SE513: Systems Identification

Lecture 2

Introductory Examples

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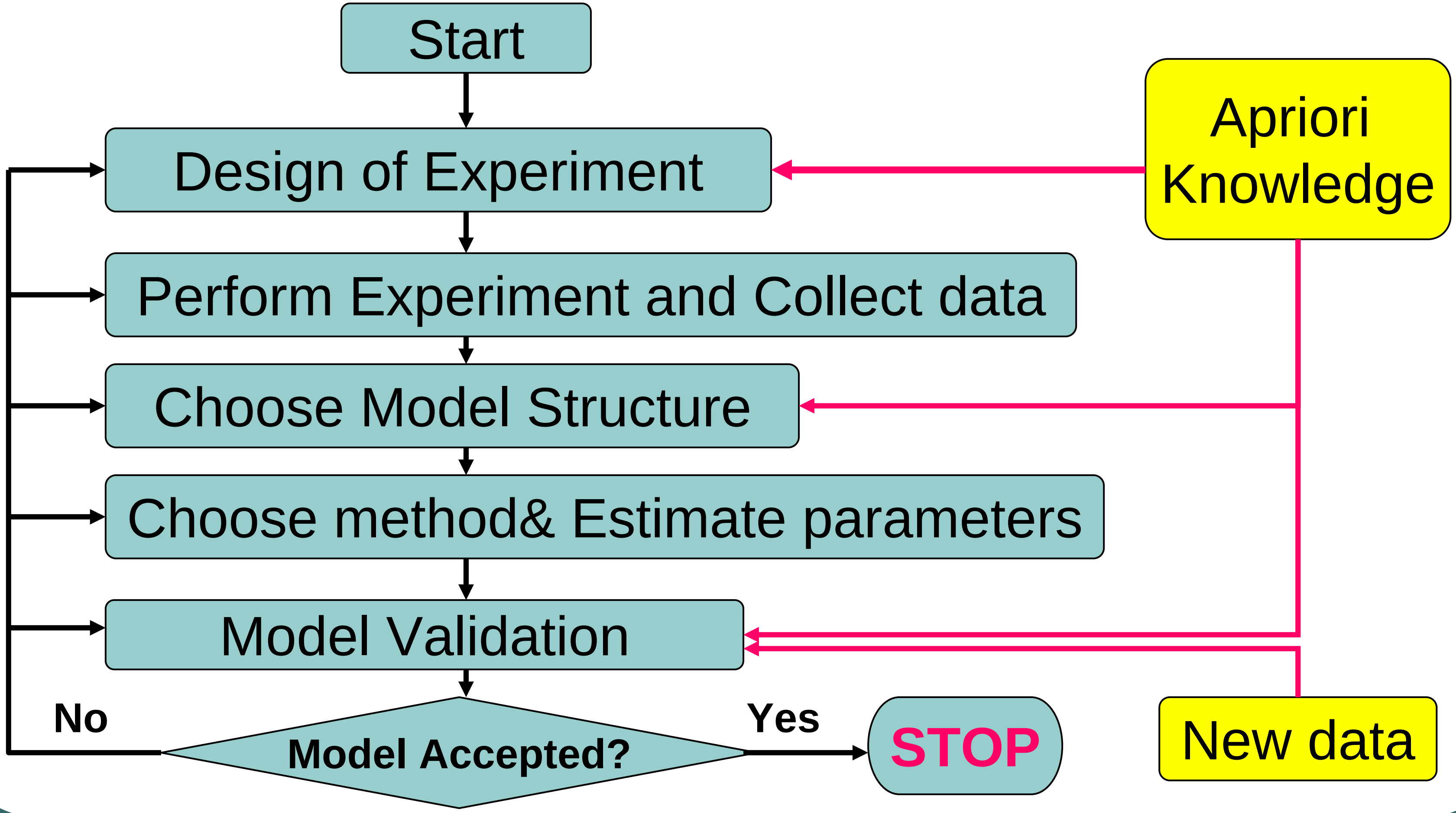
Flow Chart for System Identification



Lecture Outlines

- Important Concepts:
  - The System
  - The Model Structure
  - The Identification Method
  - Experimental Condition
- Basic Example
- Non-parametric Methods
- Parametric Methods

Four Important Concepts in Identification

System *S*	Model Structure *M*
Identification Method J	Experimental Condition *X*

The System S

- The System S is a mathematical description of the process
  - S is unknown in physical systems
    - No model can ever represent a real physical system
  - S is known in simulated systems
- The system S is specified by specifying assumptions on the signals
- We do not need to know S to identify the system
- The system S is used for theoretical investigation of behavior of different methods under different conditions

Model Structure $\mathcal{M}$

- The model structure defines the set of all candidate models parameterized by the parameter vector

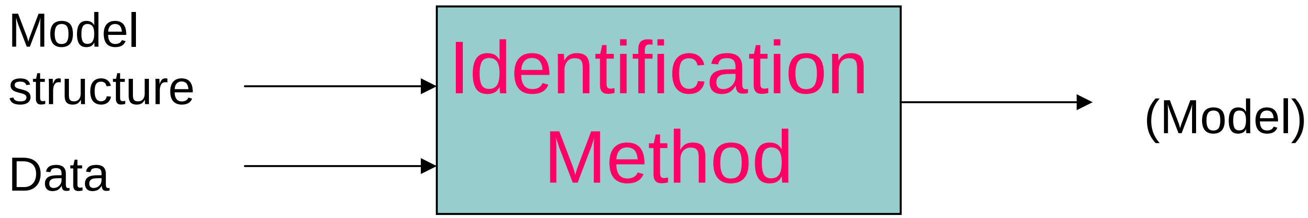
$$Example: \quad y(t) + a\,y(t-1) = b\,u(t-1) + \varepsilon(t)$$

$$\varepsilon(t): Error$$

- Candidate models are first order discrete time models with two parameters $a \;\; and \;\; b$

Identification Method

- The identification method is a procedure with witch the parameter vector is obtained.



- Large number of methods are available
- Choice depends on { model structure, data, preference,….}

Experimental Conditions X

- How to do the identification experiment?
- Major Decisions:
  - Selection of input signals
  - Selection of sample period
  - Selection of prefiltering
  - Take measurement while open loop/feedback
  - …

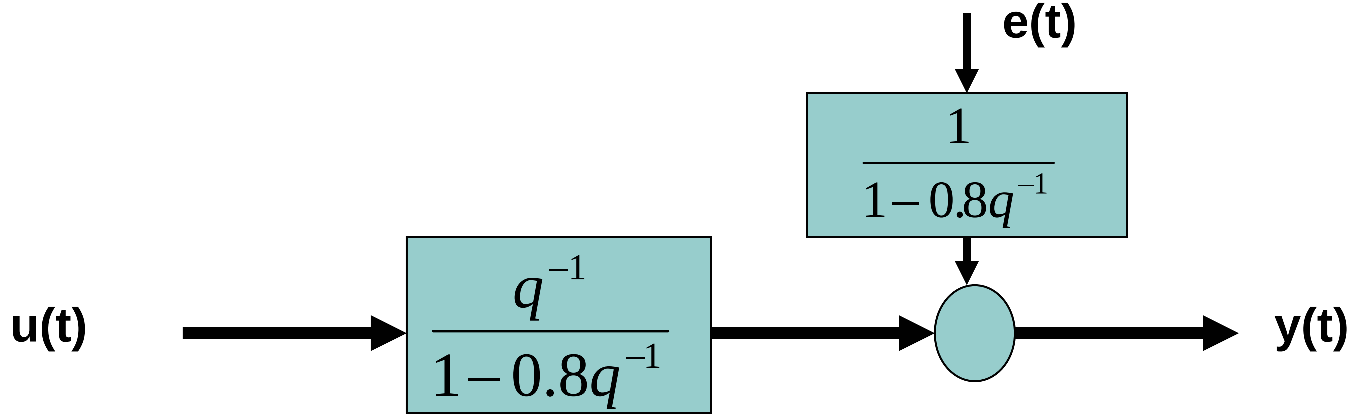
The System S is a mathematical description of the process
S is known in simulated systems

The System *S1*

- We have two simulated systems
- S 1: $y(t) - 0.8y(t-1) = u(t-1) + e(t)$

$$e : \text{i.i.d. random variable}$$

$$\text{mean}\{e(t)\} = 0, \ \text{variance}\{e(t)\} = \lambda^2$$



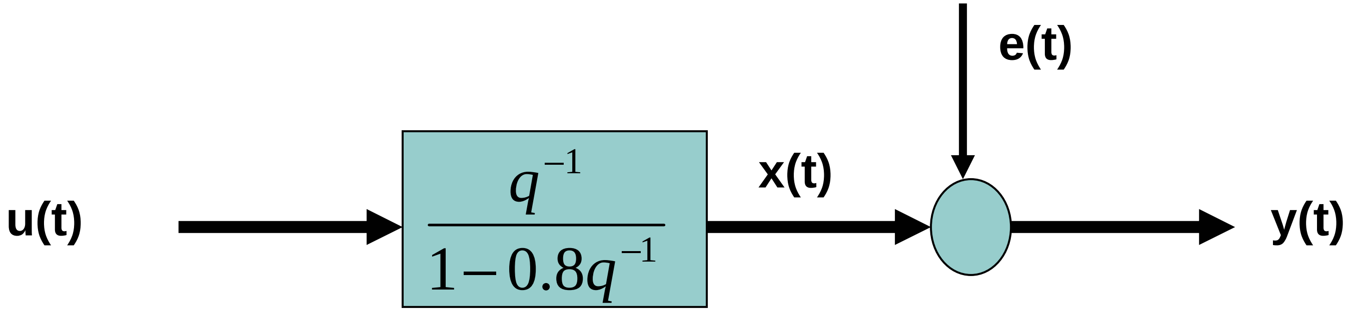
The System S is a mathematical description of the process
S is known in simulated systems

The System S2

S 2: $x(t) - 0.8x(t-1) = u(t-1)$

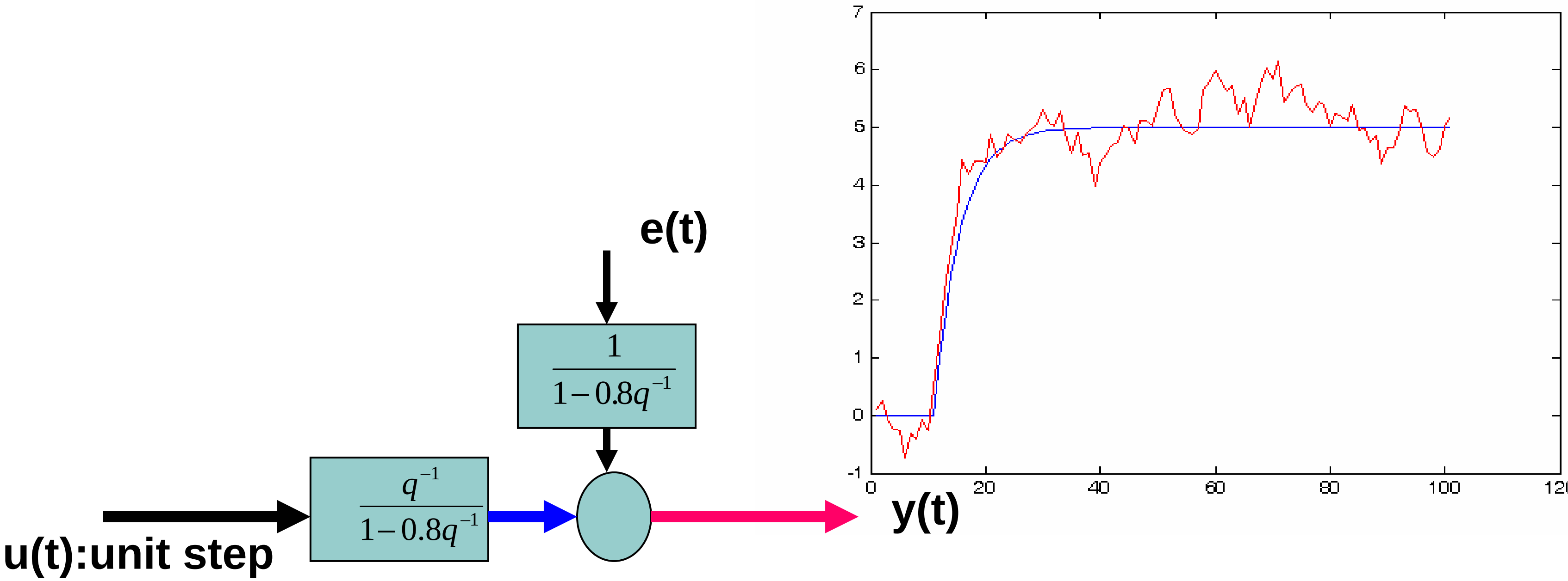
$$y(t) = x(t) + e(t)$$

$$\text{mean}\{e(t)\} = 0, \ \text{variance}\{e(t)\} = \lambda^2$$



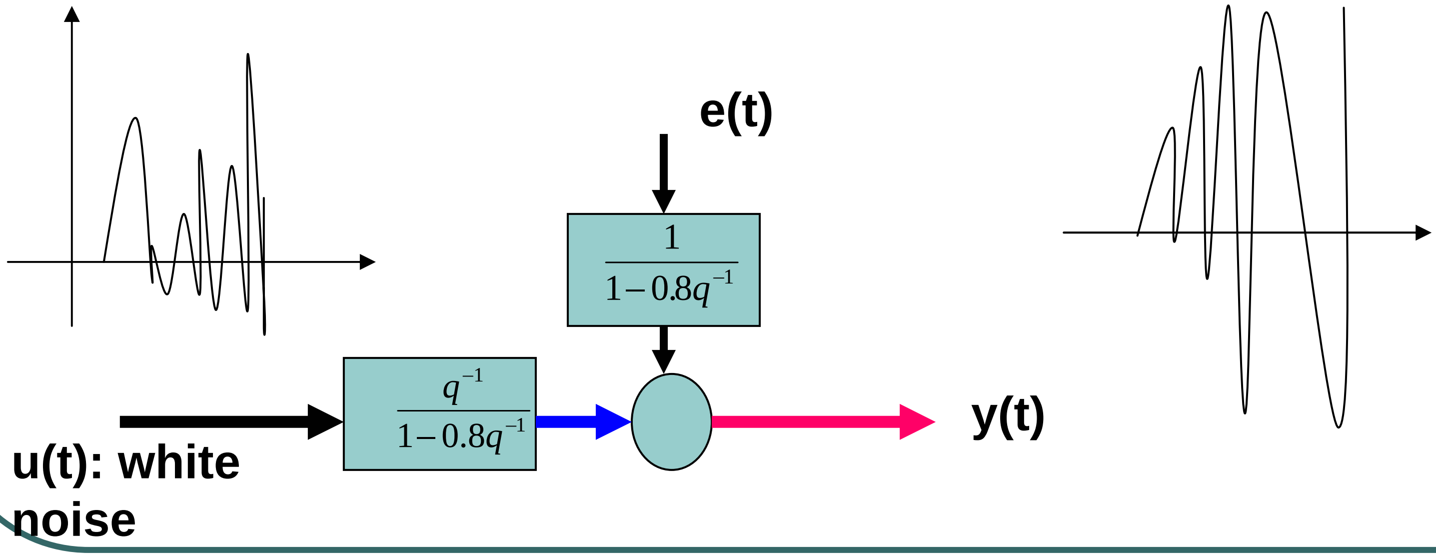
Example 1
Non-parametric Model

- Data: Step response of *S* 1
- Model: Step Response Curve.



Example 2
Non-parametric Model

- Data: input and output of *S* 1
- Model: Weighting Sequence Curve.

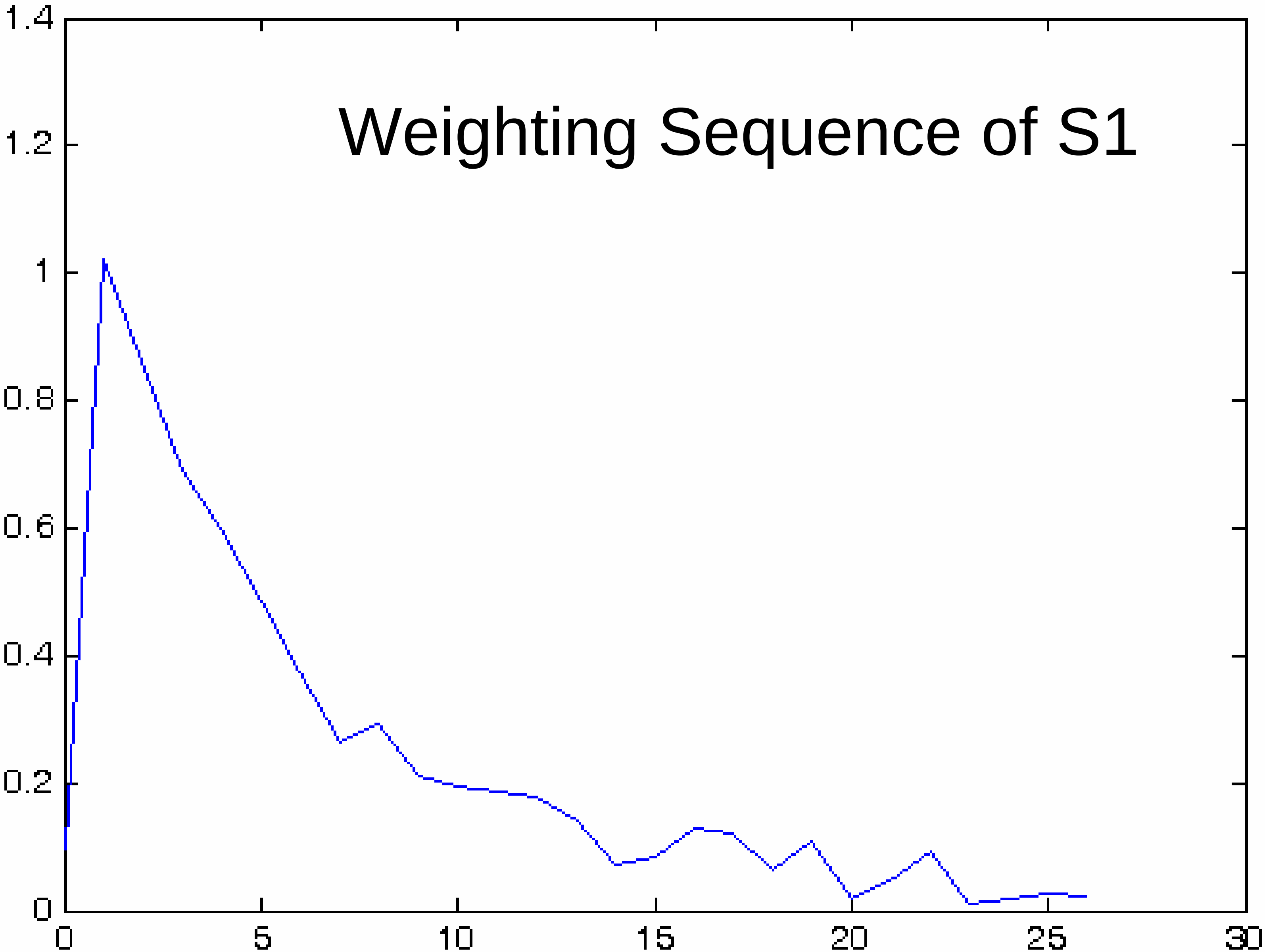


Example 2
Non-parametric Model

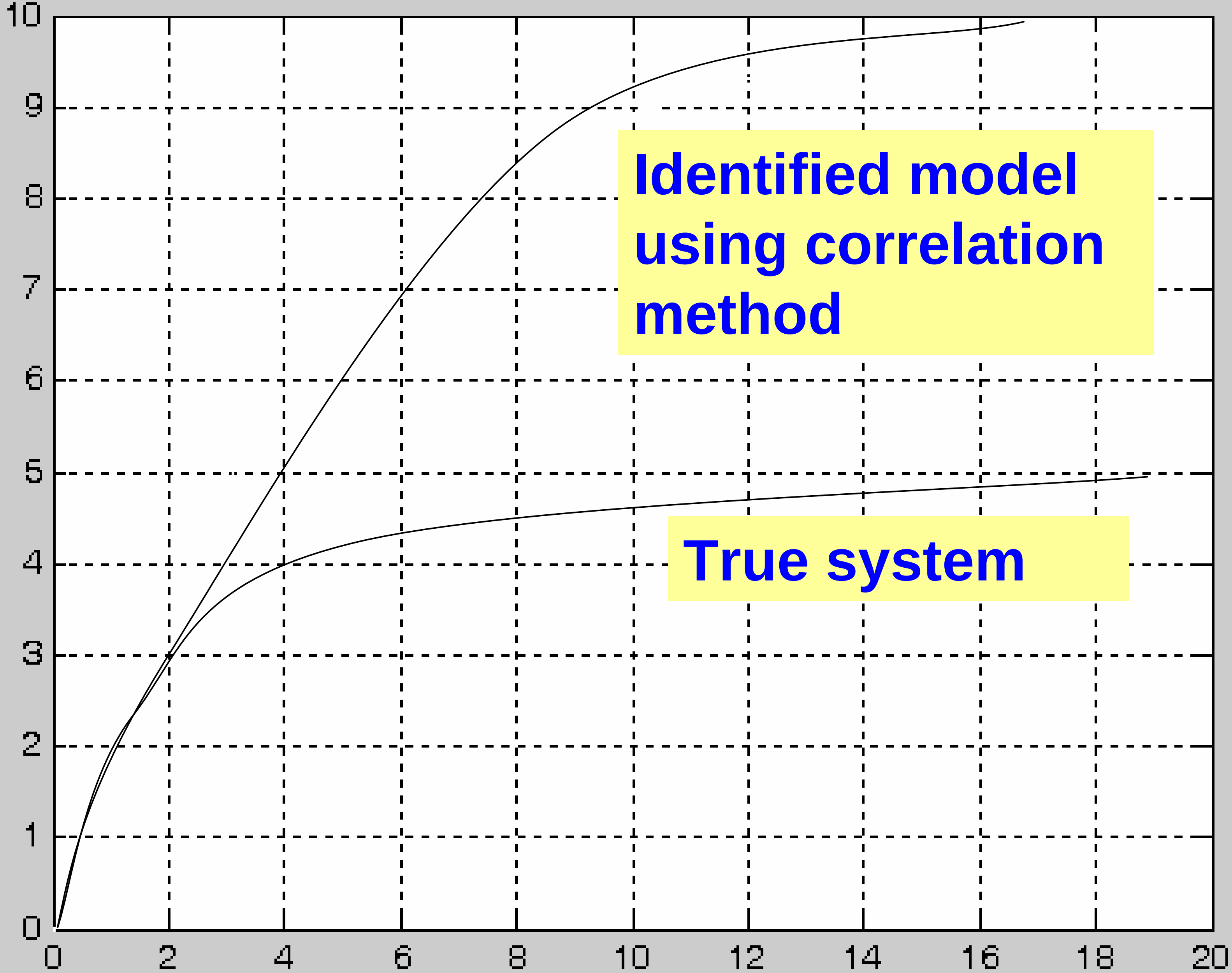
- **Data:** u(t): white noise, y: response of *S* 1
- **Model:** Weighting Sequence Curve.

$$h(k) = \frac{\frac{1}{N}\sum_{t=1}^{N-k} y(t+k)u(t)}{\frac{1}{N}\sum_{t=1}^{N} u^2(t)}$$

This is an approximate formula for the weighting sequence.


Weighting Sequence of S1
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30

True verses identified model (noise free step response)



Remarks on the obtained model

- The obtained model is not accurate
- The steady state gain is almost double the true value
- The time constant and rise time are very close to the true value

Parametric Model

- True System S 1
- Model Structure M1: $y(t) + ay(t-1) = bu(t-1) + \varepsilon(t)$
- Two unknown parameters $\theta = [a \quad b]^T$
- Why do we include ε in the model?
- ε is often called equation error

Least Square Identification Method

- Determine the parameters of M such that the sum of the square of the error is minimized.
- The loss function (to be minimized) is

$$V(\theta) = \sum_{t=1}^{N} \varepsilon^2(t)$$

$$= \sum_{t=1}^{N} \left[y(t) + ay(t-1) - bu(t-1) \right]^2$$

- **How do we minimize V( )?**

Least Square Identification Method

$$\left.\begin{aligned}\frac{\partial}{\partial a}V(\theta)=0\\ \frac{\partial}{\partial b}V(\theta)=0\end{aligned}\right\}\text{The normal equations}$$

$$\Rightarrow$$

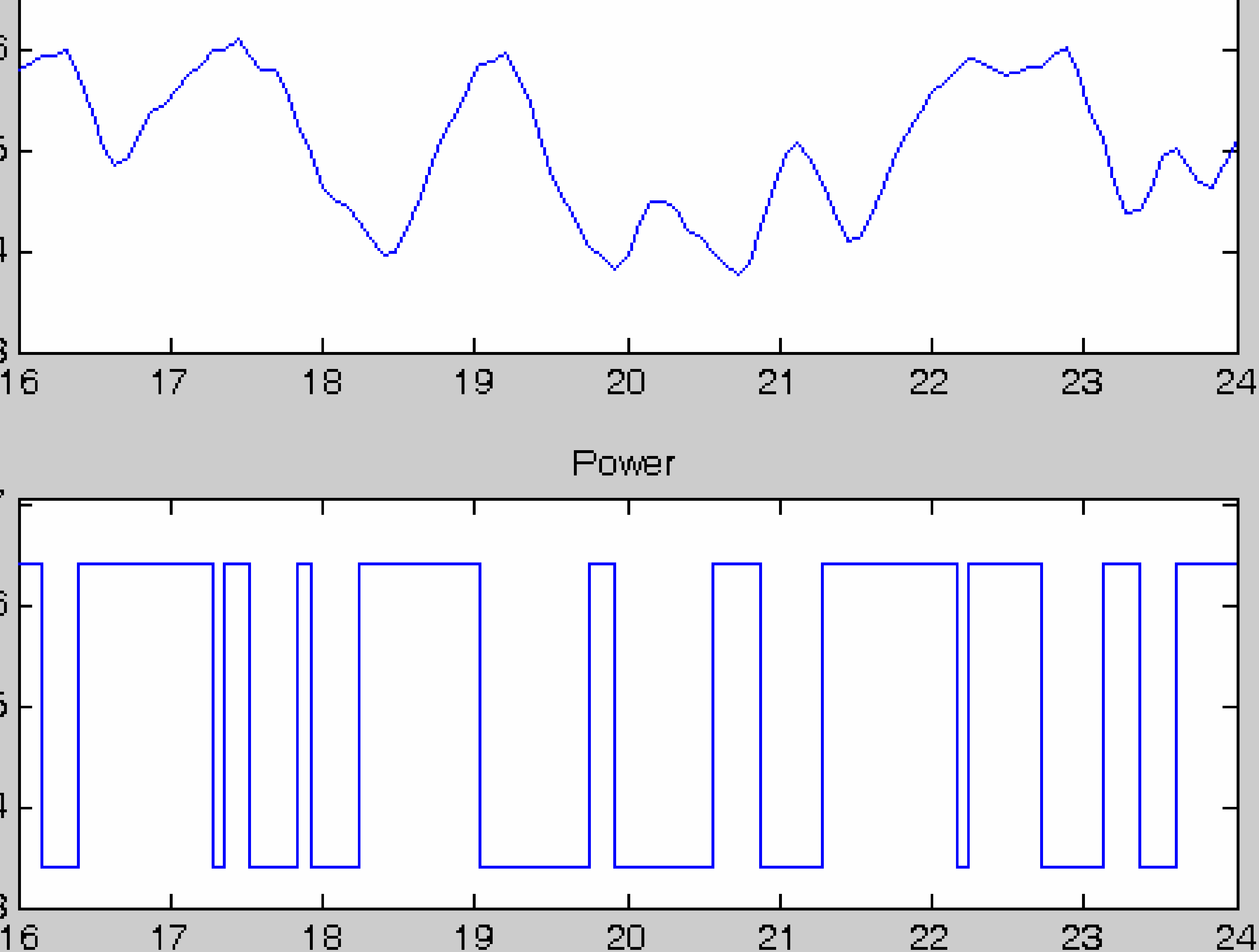
$$\begin{bmatrix}\sum_{t=1}^{N}y^2(t-1) & -\sum_{t=1}^{N}y(t-1)u(t-1)\\ -\sum_{t=1}^{N}y(t-1)u(t-1) & \sum_{t=1}^{N}u^2(t-1)\end{bmatrix}\begin{pmatrix}\hat{a}\\ \hat{b}\end{pmatrix}=\begin{pmatrix}-\sum_{t=1}^{N}y(t)y(t-1)\\ \sum_{t=1}^{N}y(t)u(t-1)\end{pmatrix}$$

Least Square Identification Method

- To find the unknown parameters, we need to solve two equations in two unknowns

Experimental Conditions *X*

- Selection of input signals
  - Step
  - Impulse
  - White Noise
  - PRBS (Pseudo Random Binary Sequence)
- Selection of sample period
- Selection of the number of samples N
- Selection of prefiltering
- Take measurement while open loop/feedback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Power
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Experimental Conditions *X*

- Different input sequences leads to different identified models.
- Which one is better?
  - Step
  - Impulse
  - Sum of sinusoidal
  - PRBS (Pseudo Random Binary Sequence)

Reading Material

- Chapter 2 (Soderstrom & Stoica)