2-9. The plate is subjected to the forces acting on members *A* and *B* as shown. If $\theta = 60^{o}$, determine the magnitude of the resultant of these forces and its direction measured clockwise from the positive *x* axis.

2.17. The cable exerts a force of 600 N on the frame. Resolve this force into components acting *(a)* along the *x* and *v* axes and *(b)* along the *y* and *u* axes. What is the magnitude of each component?

2-25. The jet aircraft is being towed by two trucks *B* and *C*. Determine the magnitude of the two towing forces $\mathbf{F}_B$ and $\mathbf{F}_C$ if the resultant force has a magnitude $F_R = 10$ kN and is directed along the positive *x* axis. Set $\theta = 15^o$.

2-9

Cosine law:

$$F_R = \sqrt{500^2 + 400^2 - 2(500)(400)\cos 60^\circ} = 458 \text{ lb} \qquad \text{Ans}$$

Sine law:

$$\frac{\sin 60^\circ}{458} = \frac{\sin(60^\circ - \alpha)}{400} \qquad \alpha = 10.9^o \qquad \text{Ans}$$

2-19. The horizontal force F = 500 N acts to the left at A on the two-member frame. Determine the magnitudes of the two components of **F** directed along the axes of members AB and AC.

2-17

(a)

Sine law:

$$\frac{F_x}{\sin 45°} = \frac{600}{\sin 60°} \qquad F_x = 490 \text{ N} \qquad \text{Ans}$$

$$\frac{F_y}{\sin 75°} = \frac{600}{\sin 60°} \qquad F_y = 669 \text{ N} \qquad \text{Ans}$$

(b)

Sine law:

$$\frac{F_u}{\sin 15°} = \frac{600}{\sin 120°} \qquad F_u = 179 \text{ N} \qquad \text{Ans}$$

$$\frac{F_y}{\sin 45°} = \frac{600}{\sin 120°} \qquad F_y = 490 \text{ N} \qquad \text{Ans}$$

2-19

Sine law:

$$\frac{F_{AC}}{\sin 45^\circ} = \frac{500}{\sin 75^\circ} \qquad F_{AC} = 366 \text{ N} \qquad \text{Ans}$$

$$\frac{F_{BA}}{\sin 60^\circ} = \frac{500}{\sin 75^\circ} \qquad F_{BA} = 448 \text{ N} \qquad \text{Ans}$$

2-24. Three chains act on the bracket such that they create a resultant force having a magnitude of 500 lb. If two of the chains are subjected to known forces, as shown, determine the orientation θ of the third chain, measured clockwise from the positive x axis, so that the magnitude of force **F** in this chain is a *minimum*. All forces lie in the x-y plane. What is the magnitude of **F**? *Hint* First find the resultant of the two known forces. Force **F** acts in this direction.

2.25. The jet aircraft is being towed by two trucks B and C. Determine the magnitudes of the two towing forces $\mathbf{F}_B$ and $\mathbf{F}_C$ if the resultant force has a magnitude $F_R = 10$ kN and is directed along the positive x axis. Set $\theta = 15^o$.

2.26. If the resultant $\mathbf{F}_R$ of the two forces acting on the jet aircraft is to be directed along the positive x axis and have a magnitude of 10 kN, determine the angle θ of the cable attached to the truck at B such that the force $\mathbf{F}_B$ in this cable is a *minimum*. What is the magnitude of force in each cable when this occurs?

2-25

Sine law:

$$\frac{F_C}{\sin 15°} = \frac{10}{\sin 145°} \qquad F_C = 4.51 \text{ kN} \qquad \text{Ans.}$$

$$\frac{F_B}{\sin 20°} = \frac{10}{\sin 145°} \qquad F_B = 5.96 \text{ kN} \qquad \text{Ans.}$$

2.4 Addition of a System of Coplanar Forces

- Resultant of more than two forces by

    1. Successive application of Parallelogram Law

    2. • Finding components of each force along specific axes
       • Adding these components algebraically
       • Forming the resultant

• Finding components

    - Resolve each force into its <u>rectangular</u> components

        $\vec{F}_x$ & $\vec{F}_y$, which lies along *x* & *y* axes.

    x & *y* axes : Horizontal & vertical
    or inclined but orthogonal perpendicular to one another

By parallelogram law:

$$\vec{F} = \vec{F}_x + \vec{F}_y$$

$$\vec{F}' = \vec{F}'_{x'} + \vec{F}_{y'}$$

- sense of direction of each components is represented graphically by the arrowhead.

For analytical work:

Need a <u>notation</u> to represent the sense of the direction.

- notations → Scalar
  → Cartesian vector

Scalar Notation:

- o for computational purposes not for graphical representation.

Cartesian Vector Notation:

Cartesian unit vectors

$\hat{i}, \hat{j}$: directions of *x* , *y* axes, respectively.

- dimensionless magnitude of unity
- sense (arrowhead) ------ analytically + or – depending on … pointing along + or – (ve) axes.

$\vec{F}_x = F_x \cdot \hat{i}$

$\vec{F}_y = F_y \cdot \hat{j}$

$\vec{F} = F_x \hat{i} + F_y \hat{j}$

$\vec{F}' = F_x' \hat{i} + F_y (-\hat{j})$

$= F_x' \hat{i} - F_y \cdot \hat{j}$

Coplanar Force Resultants:

Resultant of coplanar forces

1. resolve each force into its *x* & *y* components.
2. add respective components using scalar algebra
   - sketch components on *x* & *y* axes ∴ they are collinear
3. resultant is formed by adding resultants of *x* & *y* components using parallelogram law.

Ex.

* Using Cartesian vector notation

$$\vec{F}_1 = F_{1x}\,\hat{i} + F_{1y}\,\hat{j}$$

$$\vec{F}_2 = -F_{2x}\,\hat{i} + F_{2y}\,\hat{j}$$

$$\vec{F}_3 = F_{3x}\,\hat{i} - F_{3y}\,\hat{j}$$

Resultant = $\vec{F}_R = F_{Rx}\,\hat{i} + F_{Ry}\,\hat{j}$

$$F_{Rx} = (F_{1x} - F_{2x} + F_{3x})$$

$$F_{Ry} = (F_{1y} + F_{2y} - F_{3y})$$

* Using Scalar Notation ($\xrightarrow{x} \uparrow y$) positive

$$F_{Rx} = F_{1x} - F_{2x} + F_{3x}$$

$$F_{Ry} = F_{1y} + F_{2y} - F_{3y}$$

Resultant $\rightarrow$ $F_R = \vec{F}_{Rx} + \vec{F}_{Ry}$

In general $\vec{F}_R = F_{Rx}\,\hat{i} + F_{Ry}\,\hat{j}$

where

$$F_{Rx} = \sum F_x$$

$$F_{Ry} = \sum F_y$$

Sign Convention: components with directional sense along (+ve) axis are positive scalar.

components with directional sense along (−ve) axis are negative scalar.

and vice versa $\Rightarrow$ the component means along +ve axis.

Signs of the components of the resultant $\Rightarrow$ specify their sense.

- Sketch components F_{Rx} & F_{Ry}

Using Pythagorean theorem

Resultant magnitude

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

Direction angle θ orientation – direction & sense

from trigonometry

$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$