3-1 Determine the magnitude of $\mathbf{F}_1$ and $\mathbf{F}_2$ so that the particle is in equilibrium.

3-8 Determine the force in cables *AB* and *AC* necessary to support the 15-kg light fixture.

3-17 Determine the stretch of each spring for equilibrium of the 20-kg block. The springs are shown in their equilibrium position.

3-1

$+\uparrow \sum F_y = 0;$ $F_1 \sin 45^{\circ} - F_2 \sin 30^{\circ} = 0$

$F_2 = 1.414F_1$ [1]

$\xrightarrow{+} \sum F_x = 0;$ $F_1 \cos 45^{\circ} + f_2 \cos 30^{\circ} - 500 = 0$ [2]

Solving Eqs. [1] and [2] yields:

$F_1 = 259$ N $F_2 = 366$ N Ans

3-8

$\xrightarrow{+} F_x = 0;$ $F_{AC} \cos 45^{\circ} - F_{AB} \cos 30^{\circ} = 0$ [1]

$+\uparrow \sum F_y = 0;$ $F_{AC} \sin 45^{\circ} + F_{AB} \sin 30^{\circ} - 15(9.81) = 0$ [2]

Solving Eqs. [1] and [2] yields:

$F_{AB} = 108$ N $F_{AC} = 132$ N Ans

3-17

Equilibrium:

Spring *AD*

$$+\uparrow \sum F_y = 0; \qquad F_{AD} - 20(9.81) = 0 \qquad F_{AD} = 196.2 \text{ N}$$

Spring *AB* and *AC*

$$+\uparrow \sum F_y = 0; \qquad \tfrac{4}{5} F_{AB} - F_{AC} \sin 45^\circ = 0 \qquad [1]$$

$$\xrightarrow{+} \sum F_x = 0; \qquad \tfrac{3}{5} F_{AB} + F_{AC} \cos 45^\circ - 196.2 = 0 \qquad [2]$$

Solving Eq. [1] and [2] yields:

$F_{AC} = 158.55$ N $\qquad F_{AB} = 140.14$ N

Spring elongation: $\qquad x = \dfrac{F}{k}$

$$x_{AB} = \frac{140.14}{300} = 0.467 \text{ m} \qquad \text{Ans}$$

$$x_{AC} = \frac{158.55}{200} = 0.793 \text{ m} \qquad \text{Ans}$$

$$x_{AD} = \frac{196.2}{400} = 0.490 \text{ m} \qquad \text{Ans}$$

3.4 3-D Force Systems

Particle equilibrium requires $\sum \vec{F} = \vec{0}$

if forces acting on the particle are resolved into $\hat{i}$, $\hat{j}$, $\hat{k}$ components

$$\Rightarrow \qquad \sum F_x \hat{i} + \sum F_y \hat{j} + \sum F_z \hat{k} = 0$$

$\Rightarrow$ Three scalar component equations:

$$\left.\begin{array}{l} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma F_z = 0 \end{array}\right\}$$ algebraic sums of x,y,z components of forces acting on a particle.

3 equation $\Rightarrow$ solve for 3 unknowns.

angles & magnitudes of forces on a F.B.D.

Procedure for analysis:

1. Draw a F.B.D – label all known & unknown forces
2. Establish x, y, z coordinate axes with origin located at the particle
3. Apply equations of equilibrium

scalar $\Sigma F_x = \Sigma F_y = \Sigma F_z = 0$ in cases easy to resolve forces to x, y, z components

Cartesian vector – express each force in Cartesian vector form

$$\sum \vec{F} = \vec{0}$$

set respected $\hat{i}$, $\hat{j}$, $\hat{k}$ components to 0

- Additional eqn. for springs $F = k.s$

3-38 The three cables are used to support the 8-kg lamp. Determine the force developed in each cable for equilibrium.

3-38

Force vector:

$$\mathbf{F}_{AD} = F_{AD}\left(\frac{-2\mathbf{i}-4\mathbf{j}+4\mathbf{k}}{\sqrt{(-2)^2+(-4)^2+4^2}}\right) = -\tfrac{1}{3}F_{AD}\,\mathbf{i}-\tfrac{2}{3}F_{AD}\,\mathbf{j}+\tfrac{2}{3}F_{AD}\,\mathbf{k}$$

$\mathbf{F}_{AB} = F_{AB}\ \mathbf{j}$

$\mathbf{F}_{AC} = F_{AC}\,\mathbf{i}$

$\mathbf{F} = -8(9.81)\mathbf{k} = \{-78.48\mathbf{k}\}$ N

Equilibrium:

$$\Sigma\,\mathbf{F} = \mathbf{0}\,; \qquad \mathbf{F}_{AD} = \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F} = \mathbf{0}$$

$$\left(-\tfrac{1}{3}F_{AD}\mathbf{i}-\tfrac{2}{3}F_{AD}\mathbf{j}+\tfrac{2}{3}F_{AD}\mathbf{k}\right)+(F_{AB}\mathbf{j})+(F_{AC}\mathbf{i})+(-78.48\mathbf{k}) = \mathbf{0}$$

$$\left(-\tfrac{1}{3}F_{AD}+F_{AC}\right)\mathbf{i}+\left(-\tfrac{2}{3}F_{AD}+F_{AB}\right)\mathbf{j}+\left(\tfrac{2}{3}F_{AD}-78.48\right)\mathbf{k} = \mathbf{0}$$

Equating the respective $\mathbf{i},\mathbf{j},\mathbf{k}$ components to zero yields:

$$\sum F_x = 0; \qquad -\tfrac{1}{3}F_{AD}+F_{AC} = 0 \qquad [1]$$

$$\sum \mathrm{F_y} = 0; \qquad -\tfrac{2}{3}F_{AD}+F_{AB} = 0 \qquad [2]$$

$$\sum F_z = 0; \qquad \tfrac{2}{3}F_{AD}-78.48 = 0 \qquad [3]$$

Solving Eqs. [1], [2] and [3] yields:

F_{AD} = 118 N $\qquad F_{AB}$ = 78.5 N $\qquad F_{AC}$ = 39.2 N $\qquad$ Ans

3-41 The 25-kg pot is supported at *A* by the three cables. Determine the force in each cable for equilibrium.

3-41

Force vector:

$$\mathbf{F}_{AD} = F_{AD}\,(\sin 30^{\circ}\mathbf{i} - \cos 30^{\circ}\sin 60^{\circ}\mathbf{j} + \cos 30^{\circ}\cos 60^{\circ}\mathbf{k})$$
$$= 0.5\,F_{AD}\,\mathbf{i} - 0.75\,F_{AD}\,\mathbf{j} + 0.4330\,F_{AD}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\,(-\sin 30^{\circ}\mathbf{i} - \cos 30^{\circ}\sin 60^{\circ}\mathbf{j} + \cos 30^{\circ}\cos 60^{\circ}\mathbf{k})$$
$$= -0.5\,F_{AC}\,\mathbf{i} - 0.75\,F_{AC}\,\mathbf{j} + 0.4330\,F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AB} = F_{AB}\,(\sin 45^{\circ}\mathbf{j}) = \cos 45^{\circ}\mathbf{k}) = 0.7071\,F_{AB}\,\mathbf{j} + 0.7071\,F_{AB}\,\mathbf{k}$$

$$\mathbf{F} = -25(9.81)\mathbf{k} = \{-245.25\,\mathbf{k}\}\text{ N}$$

Equilibrium:

$$\sum \mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AD} + \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F} = 0$$

$$(0.5\,F_{AD}\,\mathbf{i} - 0.75F_{AD}\,\mathbf{j} + 0.4330F_{AD}\,\mathbf{k}) - (0.7071F_{AB}\,\mathbf{j} + 0.7071F_{AB}\,\mathbf{k})$$
$$+ (-0.5F_{AC}\,\mathbf{i} - 0.75F_{AC}\,\mathbf{j} + 0.4330F_{AC}\,\mathbf{k}) + (-245.25\mathbf{k}) = \mathbf{0}$$

$$(0.5F_{AD} - 0.5F_{AC})\mathbf{i} + (-0.75F_{AD} + 0.7071F_{AB} - 0.75F_{AC})\mathbf{j}$$
$$+ (0.4330F_{AD} + 0.7071F_{AB} + 0.4330F_{AC} - 245.25)\mathbf{k} = \mathbf{0}$$

Equating the respective **i,j,k** components to zero yields:

$$\sum F_x = 0; \qquad 0.5F_{AD} - 0.5F_{AC} = 0 \qquad [1]$$

$$\sum F_y = 0; \qquad -0.75F_{AD} + 0.7071F_{AB} - 0.75F_{AC} = 0 \qquad [2]$$

$$\sum F_z = 0; \qquad 0.4330F_{AD} + 0.7071F_{AB} + 0.4330F_{AC} - 245.25 = 0 \qquad [3]$$

Solving Eqs. [1], [2] and [3] yields:

$F_{AD} = F_{AC} = 104$ N $\qquad F_{AB} = 220$ N $\qquad$ Ans

3-42 Determine the force in each of the three cables needed to lift the tractor which has a mass of 8 Mg.

3-42

Force vector:

$$\mathbf{F}_{AD} = F_{AD}\left(\frac{-1\mathbf{j}-3\mathbf{k}}{\sqrt{(-1)^2+(-3)^2}}\right) = -0.3162F_{AD}\mathbf{j} - 0.9487F_{AD}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{-1.25\mathbf{i}+2\mathbf{j}-3\mathbf{k}}{\sqrt{(-1.25)^2+2^2+(-3)^2}}\right) = -0.327F_{AC}\mathbf{i} + 0.5241F_{AC}\mathbf{j} - 0.7861F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{1.25\mathbf{i}+2\mathbf{j}-3\mathbf{k}}{\sqrt{1.25^2+2^2+(-3)^2}}\right) = 0.3276F_{AB}\mathbf{i} + 0.5241F_{AB}\mathbf{j} - 0.7861F_{AB}\mathbf{k}$$

$\mathbf{F} = 8(10)^3\,(9.81)\mathbf{k} = \{78.48\mathbf{k}\}$ kN

Equilibrium:

$$\sum \mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AD} + \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F} = \mathbf{0}$$

$$(-0.3162F_{AD}\mathbf{j} - 0.9487F_{AD}\mathbf{k}) + (0.3276F_{AB}\mathbf{i} + 0.5241F_{AB}\mathbf{j} - 0.7861F_{AB}\mathbf{k})$$
$$+ (-0.3276F_{AC}\mathbf{i} + 0.5241F_{AC}\mathbf{j} - 0.7861F_{AC}\mathbf{k}) + (78.48\mathbf{k}) = \mathbf{0}$$

$$(0.3276F_{AB} - 0.3276F_{AC})\mathbf{i} + (-0.3162F_{AD} + 0.5241F_{AB} + 0.5241F_{AC})\mathbf{j}$$
$$+ (-0.9487F_{AD} - 0.786F_{AB} - 0.7861F_{AC} + 78.48)\mathbf{k} = \mathbf{0}$$

Equating the respective **i,j,k** components to zero yields:

$$\sum F_x = 0; \qquad 0.3276F_{AB} - 0.3276F_{AC} = 0 \qquad 1]$$

$$\sum F_y = 0; \qquad -0.3162F_{AD} + 0.5241F_{AB} + 0.5241F_{AC} = 0 \qquad 2]$$

$$\sum F_z = 0; \qquad -0.9487F_{AD} - 0.7861F_{AB} - 0.7861F_{AC} + 78.48 = 0 \qquad [3]$$

Solving Eqs. [1], [2] and [3] yields:

$F_{AB} = F_{AC} = 16.6$ kN $\qquad F_{AD} = 55.2$ kN $\qquad$ Ans

Ch. 4 Force System Resultants

- Equilibrium of a particle
  (concurrent force system) $\Rightarrow$ Resultant = 0
  $$\sum \vec{F} = 0$$

- Equilibrium of a rigid body

  $\sum \vec{F} = 0$ is a necessary condition, but <u>not</u> sufficient

  Non-concurrent force system $\Rightarrow$ moment

<u>Outline</u>

1. Ref. of a moment

2. Ways of finding moment of a force about a point or axis

3. Resultant of non-concurrent force system

   force-system simplification $\equiv$ rigid body

   influence of resultant same as force-system

4.1 Cross Product

To find moment of a force, ----- need vector algebra, cross product.

Cross product of two vectors $\vec{A}$ & $\vec{B}$, is a vector, $\vec{C}$.

$$\vec{C} = \vec{A} \times \vec{B}$$

($\vec{C}$ equals $\vec{A}$ cross $\vec{B}$)

$\vec{C}$

- magnitude ---- $C = AB \sin\theta$, θ angle between tails of $\vec{A}$ and $\vec{B}$, $0^o \le \theta \le 180^0$
- direction ---- perpendicular to the plane containing $\vec{A}$ & $\vec{B}$, according to right-hand rule
- sense (curling fingers from $\vec{A}$ cross to $\vec{B}$, thumb points in the direction of $\vec{C}$)

$\vec{C} = \vec{A} \times \vec{B} = (AB \sin\theta) \cdot \vec{u}_c$, $\vec{u}_c$: unit vector defines the direction of $\vec{C}$.

Laws of Operation:

1. Cumulative law is not valid

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

$$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$$

2. Multiplication by a scalar, a

$$a\,(\vec{A}\times\vec{B}) = (a\,\vec{A}) \times \vec{B} = \vec{A} \times (a\,\vec{B}) = (\vec{A}\times\vec{B})\,a$$

affect only magnitude $\Rightarrow$ $|a|\ AB \sin\theta$.

3. Distributive Law $\quad \vec{A}\times(\vec{B}+\vec{D}) = (\vec{A}\times\vec{B}) + (\vec{A}\times\vec{D})$

maintains proper order $\Rightarrow$ (1)

Cartesian Vector Formulation: $\quad \hat{i},\ \hat{j},\ \hat{k}$

$$\hat{i}\times\hat{j} = |i|\ \sin\ 90\cdot\bar{u} = 1\cdot 1\cdot 1\cdot\ \hat{k} = \hat{k}$$

$$\hat{j}\times\hat{k} = \hat{i}$$

$$\hat{k}\times\hat{i} = \hat{k}$$

$$\hat{i}\times\hat{k} = -\hat{j} \qquad \hat{i}\times\hat{i} = 1\cdot 1\cdot \sin\ 0\cdot\bar{u} = 0$$

$$\hat{j}\times\hat{i} = -\hat{k} \qquad \hat{j}\times\hat{j} = 0$$

$$\hat{k}\times\hat{j} = -\hat{i} \qquad \hat{k}\times\hat{k} = 0$$

Let $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

and

$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

$$\vec{A} \times \vec{B} = A_x B_x (\hat{i} \times \hat{i}) + A_x B_y (\hat{i} \times \hat{j}) + A_x B_z (\hat{i} \times \hat{k})$$

$$+ A_y B_x (\hat{j} \times \hat{i}) + A_y B_y (\hat{j} \times \hat{j}) + A_y B_z (\hat{j} \times \hat{k})$$

$$+ A_z B_x (\hat{k} \times \hat{i}) + A_z B_y (\hat{k} \times \hat{j}) + A_z B_z (\hat{k} \times \hat{k})$$

$$= (A_y B_z - A_z B_y)\hat{i}$$

$$- (A_x B_z - A_z B_x)\hat{j}$$

$$+ (A_x B_y - A_y B_x)\hat{k}$$

in a matrix form $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

cross product = determinant of a matrix with 1st row $\hat{i}$, $\hat{j}$, & $\hat{k}$

2nd components of 1st vector

3rd components of 2nd vector

4.2 Moment of a Force – Scalar Formulation

Def. : moment of a force (moment, torque) is the tendency for rotation caused by a force. Rotation about an axis.

$(M_o)_z$	$(M_o)_x$	$(M_o)_y = 0$, ---
(rotation)	(tendency for rotation)	Line of action of F_y passes through 0 (no tendency for rotation)

$M_o \uparrow$ as $F_x \uparrow$ or $d_y \uparrow$

moment axis (z) $\perp$ plane (x-y) containing F_x and d_y)

Let $\vec{F}$orce $\vec{F}$ and point O lies in same plane

Moment of the force $\hat{F}$ about

point O (about an axis passing through O $\perp$ plane) is

$\vec{M}_o$, which is a vector quantity $\Rightarrow$ magnitude, direction, sense $\Rightarrow$ vector algebra parallelogram law

$\vec{M}_o$
- magnitude
  - $M_o = Fd$
  - $F = |\vec{F}|$
  - d = moment arm = perpendicular distant from point O to the line of action of the force $\vec{F}$
  - units: N.m , Lb.ft
- direction

$\rightarrow$ direction : right-hand rule
&
sense figures are curled as if $\vec{F}$ is rotating about point O.

Thumb points along moment axis.

Moment $\Rightarrow$ sliding vector @ any point along moment axis

Resultant Moment of a System of Coplanar Forces:

- System of forces, all lie in x-y plane

  $\vec{M}_o$ along z-axis

Resultant moment $(\vec{M}_R)_o$ = algebraic summation of moments of each force

$\therefore$ moment in vectors are colinear

+ $M_{R\,o} = \Sigma\, Fd$

scalar sign convention