
- 5. If σ_Z)@ $Z = \varrho > q_{all}$, then assume a new ℓ_2 .
- 6. If σ_Z)@ $Z = \ell^2 \le q_{all}$, determine τ along perimeter of pile

$$\tau = \frac{1}{2 \pi a \ell_2} \left[\left(1 - \frac{\sigma_Z)_{@\ell_2}}{\sigma_{max}} \right) P \right]$$

- 7. Repeat the procedure to find the shortest " ℓ " for which $\sigma)_{@\,\ell} \leq q_{all} \quad \text{and} \quad \tau \leq \tau_{all}$
- $q_{all} \ ??? \ \therefore$ for shallow foundation. No vertical stress

$$q_{all} = \frac{q_u (N_{\phi} + 1)}{F.S.}$$

For Deep Foundation

Since q_{all} as determined for shallow foundations would be very conservative, \therefore use a smaller factor of safety, say 1.5 to 2.

Final: open book & notes.

- (1) Geological Rock = texture
- (2) Permeability

- (3) Testing <u>Brazilian</u> <u>2 point load</u>
- (4) Triaxial

$$\begin{split} \sigma_3 &= \\ \sigma_1 &= \frac{\Delta V}{V} = \epsilon_1 \!=\! \epsilon_2 \!+\! \epsilon_3 \\ \Delta L_1 \\ \phi &V &\& \epsilon_2 \!=\! \epsilon_3 \!=\! \underline{\hspace{1cm}} \\ L_o \end{split}$$

$$E \qquad \qquad \epsilon_1 = f \left(\sigma_1 \; \sigma_2 \; \sigma_3 \; E, \; Y \right)$$

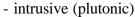
$$\epsilon_2 =$$

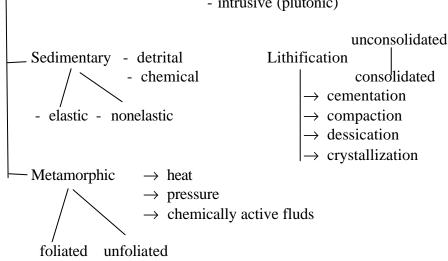
$$E, \; V \; \rightarrow \; G$$

$$G = \frac{E}{2 \left(1 + \nu \right)}$$

(5) Denatoric - stress - strain

- <u>Disc.</u> Faults \rightarrow shears \rightarrow joints \rightarrow fissures \rightarrow microfissures
- Geo. class \rightarrow Geometric \rightarrow Igneous \rightarrow extrusive (volcanic)





shale \downarrow slate \downarrow phyllite schist \downarrow gneiss

•
$$n = \frac{V_P}{V_t}$$

$$\bullet \quad G_{s,r} \sum_{i=1}^{n} G_{s,i} V_{i}$$

•
$$\gamma_{dry} = G_{s,r} \gamma_w (1-n)$$

$$\begin{aligned} \bullet & \quad k = \text{K.F.} = \text{K} \, \frac{\gamma_w}{\mu} & \quad q_i = k_i \, \frac{dh}{dx_i} \, A_L \\ k = \frac{q \cdot \ell n \, (R_2 / R_1)}{dx_i} \\ k = \frac{\gamma_w}{6 \mu} \left(\frac{e^2}{S} \right) \end{aligned}$$

Point load test

$$d = 54 \text{ mm}$$

$$L = 1.5 \text{ D}$$

$$I_s = P/D^2$$

$$Q_u = C I_s$$

•
$$V_p^2 = E/\delta$$

$$\frac{1}{V_\ell^*} = \sum_{i=1}^n \frac{C_i}{V\ell_i} \implies I_Q \% = \frac{V_\ell}{V_\ell^*} * 100\% \text{ no fissure } I_Q = 100 - 1.6 \text{ n } \%$$

•
$$RQD = \frac{\sum LX''}{\sum L}$$

- Failure \rightarrow Tension
 - \rightarrow Shear
 - → Compaction

• Unconfined comp. test
$$\frac{L}{D} = 2 - 3$$
 \Rightarrow $q_u = \frac{P_{max}}{A}$

- Triaxial Testing: strength = f (confining p.)
- Brazilian split cylinder test $\sigma_t = \frac{2P}{\pi dt}$

• Flex
$$\sigma_{t} = \frac{16}{3} \frac{PL}{\pi d^{3}} \qquad I = \frac{\pi 6^{4}}{64}$$

• Ring shear test
$$\tau_{\text{max}} = \frac{2P}{\pi d^2}$$

$$\begin{array}{c} * \quad \sigma_{ij} = \frac{1}{3} \, \sigma_{kk} \, \, \delta_{ij} + \tau_{ij} \\ \\ D_1 = \frac{\Delta \overline{\sigma}}{\Delta \overline{\epsilon}} = 3 \, K \quad \Rightarrow \\ \\ D_2 = \frac{\Delta \sigma_{1, \, dev}}{\Delta \epsilon_{1, \, dev}} = 2 G \quad \Rightarrow \end{array} \right\} \qquad E = \frac{3 D_1 \, D_2}{D_2 + 2 \, D_1} \quad , \quad \nu = \frac{D_1 - D_2}{2 D_1 + D_2}$$

$$G_{yx} = \frac{E_{x}}{2(1+v_{yx})} \implies G = \frac{E}{2(1+v)} \implies 2G = \frac{E}{1+v}$$

$$K = \frac{E}{3(1-2v)} \implies 3K = \frac{E}{(1-2v)}$$

$$\lambda = \frac{Ev}{(1+v)(1-2v)}$$

$$\begin{cases} \boldsymbol{\epsilon}_{x} \\ \boldsymbol{\epsilon}_{y} \\ \boldsymbol{\epsilon}_{z} \\ \boldsymbol{\gamma}_{xy} \\ \boldsymbol{\gamma}_{yz} \\ \boldsymbol{\gamma}_{zx} \end{cases} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{-\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} \end{cases} \begin{bmatrix} \boldsymbol{\sigma}_{x} \\ \boldsymbol{\sigma}_{y} \\ \boldsymbol{\sigma}_{z} \\ \boldsymbol{\tau}_{xy} \\ \boldsymbol{\tau}_{yz} \\ \boldsymbol{\tau}_{zx} \end{bmatrix}$$

$$\begin{cases} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{cases} = \begin{bmatrix} \lambda + 2G & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2G & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2G & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & G & G \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix}$$

(1) Failure

(2)

(3)

(4)

1) Road

2) Foundation

Multistage

- $I \qquad \sigma_3 \qquad \sigma_1$
- $II \qquad \sigma_3 \qquad \sigma_1$
- $III \qquad \sigma_3 \qquad \sigma_1$
 - 1) Draw joint failure
 - 2) He gave us $\tau {=} S_i + \sigma \tan \varphi$ for intact rock.

For failure occurs on the joint $\;\beta_1<\beta<\beta_2\;$

For failure occurs on the intact $\beta < \beta_1 \\ \beta > \beta_2$

Disadvantage of overcoring:
1. The linear dependence of the stresses upon, the one elastic const. t .
2. Large drill cores to make the rock not crack.