-100. Determine the angle θ between the two position vectors.
-105. A force of $F = 80$ N is applied to the handle of the wrench. Determine the agnitudes of the components of the force acting along the axis AB of the wrench andle and perpendicular to it.
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2-116. The force $\mathbf{F} = \{25\mathbf{i} - 50\mathbf{j} + 10\mathbf{k} \text{ N acts at the end } A \text{ of the pipe assembly.}$ Determine the magnitudes of the components \mathbf{F}_1 and \mathbf{F}_2 which act along the axis of AB and perpendicular to it.

Position vector:

$$\mathbf{r}_{1} = [0 - (-8)]\mathbf{i} + (15 - 5)\mathbf{j} + (10 - 8)\mathbf{k}$$

$$= \{8\mathbf{i} + 10\mathbf{j} - 8\mathbf{k}\} \text{ ft}$$

$$r_{1} = \sqrt{8^{2} + 10^{2} + (-8)^{2}} = \sqrt{228 \text{ ft}} \approx 15.1 \text{ ft}$$

$$\mathbf{r}_{2} = [0 - (-8)]\mathbf{i} + (0 - 5)\mathbf{j} + (6 - 8)\mathbf{k}$$

$$= \{8\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}\} \text{ ft}$$

$$r_{2} = \sqrt{8^{2} + (-5)^{2} + (-2)^{2}} = \sqrt{93 \text{ ft}} \approx 9.64 \text{ ft}$$

$$\cos \theta = \frac{\mathbf{r}_{1} \cdot \mathbf{r}_{2}}{r_{1} r_{2}}$$

$$= \frac{(8\mathbf{i} + 10\mathbf{j} - 8\mathbf{k}) \cdot (8\mathbf{i} - 5\mathbf{j} - 2\mathbf{k})}{15.1 * 9.64}$$

$$= \frac{8(8) + 10(-5) + (-8)(-2)}{145.6} = 0.2060$$

$$\theta = 78.1^{\circ}$$
Answer

Ans

$$\mathbf{u}_{F} = -\cos 30^{\circ} \sin 45^{\circ} \mathbf{i} + \cos 30^{\circ} \cos 45^{\circ} \mathbf{j} + \sin 30^{\circ} \mathbf{k}$$

$$= -0.6124 \mathbf{i} + 0.6124 \mathbf{j} + 0.5 \mathbf{k}$$

$$\mathbf{u}_{AB} = -\mathbf{j}$$

$$\mathbf{F} = F \mathbf{u}_{F} = 80(-0.6124 \mathbf{i} + 0.6124 \mathbf{j} + 0.5 \mathbf{k})$$

$$= (-48.990 \mathbf{i} + 48.99 \mathbf{j} + 40 \mathbf{k}) \text{ N}$$

$$F_{AB} = \mathbf{F} \cdot \mathbf{u}_{AB} = |(-48.990 \mathbf{i} + 48.99 \mathbf{j} + 40 \mathbf{k}) \cdot (-\mathbf{j})|$$

$$= 49.0 \text{ N}$$
Ans

Negative sign indicates that F_{AB} acts in the direction opposite to that of \mathbf{u}_{AB} .

$$F_{per.} = \sqrt{F^2 - F_A^e B}$$

= $\sqrt{80^2 - (-49.0)^2} = 63.2 \text{ N}$ Ans

Also, from prob. 2-104
$$\theta = 128^{\circ}$$
 $F_{AB} = F \cos \theta = |80 \cos 128^{\circ}| = 49.0 \text{ N}$ $F_{per.} = F \sin \theta = |80 \sin 128^{\circ}| = 63.2 \text{ N}$

$$\mathbf{u}_{AB} = \frac{(0-0)\mathbf{i} + (5-9)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(0-0)^2 + (5-9)^2 + (0-6)^2}} = \frac{-4\mathbf{j} - 6\mathbf{k}}{\sqrt{52}}$$

$$F_1 = \mathbf{F} \cdot \mathbf{u}_{AB} = (25\mathbf{i} - 50\mathbf{j} + 10\mathbf{k}) \cdot \left(\frac{-4\mathbf{j} - 6\mathbf{k}}{\sqrt{52}}\right)$$

$$= \frac{25(0) + (-50)(-4) + (10)(-6)}{\sqrt{52}}$$

$$= 19.4 \text{ N} \qquad \text{Ans}$$

$$F = \sqrt{25^2 + (-50)^2 + 10^2} = 56.79 \text{ N}$$

$$F_2 = \sqrt{F^2 - F_1^2}$$

$$= \sqrt{56.79^2 - 19.41^2} = 53.4 \text{ N} \qquad \text{Ans}$$

Ch. 3 Equilibrium of a Particle

- Resolve a force into components
- Express a force as a Cartesian vector
- 3.1 Conditions for the equilibrium of a Particle

A particle is in equilibrium:

- \rightarrow at rest, if originally at rest.
- \rightarrow has constant velocity, if originally in motion.

Static equilibrium or equilibrium, mostly at rest, if originally at rest.

• To maintain a state of equilibrium, it is necessary to satisfy first Law of Motion. [if resultant force acting on a particle is zero, then particle is in equilibrium].

$$\sum \vec{F} = 0$$
 vector sum of all the forces acting on the particle necessary condition of equilibrium & sufficient $\rightarrow \sum \vec{F} = m\vec{a}$ satisfied $\Rightarrow m\vec{a} = 0 \Rightarrow \vec{a} = 0$ constant velocity or at rest.

3.2 Free-Body Diagram

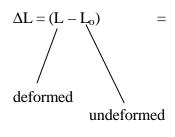
 Σ F includes known & unknown forces acting on the particle.

- (1) Draw the <u>free body diagram</u> of the particle sketch of the particle, which represents it as being isolated or cut "force" from its surroundings.
 - * on sketch show all forces acting on the particle (action & reaction forces)
- (2) Apply equation of equilibrium

Connections:

1. Springs: Linear elastic spring support

change in length α force acting on it.



S α F



spring constant or stiffness

$$F = KS$$

$$= Elongation or reduction$$

$$+ve \Rightarrow F \text{ is pull}$$

$$-ve \Rightarrow F \text{ is push}$$

$$K = 500 \text{ N/m}$$

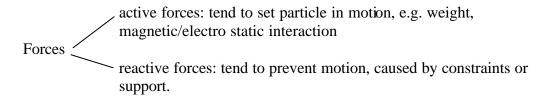
$$F_1 = 500 * .2 = 100 N$$

$$F_2 = 500 * (-2) = -100 N$$

- 2. Cables & pulleys
 - Assume 1) cable's weight is negligible
 - 2) cable's can't be stretched
- Cable can support <u>only</u> tension (pulling) force
 - * acting in the direction of the cable
 - * tension in a <u>continuous cable</u> passing over frictionless pulley has <u>constant</u> magnitude, to keep cable in equilibrium.
 - T regardless of θ
 - * cable is subjected to a constant tension T throughout its length.
- 3. Others supports, later

Procedure for Drawing a Free-Body Diagram

- 1. Imagine the particle isolated, cut, free, from its surroundings. Draw / sketch its outline shape.
- 2. On the sketch, indicate all forces acting on the particle.



- o trace around the particle's boundary, noting each force acting on it.
- 3. Label known forces with their proper magnitudes & directions
 - Represent unknown forces by letters
 - If a force has a known line of action (direction), a sense can be assumed. Then found
 - \therefore mag. of \vec{F} is +ve. \Rightarrow -ve \equiv opposite sense

Equilibrium

- Apply equilibrium eqn. on the free-body diagram.

$$\Sigma \vec{F} = \sum \Sigma F_x = 0$$

$$= \sum F_x \hat{i} + \sum F_y \hat{j} + \sum F_z \hat{k}$$

$$\Sigma F_z = 0$$

Objectives:

- 1. Introduce the concept of F.B.D. for a particle.
- 2. Solve particle problems using the equation of equilibrium

3.3 Coplanar Force Systems

$$\begin{split} \Sigma \, \bar{F} &= 0 \\ \Sigma \, F_x \, \hat{i} \, + \, \Sigma_y \, \hat{j} &= 0 \\ \\ \Rightarrow \, \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \end{split} \right\} \quad \text{scalar equilibrium eqns.} \\ \Rightarrow \quad \text{algebraic sum of x \& y components} \\ \quad \text{of all the forces acting on} \\ \quad \text{the particle} &= 0. \end{split}$$

2 equations & 2 unknowns (mag. & direction)

Scalar Notation:

sense
$$\Rightarrow$$
 algebraic sign \equiv arrowhead magnitude & sense can be assmed

 \vec{F} is always +ve, \Rightarrow if (-ve) result \equiv sense is opposite to that which was assumed.

F

→•→ 10 N

$$\Sigma F_x = 0 \implies F + 10 = 0 \implies F = -10 \text{ N}$$
opposite sense

<u>Procedure for Analysis:</u> (planar forces & particle equilibrium)

- 1. Free body diagram:
 - identify the body / particle
 - draw F.B.D. of the particle
 - isolation
 - known & unknown forces (mag. & angles) to be labeled)
 - assume sense for unknown forces
- 2. Equations of Equilibrium
 - x-y axes in any direction

$$\begin{array}{ccc} \bullet & \xrightarrow{^+} & \Sigma \ F_x = 0 \\ & + \uparrow \Sigma \ F_y = 0 \end{array}$$

• Spring
$$F = Ks$$