King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 001 Class Test II
Textbook Sections: 1.1 to 2.3
Term 171
Time Allowed: 90 Minutes

Time: 7:20 pm – 8:50 pm

Student's Name: …………… ………………………………………………
ID #:………………………. **Section:** ……. **Serial Number:** …….

Provide neat and complete solutions.
Show all necessary steps for full credit and write the answer in simplest form.

No Calculators, Cameras, or Mobiles are allowed during this exam.

Question	**Points**	Student's Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
8	5	
9	5	
10	5	
Total	**50**	____ 50
		____ 100

Q1. (5 points): If $M\left(5,\frac{3}{2}\right)$ is the midpoint of the line segment joining the points $P_1(x,8)$ and $P_2(3,y)$, then find the distance between P_1 and P_2.

(A) $\sqrt{305}$
(B) $\sqrt{215}$
(C) $\sqrt{185}$
(D) 5
(E) 15

Solution:

$$\left(\frac{3+x}{2},\frac{y+8}{2}\right)=\left(5,\frac{3}{2}\right) \Rightarrow \frac{3+x}{2}=5 \quad \text{and} \quad \frac{y+8}{2}=\frac{3}{2}$$

$$\Rightarrow 3+x=10 \quad \text{and} \quad 2y+16=6$$

$$\Rightarrow x=7 \quad \text{and} \quad y=-5$$

$$P_1(x,8)=(7,8) \;,\; P_2(3,y)=(3,-5)$$

$$d(P_1,P_2)=\sqrt{(7-3)^2+(8+5)^2}=\sqrt{16+169}=\sqrt{185}$$

Q2. (5 points): Find the **center** and **radius** of the equation of the circle then **sketch its graph**.
$2x^2-20x+2y^2+12y=-50$. Find the **domain** and the **range**.

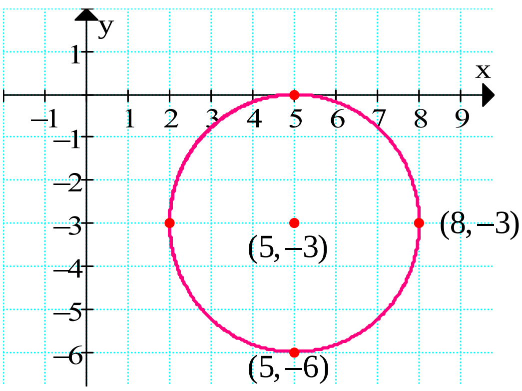
Solution:

$$2x^2-20x+2y^2+12y=-50$$
$$x^2-10x+y^2+6y=-25$$
$$x^2-10x+5^2+y^2+6y+3^2=-25+25+9$$
$$(x-5)^2+(y+3)^2=3^2$$

It is equation of a circle with center $(5,-3)$ and radius 3.

$Center=(5.-3)$

$Radius=3$



$Domain=[2,8]$, $Range=[-6,0]$

Q3. (5 points):

If the lines $kx + 4y = 24$ and $y = -\frac{3}{k+1}x + \frac{15}{4}$ are parallel, then find the values of k.

Solution: $kx + 4y = 24 \Rightarrow 4y = -kx + 24 \Rightarrow y = \frac{-k}{4}x + 6 \Rightarrow \boxed{m_1 = \frac{-k}{4}}$

$y = -\frac{3}{k+1}x + \frac{15}{4} \Rightarrow \boxed{m_2 = -\frac{3}{k+1}}$

Since the lines are **parallel** then the slopes are equal $m_1 = m_2$

$\frac{-k}{4} = -\frac{3}{k+1} \Rightarrow \frac{k}{4} = \frac{3}{k+1} \Rightarrow 12 = k^2 + k \Rightarrow k^2 + k - 12 = 0 \Rightarrow (k+4)(k-3) = 0$

$\boxed{k = -4}$, $\boxed{k = 3}$

Q4. (5 points)(1.4 Textbook exercise 68): The sum of the squares of two consecutive even integers is 1252. Find the integers.

Solution:

68. Let n be one even number. Then the next even number is $n+2$. Thus we get the equation $n^2 + (n+2)^2 = 1252 \Leftrightarrow n^2 + n^2 + 4n + 4 = 1252 \Leftrightarrow 0 = 2n^2 + 4n - 1248 = 2\left(n^2 + 2n - 624\right) = 2(n-24)(n+26)$. So $n = 24$ or $n = -26$. Thus the consecutive even integers are 24 and 26 or -26 and -24.

Q5. (5 points): If $z = \frac{\sqrt[3]{-8} + i^{103} - \sqrt{2}\left(\sqrt{-2}\right)}{i - (2i-1)(-2i-1)} = a + bi$, then $a = ?$, $b = ?$ and $\bar{z} = ?$

Solution:

$$z = \frac{\sqrt[3]{-8} + i^{103} - \sqrt{2}\left(\sqrt{-2}\right)}{i - (2i-1)(-2i-1)} = \frac{-2 + i\left(i^{102}\right) - 2i}{i-5} = \frac{-2 + i\left(i^2\right)^{51} - 2i}{i-5} = \frac{-2 - i - 2i}{i-5}$$

$$= \frac{-2-3i}{i-5} = \frac{-2-3i}{i-5} \cdot \frac{-i-5}{-i-5} = -\frac{2i + 10 - 3 + 15i}{1+25} = \frac{17i+7}{26} = \frac{7}{26} + \frac{17}{26}i$$

$$a = \frac{7}{26},\ b = \frac{17}{26} \text{ and } \bar{z} = \frac{7}{26} - \frac{17}{26}i$$

Q6. (5 points)(1.6 Textbook 45): Solve $x - \sqrt{x-1} = 3$

Solution: $x - \sqrt{x-1} = 3$

$x - 3 = \sqrt{x-1} \Rightarrow (x-3)^2 = \left(\sqrt{x-1}\right)^2 \Rightarrow x^2 - 6x + 9 = x - 1$

$\Rightarrow x^2 - 7x + 10 = 0 \Rightarrow (x-2)(x-5) = 0 \Rightarrow x = 2$ or $x = 5$

We must check (because we squared both sides of the equation):

Check $x = 2$:

$2 - \sqrt{2-1} \stackrel{?}{=} 3$

$2 - \sqrt{1} \stackrel{?}{=} 3$ false $\Rightarrow x = 2$ is rejected

Check $x = 5$

$5 - \sqrt{5-1} \stackrel{?}{=} 3$

$5 - \sqrt{4} \stackrel{?}{=} 3$ true $\Rightarrow \boxed{x = 5}$ $\quad ss = \{5\}$

Q7. (5 points)(1.7 Textbook Exercise 62): Solve the inequality $\dfrac{x}{x+1} > 3x$

Solution:

62. $\dfrac{x}{x+1} > 3x \Leftrightarrow \dfrac{x}{x+1} - 3x > 0 \Leftrightarrow \dfrac{x}{x+1} - \dfrac{3x(x+1)}{x+1} > 0 \Leftrightarrow \dfrac{-2x-3x^2}{x+1} > 0 \Leftrightarrow \dfrac{-x(2+3x)}{x+1} > 0$. The expression on the left of the inequality changes sign when $x = 0$, $x = -\frac{2}{3}$, and $x = -1$. Thus we must check the intervals in the following

Interval	$(-\infty, -1)$	$\left(-1, -\frac{2}{3}\right)$	$\left(-\frac{2}{3}, 0\right)$	$(0, \infty)$
Sign of $-x$	+	+	+	−
Sign of $2+3x$	−	−	+	+
Sign of $x+1$	−	+	+	+
Sign of $\dfrac{(2-x)(2+x)}{x}$	+	−	+	−

From the table, the solution set is $\left\{x \mid x < -1 \text{ or } -\frac{2}{3} < x < 0\right\}$. Interval: $(-\infty, -1) \cup \left(-\frac{2}{3}, 0\right)$.

$$SS = (-\infty, -1) \cup \left(-\frac{2}{3}, 0\right)$$

Q8. (5 points) (1.8 Textbook Exercise 48):

Find the solution set in interval notation of the inequality of $\dfrac{1}{|2x-3|} \le 5$

Solution:

48. $\dfrac{1}{|2x-3|} \le 5 \Leftrightarrow \frac{1}{5} \le |2x-3|$, since $|2x-3| > 0$, provided $2x - 3 \ne 0 \Leftrightarrow x \ne \frac{3}{2}$. Now for $x \ne \frac{3}{2}$, we have $\frac{1}{5} \le |2x-3|$ is equivalent to either $\frac{1}{5} \le 2x - 3 \Leftrightarrow \frac{16}{5} \le 2x \Leftrightarrow \frac{8}{5} \le x$; or $2x - 3 \le -\frac{1}{5} \Leftrightarrow 2x \le \frac{14}{5} \Leftrightarrow x \le \frac{7}{5}$.

Interval: $\left(-\infty, \frac{7}{5}\right] \cup \left[\frac{8}{5}, \infty\right)$.

$$SS = \left(-\infty, \frac{7}{5}\right] \cup \left[\frac{8}{5}, \infty\right)$$

Q9. (5 points): (2.1 Textbook Exercise 49): If $f(x) = 3 - 5x + 4x^2$, then find $\dfrac{f(a+h) - f(a)}{h} = ?$

(Show all necessary steps)

Solution:

$f(a) = 3 - 5a + 4a^2$;

$$f(a+h) = 3 - 5(a+h) + 4(a+h)^2 = 3 - 5a - 5h + 4\left(a^2 + 2ah + h^2\right)$$
$$= 3 - 5a - 5h + 4a^2 + 8ah + 4h^2;$$

$$\frac{f(a+h) - f(a)}{h} = \frac{\left(3 - 5a - 5h + 4a^2 + 8ah + 4h^2\right) - \left(3 - 5a + 4a^2\right)}{h}$$
$$= \frac{3 - 5a - 5h + 4a^2 + 8ah + 4h^2 - 3 + 5a - 4a^2}{h} = \frac{-5h + 8ah + 4h^2}{h}$$
$$= \frac{h(-5 + 8a + 4h)}{h} = -5 + 8a + 4h.$$

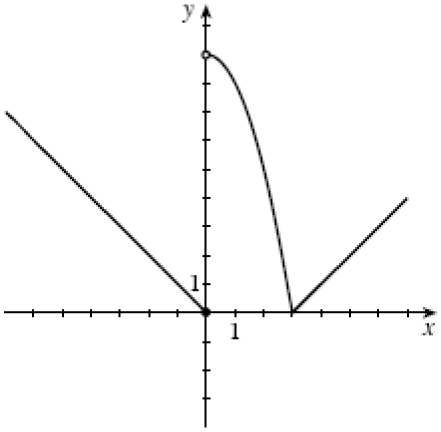
Q10. **(5 points) (2.2 Textbook Exercise 46):** Give the function $f(x)=\begin{cases} -x & \text{if } x \le 0 \\ 9-x^2 & \text{if } 0<x\le 3 \\ x-3 & \text{if } x>3 \end{cases}$

(a): Sketch the graph of the function.
(b): Find the intervals where the function decreasing.
(c): Find the interval where the function increasing.
(d): Find the range.

Solution:

$$f(x)=\begin{cases} -x & \text{if } x \le 0 \\ 9-x^2 & \text{if } 0<x\le 3 \\ x-3 & \text{if } x>3 \end{cases}$$

(a):



(b): $(-\infty,0]$ and $(0,3]$

(c): $(3,\infty)$

(d): $[0,\infty)$