## Problem S4.6

Obtain the inverse Laplace transform of $F(s)=\dfrac{1}{(s+1)(s+2)^3}$

Solution:

$$\frac{1}{(s+1)(s+2)^3}=\frac{A}{s+1}+\frac{B}{(s+2)^3}+\frac{C}{(s+2)^2}+\frac{D}{s+2}$$

$$A=(s+1)\frac{1}{(s+1)(s+2)^3}\bigg|_{s=-1}=1$$

$$B=(s+2)^3\frac{1}{(s+1)(s+2)^3}\bigg|_{s=-2}=\frac{1}{s+1}\bigg|_{s=-2}=-1$$

$$C=\frac{d}{ds}\left(\frac{1}{s+1}\right)\bigg|_{s=-2}=\frac{-1}{(s+1)^2}\bigg|_{s=-2}=-1$$

$$D=\frac{1}{2!}\ \frac{d^2}{ds^2}\left(\frac{1}{s+1}\right)\bigg|_{s=-2}=\frac{1}{2!}\ \frac{d}{ds}\left(\frac{-1}{(s+1)^2}\right)\bigg|_{s=-2}=-1$$

$$\frac{1}{(s+1)(s+2)^3}=\frac{1}{s+1}+\frac{-1}{(s+2)^3}+\frac{-1}{(s+2)^2}+\frac{-1}{s+2}$$

$$f(t)=e^{-t}-\frac{1}{2}t^2e^{-2t}-te^{-2t}-e^{-2t}$$