Problem S4.3

Obtain the inverse Laplace transform of $F(s)=\dfrac{1}{s(s^2+4)}$

Solution:

F(s) has three simple poles ($s=0,+2i\ \ and\ -2i$).

$$F(s)=\frac{a_1}{s}+\frac{a_2}{s+2i}+\frac{a_3}{s-2i}$$

$$a_1=s\ \left.\frac{1}{s(s^2+4)}\right|_{s=0}=\left.\frac{1}{(s^2+4)}\right|_{s=0}=\frac{1}{4}$$

$$a_2=(s+2i)\ \left.\frac{1}{s(s+2i)(s-2i)}\right|_{s=-2i}=\frac{1}{-2i(-4i)}=\frac{-1}{8}$$

$$a_3=(s-2i)\ \left.\frac{1}{s(s+2i)(s-2i)}\right|_{s=2i}=\frac{1}{2i(4i)}=\frac{-1}{8}$$

$$F(s)=\frac{0.25}{s}+\frac{-0.125}{s+2i}+\frac{-0.125}{s-2i}$$

$$f(t)=0.25-0.125e^{-2it}-0.125e^{2it}$$

Using Euler identity $\cos(\omega t)=\dfrac{e^{j\omega t}+e^{-j\omega t}}{2}$, the expression of f(t) is simplified as

$$f(t)=0.25-0.125e^{-2it}-0.125e^{2it}=0.25-0.25\cos(2t)\ .$$