

Serial #: _____ ID _____ NAME _____

Show all necessary steps for full marks.**Question 1: (5 points):** Find all values of k for which the following equation has only one solution:

$$kx^2 + 2(k+4)x + 25 = 0$$

Solution: $b^2 - 4ac = 0$

$$[2(k+4)]^2 - 4k(25) = 0$$

$$4(k+4)^2 - 4k(25) = 0$$

$$(k+4)^2 - 25k = 0$$

$$k^2 + 8k + 16 - 25k = 0$$

$$k^2 - 17k + 16 = 0$$

$$(k-1)(k-16) = 0$$

$$k = 1, k = 16$$

Question 2: (5 points): If the equation $(3x-4)(x+1) = -2$ is written in the form $(x+m)^2 = n$, then find $m+n = ?$ **Solution:** $3x^2 + 3x - 4x - 4 = -2$

$$3x^2 - x = 2$$

$$x^2 - \frac{1}{3}x = \frac{2}{3}$$

$$x^2 - \frac{1}{3}x + \left(\frac{1}{2} \cdot \frac{1}{3}\right)^2 = \frac{2}{3} + \left(\frac{1}{6}\right)^2$$

$$x^2 - \frac{1}{3}x + \left(\frac{1}{6}\right)^2 = \frac{2}{3} + \frac{1}{36}$$

$$\left(x - \frac{1}{6}\right)^2 = \frac{2(12)}{3(12)} + \frac{1}{36}$$

$$\left(x - \frac{1}{6}\right)^2 = \frac{24+1}{36}$$

$$m+n = -\frac{1}{6} + \frac{25}{36} = \frac{-6+25}{36} = \frac{19}{36}$$

Question 3: (5 points): If $\frac{3}{2-3i} - \frac{2}{3-2i} = A + Bi$ then find the value of A and B

$$\begin{aligned} \text{Solution: } \frac{3}{2-3i} - \frac{2}{3-2i} &= \frac{9-6i-2(2-3i)}{6-4i-9i-6} = \frac{9-6i-4+6i}{6-4i-9i-6} = \frac{5}{-13i} \\ &= \frac{5}{-13i} \cdot \frac{i}{i} = \frac{5i}{13} = 0 + \frac{5}{13}i \end{aligned}$$

$$\Rightarrow A = 0, B = \frac{5}{13}$$

Question 4: (5 points): Solve $(x+1)^{2/3} - (x+1)^{1/3} - 2 = 0$

Solution: Let $u = (x+1)^{1/3}$.

$$u^2 - u - 2 = 0$$

$$(u-2)(u+1) = 0$$

$$u-2=0 \quad \text{or} \quad u+1=0$$

$$u=2 \quad \text{or} \quad u=-1$$

$$(x+1)^{1/3} = 2 \quad \text{or} \quad (x+1)^{1/3} = -1$$

$$\left[(x+1)^{1/3}\right]^3 = 2^3 \quad \text{or} \quad \left[(x+1)^{1/3}\right]^3 = (-1)^3$$

$$x+1=8 \quad \text{or} \quad x+1=-1$$

$$x=7 \quad \text{or} \quad x=-2$$

Check $x=7$:

$$(7+1)^{2/3} - (7+1)^{1/3} - 2 = 0 \quad ?$$

$$8^{2/3} - 8^{1/3} - 2 = 0 \quad ?$$

$$4 - 2 - 2 = 0 \quad \text{True}$$

$$SS = \{-2, 7\}$$

Check $x=-2$:

$$(-2+1)^{2/3} - (-2+1)^{1/3} - 2 = 0 \quad ?$$

$$(-1)^{2/3} - (-1)^{1/3} - 2 = 0 \quad ?$$

$$1 + 1 - 2 = 0 \quad \text{True}$$