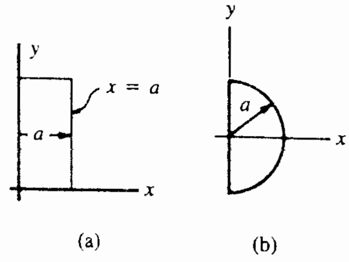
Thin-Walled Pressure Vessels.

Theory & Example

8.6 Thin-Walled Pressure Vessels. A pressure vessel can be thought of as a closed surface in three-dimensional space with a finite thickness t. Customarily, the closed surface is a surface of revolution; that is, it is a surface obtained by rotating a plane curve called the **generating curve** about a fixed axis called the **symmetry axis.** For example, a right-circular cylinder is obtained by rotating the straight line $x = a$ of Figure 8–18a about the y axis. A spherical surface is obtained by rotating the circular arc of Figure 8–18b about the y axis.

When the ratio of the wall thickness to the radius of a cylindrical or spherical pressure vessel is less than about 1/10, the pressure vessel is said to be thin. The stress distribution over the thickness of such a thin-walled pressure vessel is essentially uniform. Consequently, a spherical or cylindrical pressure vessel behaves like a thin membrane with a small thickness; that is, no bending of the walls occurs.



(a) (b)

Figure 8–18

Thin-walled cylinders. Figure 8–19a shows a right-circular cylinder of thickness t and internal radius R subjected to an internal pressure p.

Figure 8–19b shows a free-body diagram of a finite length of the cylinder. From equilibrium of forces,

$$Q = 2\sigma_c \ell t$$

where

$$Q = p(2R\ell)$$

Thus

$$\sigma_c = \frac{pR}{t} \tag{8–9}$$

Note that the force Q developed by the internal pressure is simply the pressure times the projected area of the cylindrical segment onto the diametric plane. Equation (8–9) permits the calculation of the **circumferential** or the so-called hoop stress in a thin-walled cylinder.

Figure 8–19c shows a free-body diagram that can be used to calculate the longitudinal normal stress in a thin-walled cylinder. Axial force equilibrium gives

$$(2\pi Rt)\sigma_\ell = Q$$

where

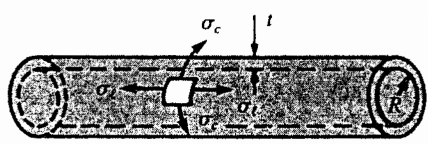
$$Q = \pi R^2 p$$

Thus

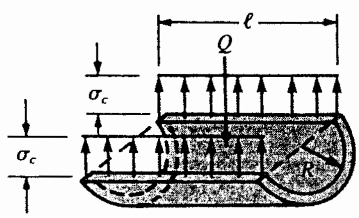
$$\sigma_\ell = \frac{pR}{2t} \tag{8–10}$$

Observe that, for cylindrical pressure vessels,

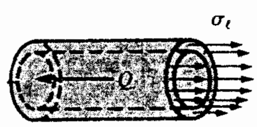
$$\sigma_c = 2\sigma_\ell \tag{8–11}$$



(a) Right-circular cylinder



(b) Free-body diagram required to determine the circumferential normal stress



(c) Free-body diagram required to determine the longitudinal normal stress

Figure 8–19

The circumferential and longitudinal stresses are shown on a differential element on the surface of the cylinder in Figure 8–19a. Note that, because of the symmetry of the pressure distribution, there is no angular distortion of the element, and consequently the shearing stresses on this element are zero. Consequently, σ_c and σ_ℓ are **principal stresses**.

Thin-walled spheres. Thin-walled spherical pressure vessels can be analyzed in a manner analogous to that used to analyze thin-walled cylindrical pressure vessels.

Figure 8–20 shows the portion of a spherical pressure vessel that has been obtained by cutting the sphere along a great circle. Denote the radius and thickness of the sphere by R and t, respectively.

Equilibrium of forces yields

$$(2\pi Rt)\sigma_c = Q$$

where

$$Q = \pi R^2 p$$

Thus

$$\sigma_c = \frac{pR}{2t} \tag{8–12}$$

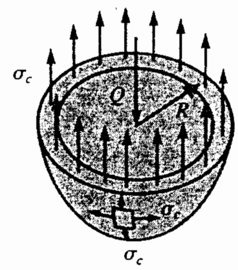


Figure 8–20

Here again, the concept of projected area has been used. Now, cutting the spherical surface along any other great circle leads to the same free-body diagram which, in turn, leads to Eq. (8–12). We conclude that the normal stress in a spherical pressure vessel is the same in all directions. This situation is shown on a differential element of material at the surface of the spherical vessel shown in Figure 8–20.

The analysis given here shows that a sphere is an optimum shape for an internally pressurized closed vessel. The maximum normal stress in a cylindrical vessel is twice that of a spherical vessel for the same internal pressure and the same R/t ratio.

EXAMPLE 8–12

A cylindrical tank 5 ft in diameter is made from steel plate ¾-in. thick and is used to store a certain gas under pressure. Determine the maximum pressure the tank can resist if the allowable stress is 20,000 psi in tension.

SOLUTION The maximum normal stress in the cylindrical pressure vessel is given by the formula

$$\sigma_c = \frac{pR}{t} \tag{a}$$

Because σ_c is the maximum normal stress in the cylinder, it cannot exceed 20,000 psi. Consequently,

$$20{,}000 = \frac{p(30)}{\frac{3}{4}}$$

or

$$p_{\max} = 500 \text{ psi} \tag{b}$$

Of course, the cylindrical tank has ends. They can be flat, hemispherical, or some other shape. Flat ends are apparently the least desirable because incompatible geometric deformations at the juncture are most pronounced in this case. This incompatibility is present for other ends also. Thus there will always be additional local stresses developed at the juncture. In the simple example given here, these local stresses were not taken into consideration.