## ORIGINAL PAPER

## Stochastic finite element analysis of the free vibration of laminated composite plates

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Abstract Using the Stochastic Finite Element Method (SFEM) to perform reliability analysis of the free vibration of composite plates with material and fabrication uncertainties has received much attention lately. In this work the stochastic analysis is performed using the First-Order Reliability Method (FORM-method 2) and the Second-Order Reliability Method (SORM). The basic random variables include laminae stiffness properties and material density, as well as the randomness in ply orientation angles. Modeling of the composite behavior utilizes a nine-noded isoparametric Lagrangian element based on the third-order shear deformation theory. Calculating the eigenvectors at the mean values of the variables proves to be a reasonable simplification which significantly increases solution speed. The stochastic finite element code is validated using comparisons with results of Monte Carlo simulation technique with importance sampling. Results show that SORM is an excellent rapid tool in the stochastic analysis of free vibration of composite plates, when compared to the slower Monte Carlo simulation techniques.

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## 1 Introduction

Dynamic behavior of composite laminates is a function of the geometrical and material properties of these laminates. Mechanical properties, density, stacking sequence, as well as the laminate dimensions determine the values of the natural frequencies. These quantities are not deterministic in nature. Rather, uncertainties in their values due to manufacturing and fabrication result in variations in the behavior characteristics of the laminate such as the values of the natural frequencies. Computer simulations of composite laminates used in aerospace applications often show closely packed or overlapping natural frequencies. In such cases even the slightest shift in characteristics of the laminate can have a pronounced effect on the response of the structure. For proper quality control of the dynamic characteristics of laminates, their sensitivities to the laminate properties need to be investigated.

Analysis of structures with deterministic characteristics to random excitations has been reported extensively in the literature; see [1] for example. This is not the case, however, for the analysis of composite structures with a comprehensive implementation of uncertainties. Sources of uncertainties range from the statistical nature of the material properties of the constituents, to the inevitable fabrication randomness in layup and curing. To implement the effects of material and manufacturing uncertainties, a set of random variables representing laminate mechanical properties, density and orientation angles is chosen. Randomness in these variables is quantified experimentally or using simulation codes. Ibrahim [2] and Manohar and Ibrahim [3] have presented a review of structural dynamics problems with such stochastic parameter variations. Oh and Librescu [4] developed a mean-centered second-moment method to study the free vibration and reliability of composite cantilevers.

