



## STOCHASTIC FINITE ELEMENT ANALYSIS OF THE FREE VIBRATION OF FUNCTIONALLY GRADED MATERIAL PLATES

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## Abstract

The superior properties of Functionally Graded Materials (FGM) are usually accompanied by randomness in their properties due to difficulties in tailoring the gradients during manufacturing processes. Using the Stochastic Finite Element Method (SFEM) proved to be a powerful tool in studying the sensitivity of the static response of FGM plates to uncertainties in their material properties. This tool is yet to be used in studying free vibration of FGM plates. The aim of this work is to use a Second Order Reliability Method (SORM), combined with a nine-noded isoparametric Lagrangian element based on the third order shear deformation theory to investigate sensitivity of the fundamental frequency of FGM plates to material uncertainties. These include uncertainties in ceramic and metal Young's modulus and Poisson's ratio, their densities and the ceramic volume fraction. The developed code utilizes MATLAB capabilities to derive the derivatives of the stiffness and mass matrices symbolically with a considerable reduction in calculation time. Calculating the eigenvectors at the mean values of the variables and updating them only at the last iteration significantly increases solution speed. The results of the stochastic finite element code are compared to published results and to the results of the well-established Monte Carlo simulation technique with importance sampling. Results show that the relative importance of variations in the constituents' properties is highly dependent on the volume fraction and is virtually independent of the frequency ratio for practical values of solution reliability. SORM is proven to be an excellent rapid tool in the stochastic analysis of free vibration of FGM plates, when compared to the slower Monte Carlo simulation techniques.

## 1. INTRODUCTION

One way to overcome the adverse effects of abrupt changes in material properties of conventional laminated composites is to use Functionally Graded Materials (FGM). In these materials, properties vary continuously across the thickness by gradually changing the volume