Serial #: ________ ID ______________ NAME ____

Show all necessary steps for full marks.

Question 1: (5 points) (Textbook Example 2): Consider the quadratic function $f(x) = 2x^2 - 12x + 13$.

(a): Express f in standard form.

(b): Find the vertex and x- and y-intercept

(c): Sketch a graph of f.

(d): Find the domain and range of f

(e): Find the minimum value of f.

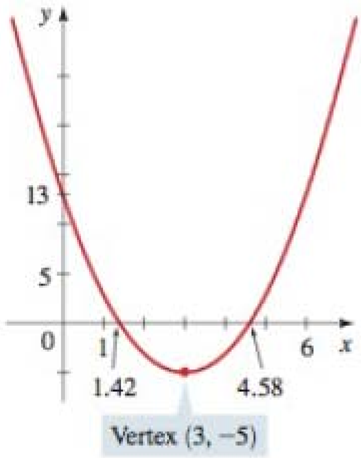
SOLUTION

(a) Since the coefficient of x^2 is not 1, we must factor this coefficient from the terms involving x before we complete the square.

$$\begin{aligned} f(x) &= 2x^2 - 12x + 13 \\ &= 2(x^2 - 6x) + 13 && \text{Factor 2 from the } x\text{-terms} \\ &= 2(x^2 - 6x + 9) + 13 - 2\cdot 9 && \text{Complete the square: Add 9 inside parentheses, subtract } 2\cdot 9 \text{ outside} \\ &= 2(x-3)^2 - 5 && \text{Factor and simplify} \end{aligned}$$

(b) From the standard form of f we can see that the vertex of f is $(3, -5)$.

(c):



(d) The domain of f is the set of all real numbers $(-\infty, \infty)$. From the graph we see that the range of f is $[-5, \infty)$.

(e): The minimum value is -5.

Question 2: (5 points) (Similar to recitation 3.1 Q#2): Find the sum of the real coefficients '***a***', '***b***', '***c***' of the quadratic function $f(x) = ax^2 + bx + c$ that has **only one** x-intercept at 3 and y-intercept at -9 is

Solution:

$y = a(x-3)^2$ only one x-intercept at 3

y-intercept is -9 $\Rightarrow$ $-9 = a(0-3)^2$ $\Rightarrow$ $9a = -9$ $\Rightarrow$ $\boxed{a = -1}$

$\Rightarrow$ $y = -1(x-3)^2 = -1(x^2 - 6x + 9) = -x^2 + 6x - 9$ $\Rightarrow$ $a + b + c = -1 + 6 - 9 = -4$

Answer: -4

Question 3: (5 points) (3.2Textbook Example 6): Let $P(x) = -2x^4 - x^3 + 3x^2$.

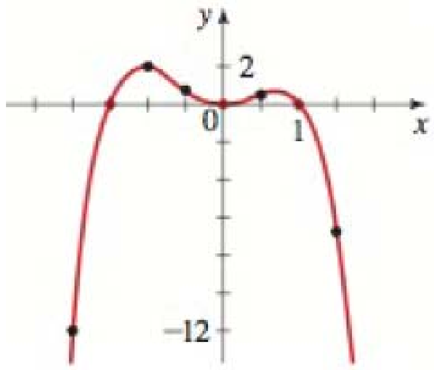
(a): Find the zeros of P. **(b):** Sketch a graph of P.

SOLUTION

(a) To find the zeros, we factor completely.

$$\begin{aligned} P(x) &= -2x^4 - x^3 + 3x^2 \\ &= -x^2(2x^2 + x - 3) && \text{Factor } -x^2 \\ &= -x^2(2x + 3)(x - 1) && \text{Factor quadratic} \end{aligned}$$

Thus the zeros are $x = 0$, $x = -\frac{3}{2}$, and $x = 1$.



Question 4: (5 points) (3.3Textbook Exercise 8):

Divide $P(x) = 2x^5 + x^3 - 2x^2 + 3x - 5$ by $x^2 - 3x + 1$ and write the results in both forms

$\dfrac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \dfrac{\text{Remainder}}{\text{Divisor}}$ and $\text{Dividend} = (\text{Divisor})(\text{Quotient}) + \text{Remainder}$

Solution:

8.

$$\begin{array}{r|l} & 2x^3 + 6x^2 + 17x + 43 \\ \hline x^2 - 3x + 1 & 2x^5 + 0x^4 + x^3 - 2x^2 + 3x - 5 \\ & 2x^5 - 6x^4 + 2x^3 \\ \hline & \quad 6x^4 - x^3 - 2x^2 \\ & \quad 6x^4 - 18x^3 + 6x^2 \\ \hline & \qquad 17x^3 - 8x^2 + 3x \\ & \qquad 17x^3 - 51x^2 + 17x \\ \hline & \qquad\quad 43x^2 - 14x - 5 \\ & \qquad\quad 43x^2 - 129x + 43 \\ & \qquad\qquad 115x - 48 \end{array}$$

Thus, the quotient is $2x^3 + 6x^2 + 17x + 43$ and the remainder is $115x - 48$ and

$$\frac{P(x)}{D(x)} = \frac{2x^5 + x^3 - 2x^2 + 3x - 5}{x^2 - 3x + 1} = \left(2x^3 + 6x^2 + 17x + 43\right) + \frac{115x - 48}{x^2 - 3x + 1}$$

$$P(x) = 2x^5 + x^3 - 2x^2 + 3x - 5 = \left(x^2 - 3x + 1\right)\left(2x^3 + 6x^2 + 17x + 43\right) + 115x - 48$$