SE311: Design of Digital Systems

Lecture 8: Gate Level Minimization II

Dr. Samir Al-Amer
(Term 041)

Outlines

- Prime Implicants
- Five Variable Maps
- Product of Sum Simplifications
- Don’t Care Condition

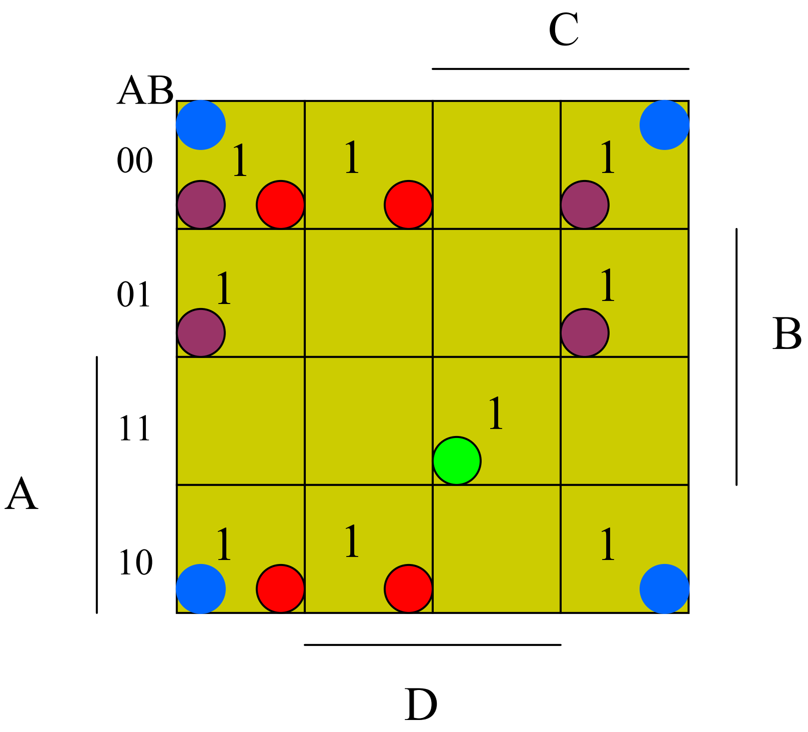
Prime implicants

- In simplifying a function make sure
  - All minterms of the function are covered
  - avoid any redundant terms
- **Prime implicant** is a product term obtained by combining the maximum number of adjacent squares in the map
- If a minterm is covered by only one prime implicant then it is called **essential prime implicant**

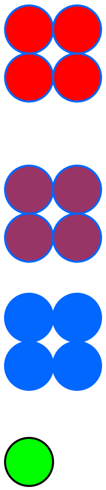
Prime implicants

- A single square is **Prime implicant** if it is not adjacent to any other 1's in the map
- 2 adjacent squares is **Prime implicant** if they can not be a part of any 4 adjacent 1's in the map.
- 4 adjacent squares is **Prime implicant** if they can not be a part of any 8 adjacent 1's in the map.

Prime Implicants



Essential prime implicants



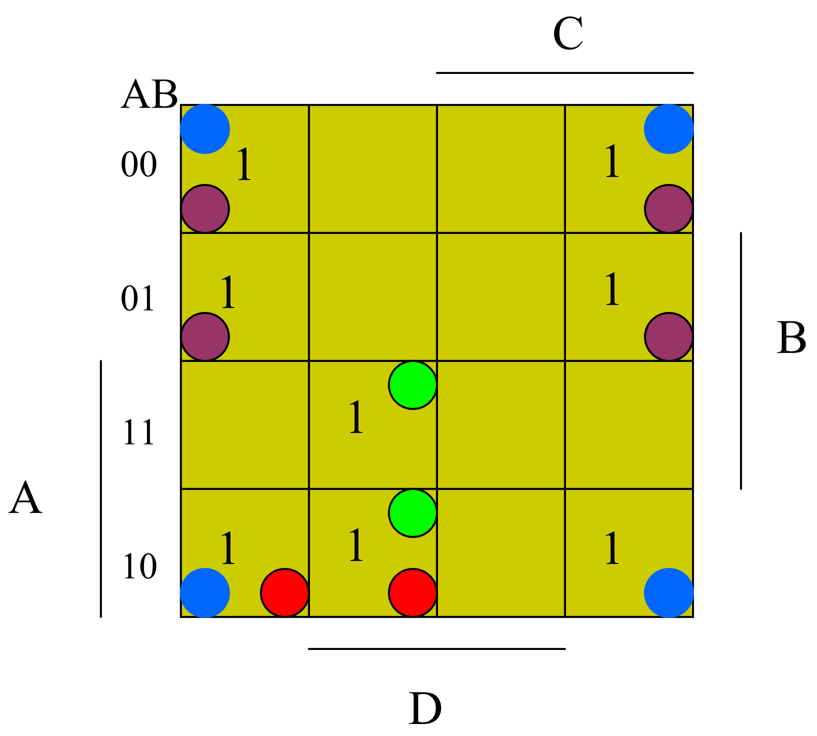
B’C’

A’D’

B’D’

ABCD

Prime Implicants



Essential prime implicants

B'D'

A'D'

AC'D

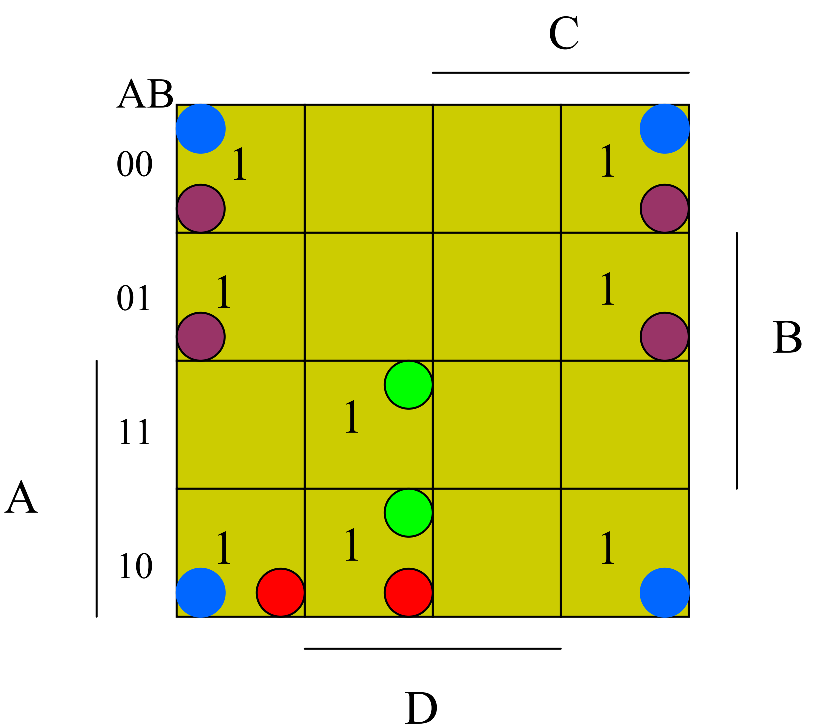
prime implicants

AB'C'

Simplification Procedure

- Determine all essential prime implicants
- The simplified expression is the sum of all essential prime implicants and the prime implicants needed to cover any remaining 1's

Simplification using Prime Implicants



Essential prime implicants

B'D'

A'D'

AC'D

prime implicants

AB'C' (not needed)

F=B'D'+A'D'+AC'D

Five Variable Maps

- 5 Variables
- 32 squares Drawn as 2 groups of 16 squares

Five Variable Maps

A=0

D

BC \ DE	00	01	11	10
00	m0	m1	m3	m2
01	m4	m5	m7	m6
11	m12	m13	m15	m14
10	m8	m9	m11	m10

B C

E

A=1

D

BC \ DE	00	01	11	10
00	m16	m17	m19	m18
01	m20	m21	m23	m22
11	m28	m29	m31	m30
10	m24	m25	m27	m26

B C

E

Adjacent Squares

Examples of five variable case

A=0

BC \ DE	00	01	11	10
00	m0	m1	m3	m2
01	m4	m5	m7	m6
11	m12	m13	m15	m14
10	m8	m9	m11	m10

D, C, B, E

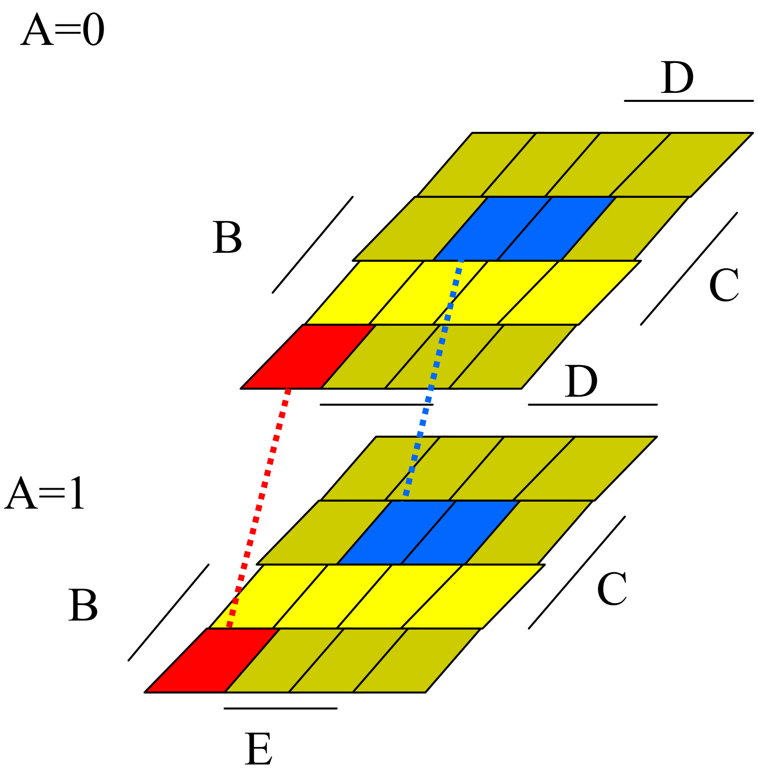
A=1

BC \ DE	00	01	11	10
00	m16	m17	m19	m18
01	m20	m21	m23	m22
11	m28	m29	m31	m30
10	m24	m25	m27	m26

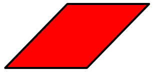
D, C, B, E

Adjacent Squares

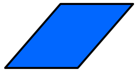
Examples of five variable case



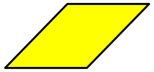
2 Adjacent squares



4 Adjacent squares

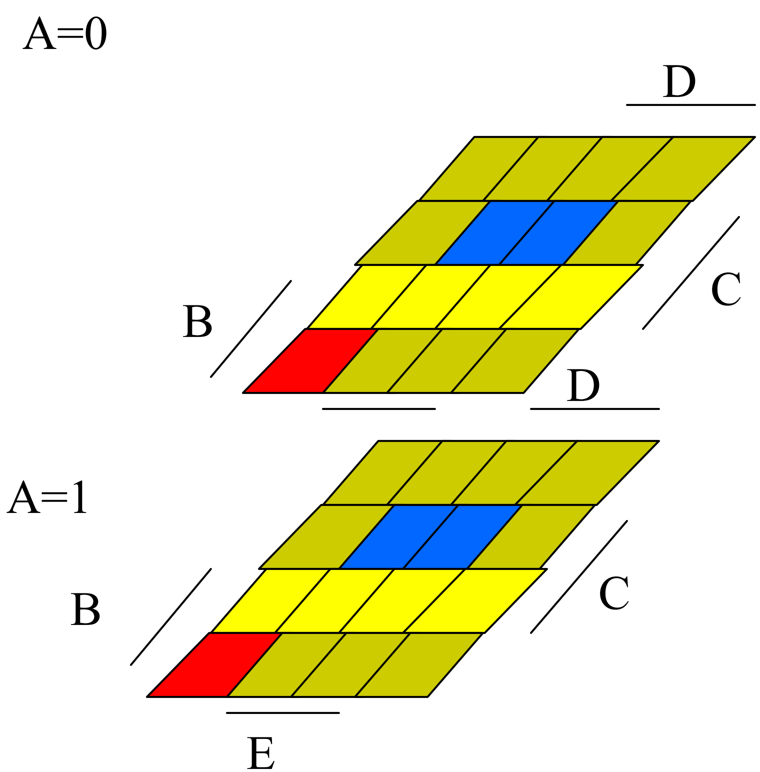


8 Adjacent squares



Adjacent Squares

Examples of five variable case



2 Adjacent squares

B C’ D’ E’

4 Adjacent squares

B’ C E

8 Adjacent squares

B C

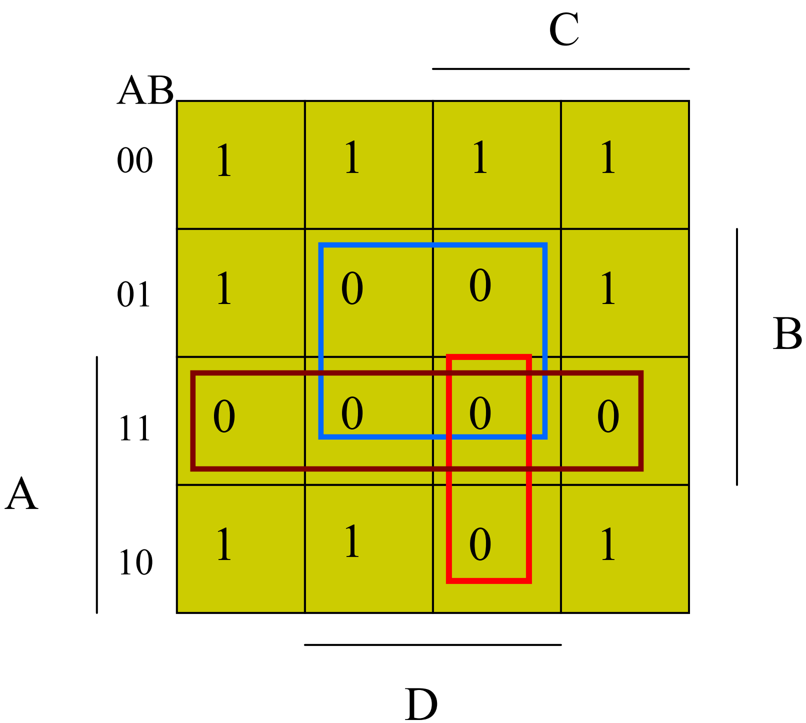
Adjacent Squares and # of literals

	Adjacent Squares				
n	2	4	8	16	32
2	1	0			
3	2	1	0		
4	3	2	1	0	
5	4	3	2	1	0

When n=3

4 adjacent squares gives a term with one literal

Product of sum simplification



Procedure

1.Mark the Map with 1’s and 0’s

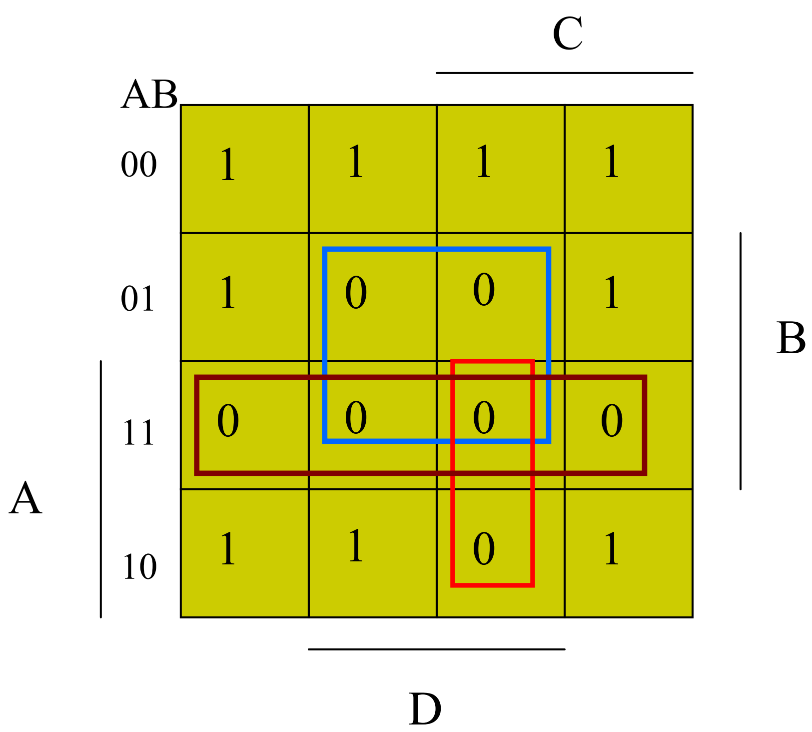
2. Obtain F’ by determining

those terms that cover 0’s

3. Obtain the complement of F’

Product of sum simplification

Example



Procedure

1.Mark the Map with 1's and 0's

2. Obtain F' by determining

those terms that cover 0's

F'=AB+BD+ACD

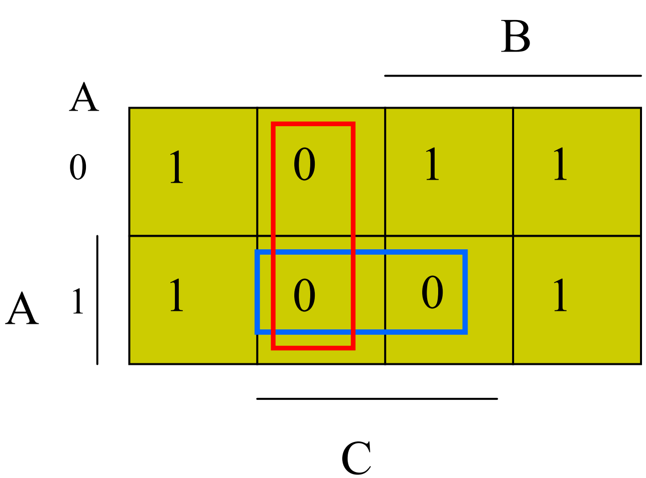
3. Obtain the complement of F'

F'=(A'+B')(B'+D')(A'+C'+D')

Product of sum simplification
Example

$F=\Sigma(0,2,3,4,6)$

F =(A’+B+C’)(A’+B’+C’)(A+B+C’)



Procedure

1. Mark the Map with 1’s and 0’s

2. Obtain F’ by determining

   those terms that cover 0’s

   F’=AC+B’ C

3. Obtain the complement of F’

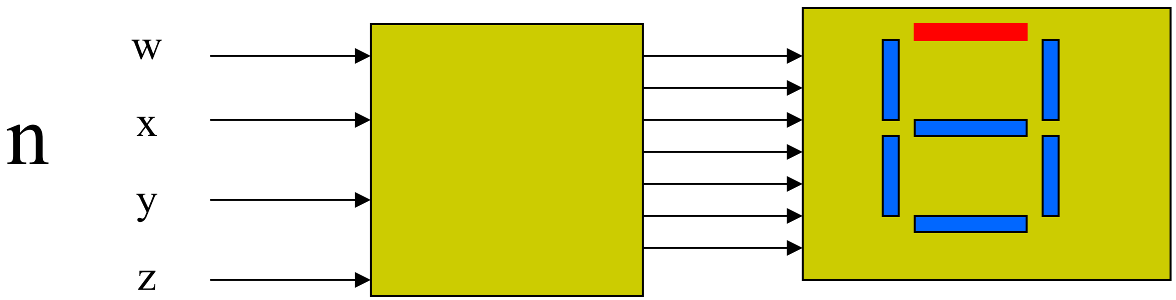
F’=(A’+C’)(B+C’)

Don’t Care Conditions

- In some applications some combinations of the variables can never occur
- Corresponding squares can not be marked as 1’s or 0’s   (the value is not specified)
- They are marked X
- For simplifying the expressions we can consider them as 1’s or as 0’s

Don’t Care Conditions

Example



is ON

if n=0,2,3

5,6,7,8,9

- n=(wxyz)B
- The following combinations of the variables can never occur

wx’yz’ (n=10), wx’yz (n=11), …, wxyz (n=15).

Don’t Care

Example

y

wx				
00	1	0	1	1
01	0	1	1	1
11	X	X	X	X
10	1	1	X	X

z

x

w

We can replace all X by 1

$$F=w+y+xz+x'z'$$

Summary

- Prime Implicants
- Five Variable Maps
  - 32 squares, 2 groups of 16 squares
- Product of Sum Simplifications
  - Obtain F' by covering all zeros in the map then complement
- Don't Care Condition
  - Replace X by 1 or 0 depending on which gives simpler expression