Serial #: ________ ID ____________ NAME ______________________________

Show all necessary steps for full marks.

Question 1: (5 points):

(a): Assume $x>0$ and $y>0$. Simplify the following expression $\left(\frac{x^{1/2}y^{2}}{2y^{1/4}}\right)^{4}\left(\frac{4x^{-2}y^{-4}}{y^{2}}\right)^{1/2}=?$

(b): If $x=\frac{1}{3}$ and $y=2$. Simplify the following expression $\left(\frac{x^{1/2}y^{2}}{2y^{1/4}}\right)^{4}\left(\frac{4x^{-2}y^{-4}}{y^{2}}\right)^{1/2}=?$

Solution (a):

$$\left(\frac{x^{1/2}y^{2}}{2y^{1/4}}\right)^{4}\left(\frac{4x^{-2}y^{-4}}{y^{2}}\right)^{1/2}=\left(x^{1/2}\right)^{4}\left(2^{-1}\right)^{4}\left(y^{2-\frac{1}{4}}\right)^{4}(4)^{1/2}\left(x^{-2}\right)^{1/2}\left(y^{-4-2}\right)^{1/2}$$

$$=x^{2}2^{-4}\left(y^{7/4}\right)^{4}2x^{-1}y^{-3}$$

$$=2^{-3}xy^{4}$$

$$=\frac{xy^{4}}{8}$$

Another Method:

$$\left(\frac{x^{1/2}y^{2}}{2y^{1/4}}\right)^{4}\left(\frac{4x^{-2}y^{-4}}{y^{2}}\right)^{1/2}=\left(\frac{x^{1/2}y^{2-\frac{1}{4}}}{2}\right)^{4}\left(\frac{4}{x^{2}y^{2}y^{4}}\right)^{1/2}=\left(\frac{x^{1/2}y^{\frac{7}{4}}}{2}\right)^{4}\left(\frac{4}{x^{2}y^{6}}\right)^{1/2}$$

$$=\frac{\left(x^{1/2}\right)^{4}\left(y^{\frac{7}{4}}\right)^{4}}{2^{4}}\frac{4^{1/2}}{\left(x^{2}\right)^{1/2}\left(y^{6}\right)^{1/2}}$$

$$=\frac{x^{2}y^{7}}{2^{4}}\frac{2}{xy^{3}}$$

$$=\frac{xy^{4}}{2^{3}}$$

$$=\frac{xy^{4}}{8}$$

(b): $$\left(\frac{x^{1/2}y^{2}}{2y^{1/4}}\right)^{4}\left(\frac{4x^{-2}y^{-4}}{y^{2}}\right)^{1/2}=\frac{xy^{4}}{8}=\frac{3(2)^{4}}{8}=\frac{\frac{1}{3}(16)}{8}=\frac{2}{3}$$

Question 2: (5 points): Simplify the expression $-3xy\sqrt[4]{32x^{5}y^{6}}+2x^{2}\sqrt[4]{2^{9}xy^{10}}$

Solution: Since the index is even, then $x>0$ and $y>0$.

$$-3xy\sqrt[4]{32x^{5}y^{6}}+2x^{2}\sqrt[4]{2^{9}xy^{10}}=-3xy\sqrt[4]{2^{4}2x^{4}xy^{4}y^{2}}+2x^{2}\sqrt[4]{2^{8}2xy^{8}y^{2}}$$

$$=-3xy\,2xy\sqrt[4]{2xy^{2}}+2x^{2}2^{2}y^{2}\sqrt[4]{2xy^{2}}$$

$$=-6x^{2}y^{2}\sqrt[4]{2xy^{2}}+8x^{2}y^{2}\sqrt[4]{2xy^{2}}$$

$$=2x^{2}y^{2}\sqrt[4]{2xy^{2}}$$

Question 3: (5 points): Perform the following indicated operations, and simplify:

(a): $\left(\sqrt{h^2+1}+1\right)\left(\sqrt{h^2+1}-1\right)$

(b): $(x+y+z)(x-y-z)$

(c): $a^x(a^x-4)(a^x+1)-(a^x-1)^3$

Solution (a): $\left(\sqrt{h^2+1}+1\right)\left(\sqrt{h^2+1}-1\right)=\left(\sqrt{h^2+1}\right)^2-1^2=h^2+1-1=h^2$

(b): $(x+y+z)(x-y-z)=[x+(y+z)][x-(y+z)]$

$$=x^2-(y+z)^2$$
$$=x^2-y^2-2yz-z^2$$

(c): $a^x(a^x-4)(a^x+1)-(a^x-1)^3=(a^{2x}-4a^x)(a^x+1)-\left[(a^x)^3-3(a^x)^2+3(a^x)-1\right]$

$$=a^{3x}+a^{2x}-4a^{2x}-4a^x-a^{3x}+3a^{2x}-3a^x+1$$
$$=a^{3x}-3a^{2x}-4a^x-a^{3x}+3a^{2x}-3a^x+1$$
$$=-7a^x+1$$

Question 4: (5 points): Factor the following expressions

(a) y^3-1-y^2+y

(b) $2(a+b)^2-5(a+b)-3$

(c) $8r^3-64t^6$

(d) $\frac{1}{2}x^{-1/2}(3x+4)^{1/2}+\frac{3}{2}x^{1/2}(3x+4)^{-1/2}$

Solution:

(a): $y^3-1-y^2+y=y^3-1^3-y(y-1)$

$$=(y-1)(y^2+y+1)-y(y-1)$$
$$=(y-1)(y^2+y+1-y)=(y-1)(y^2+1)$$

(b): $2(a+b)^2-5(a+b)-3=[2(a+b)+1][(a+b)-3]$ $\quad 2(a+b) \quad 1$

$$=(2a+2b+1)(a+b-3)$$ $\quad (a+b) \quad -3$

(c): $8r^3-64t^6=8(r^3-8t^6)=8\left[r^3-(2t^2)^3\right]=8(r-2t^2)(r^2+r2t^2+(2t^2)^2)$

$$=8(r-2t^2)(r^2+2rt^2+4t^4)$$

OR: $8r^3-64t^6=(2r)^3-(4t^2)^3=(2r-4t^2)(4r^2+2r\cdot4t^2+16t^4)$

$$=2(r-2t^2)4(r^2+2rt^2+4t^4)=8(r-2t^2)(r^2+2rt^2+4t^4)$$

(d):

$$\frac{1}{2}x^{-1/2}(3x+4)^{1/2}+\frac{3}{2}x^{1/2}(3x+4)^{-1/2}=\frac{1}{2}x^{-1/2}(3x+4)^{-1/2}[(3x+4)+3x]$$

$$=\frac{1}{2}x^{-1/2}(3x+4)^{-1/2}(6x+4)=\frac{1}{2}x^{-1/2}(3x+4)^{-1/2}2(3x+2)=\boxed{x^{-1/2}(3x+4)^{-1/2}(3x+2)}$$