4-26 The bucket boom carries a worker who has a weight of 230 lb and mass center at *G*. Determine the moment of this force about (a) point *A* and (b) point *B*.

<u>Solution</u>

$M_A = F_{dA} = 230 * 12 \cos 20 = 2594$ lb.ft

$$M_B = F_{dA} = 230 * \underbrace{(12 \cos 20 - 15 \sin 30)}_{3.776} = 869 \text{ lb.ft}$$

4-26 Using Cartesian vectors, compute the moment of each of the two forces acting on the pipe assembly about an axis passing through point *O*. Add these moments and calculate the magnitude and coordinate direction angles of the resultant moment.

4-26

Position vector:

$\mathbf{r}_1 = (0-0)\mathbf{i} + (1.2-0)\mathbf{j} + (0.8-0)\mathbf{k} = \{1.2\mathbf{j} + 0.8\mathbf{k}\}$ m
$\mathbf{r}_2 = (-0.4-0)\mathbf{i} + (1.2-0)\mathbf{j} + (0.8-0)\mathbf{k} = \{-0.4\mathbf{i} + 1.2\mathbf{j} + 0.8\mathbf{k}\}$ m

Moment about point *O*:

$\mathbf{M}_1 = \mathbf{r}_1 \times \mathbf{F}_1$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.2 & 0.8 \\ 30 & 20 & -30 \end{vmatrix}$$

$= [1.2(-30) - 20(0.8)]\mathbf{i} - [0(-30) - 30(0.8)]\mathbf{j} + [0(20) - 30(1.2)]\mathbf{k}$

$= \{-52\mathbf{i} + 24\mathbf{j} - 36\mathbf{k}\}$ N.m Ans

$\mathbf{M}_2 = \mathbf{r}_2 \times \mathbf{F}_2$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.4 & 1.2 & 0.8 \\ 60 & -10 & -35 \end{vmatrix}$$

$= [1.2(-35) - (-10)(0.8)]\mathbf{i} - [(-0.4)(-35) - 60(0.8)]\mathbf{j} + [(-0.4)(-10) - 60(1.2)]\mathbf{k}$

$= \{-34\mathbf{i} + 34\mathbf{j} - 68\mathbf{k}\}$ N.m Ans

Resultant moment about joint *O*:

$(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O$: $(\mathbf{M}_R)_O = \mathbf{M}_1 + \mathbf{M}_2$
$= (-52\mathbf{i} + 24\mathbf{j} - 36\mathbf{k}) + (-34\mathbf{i} + 34\mathbf{j} - 68\mathbf{k})$
$= \{-86\mathbf{i} + 58\mathbf{j} - 104\mathbf{k}\}$ N.m

Magnitude:

$$(M_R)_O = \sqrt{(-86)^2 + 58^2 + (-104)^2} = 147 \text{ N} \cdot \text{m}$$ Ans

Coordinate direction angles:

$\cos\alpha = \frac{-86}{147}$ $\alpha = 126^\circ$ Ans
$\cos\beta = \frac{58}{147}$ $\beta = 66.7^\circ$ Ans
$\cos\gamma = \frac{-104}{147}$ $\gamma = 135^\circ$ Ans

4-30 Find the resultant moment of the two forces acting at the end of the pipe assembly about each of the joints.

4-30

$$(\mathbf{M}_R)_B = \sum M_B ; \qquad (\mathbf{M}_R)_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 15 & 24 & -6 \\ 4 & 10 & -8 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 15 & 24 & -6 \\ 9 & -4 & -10 \end{vmatrix}$$

$$= [24(-8) - 10(-6)]\mathbf{i} - [15(-8) - 4(-6)]\mathbf{j} + [15(10) - 4(24)]\mathbf{k}$$

$$+[24(-10) - (-4)]\mathbf{i} - [15(-10) - 9(-6)]\mathbf{j} + [15(-4) - 9(24)]\mathbf{k}$$

$$= \{-396\mathbf{i} + 192\mathbf{j} - 222\mathbf{k}\} \text{ lb} \cdot \text{in.} \qquad \text{Ans}$$

$$\mathrm{M_C} = \sum \vec{r} \times \vec{F} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$$

$$\vec{r}_1 = \vec{r}_2 = 15\hat{j} + 24\hat{j} - 6\hat{k}$$

Also, one can find the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = \{13\mathbf{i} + 6\mathbf{j} - 18\mathbf{k}\}$ lb then find the moment about each point.

$$\vec{\mathrm{M}}_C = \begin{vmatrix} \hat{i} & \hat{j} & \\ 15 & 24 & -6 \\ 13 & 6 & -18 \end{vmatrix}$$

$$= \hat{i}\,(-24*18 + 6*6) - \hat{j}\,(15*-18 + 6*13) + \hat{k}\,(15*6 - 13*24)$$

$$= \{-396\,\hat{i} - 192\,\hat{j} - 222\,\hat{k}\} \text{ lb.in}$$

$$M_x \qquad M_y \qquad M_z$$

Magnitude:

$$M_C = \sqrt{(-396)^2 + (-192)^2 + (-222)^2} = 993 \text{ lb.in}$$

$$\alpha = \cos^{-1}\left(\frac{-396}{493}\right) = 143.5°$$

$$\beta = \cos^{-1}\left(\frac{-192}{493}\right) = 113°$$

$$\gamma = \cos^{-1}\left(\frac{-222}{493}\right) = 116.8°$$

$$\hat{M}_O = \vec{r} \times \vec{F}$$

mag. = r F sin θ
= F r sin θ

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

4.4

Transmissibility of a Force

Sec. 4.4

Principle of Moments

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2 = \vec{r} \times (\vec{F}_1 + \vec{F}_2) = \mathbf{r} \times \mathbf{F}$$

$$M_a = (u_a\,\mathbf{i} + u_a\,\mathbf{j} + u_a\,\mathbf{k} \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

4-50 The chain *AB* exerts a force of 20 lb on the door at *B*. Determine the magnitude of the moment of this force along the hinged axis *x* of the door.

4-50

Force Vector:

$$\mathbf{F} = 20\left(\frac{(3-0)\mathbf{i} + (0-3\cos\ 20°)\,\mathbf{j} + (4-3\ \sin\ 20°)\,\mathbf{k}}{\sqrt{(3-0)^2 + (0-3\ \cos\ 20°)^2 + (4-3\ \sin\ 20°)^2}}\right)$$

$$= \{11.814\mathbf{i} - 11.102\mathbf{j} + 11.712\mathbf{k}\}\text{lb}$$

Position Vector:

$$\mathbf{r} = 3\cos 20^{o}\,\mathbf{j} + 3\sin 20^{o}\,\mathbf{k} = \{2.8191\,\mathbf{j} + 1.0261\mathbf{k}\}\ \text{ft}$$

Magnitude of the moment along x axis:

$$M_x = \text{i}\cdot(\mathbf{r}\times\mathbf{F}) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2.8191 & 1.0261 \\ 11.814 & -11.102 & 11.712 \end{vmatrix}$$

$$= 1\ [2.8191(11.712) - (-11.102)(1.0261)] - 0 + 0$$

$$= 44.4\ \text{lb}\cdot\text{ft}$$ Ans

$$\vec{\text{M}}_x = (44.4\,\hat{\text{i}})\ \text{lb.ft}$$

$$\vec{\text{r}}\times\vec{\text{F}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2.82 & 1.03 \\ 11.81 & -11.10 & 11.71 \end{vmatrix}$$

$$= \mathbf{j}\,(2.82 * 11.71 * 1.03 * 11.10)$$

$$- \mathbf{j}\,(0 - 1.03 * 11.81)$$

$$+ \mathbf{k}\ (0 - 2.82 * 11.81)$$

$$= 44.4\hat{\text{i}} + 12.2\,\hat{\text{j}} - 33.3\hat{\text{k}}$$

4-78 The resultant couple moment created by the two couples acting on the disk is $\mathbf{M}_R = \{10\mathbf{k}\}$ kip · in. Determine the magnitude of force **T**.

$$+\ \mathbf{M}_R = \Sigma M_Z = 10 = T * 9 + T * 2 \quad \Rightarrow \quad T = .91 \text{ kips}$$

$$= T * 5 + T * 6$$

$$11\,T$$