Serial #: ________ ID ______________ NAME ____

Show all necessary steps for full marks.

Question1: (5 points) (7.1 Textbook Exercise 51): Find $\sin\theta$ if $\cos\theta = \frac{x}{x+1}$ (Simplify your answer)

Solution:

51. Find $\sin\theta$ if $\cos\theta = \frac{x}{x+1}$.

$\sin^2\theta + \cos^2\theta = 1$ and $\cos\theta = \frac{x}{x+1}$, so

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(\frac{x}{x+1}\right)^2$$

$$= 1 - \frac{x^2}{(x+1)^2} = \frac{(x+1)^2 - x^2}{(x+1)^2}$$

$$= \frac{x^2 + 2x + 1 - x^2}{(x+1)^2} = \frac{2x+1}{(x+1)^2}$$

Thus, $\sin\theta = \frac{\pm\sqrt{2x+1}}{x+1}$.

Question 2: (5 points): (7.2 Textbook Example 5): Verify the identity $\frac{\sec\alpha + \tan\alpha}{\sec\alpha - \tan\alpha} = \frac{1 + 2\sin\alpha + \sin^2\alpha}{\cos^2\alpha}$

Solution: $LHS = \frac{\sec\alpha + \tan\alpha}{\sec\alpha - \tan\alpha} = \frac{\frac{1}{\cos\alpha} + \frac{\sin\alpha}{\cos\alpha}}{\frac{1}{\cos\alpha} - \frac{\sin\alpha}{\cos\alpha}} = \frac{\cos\alpha}{\cos\alpha} \cdot \frac{\frac{1}{\cos\alpha} + \frac{\sin\alpha}{\cos\alpha}}{\frac{1}{\cos\alpha} - \frac{\sin\alpha}{\cos\alpha}} = \frac{1+\sin\alpha}{1-\sin\alpha} = \frac{1+\sin\alpha}{1+\sin\alpha} \cdot \frac{1+\sin\alpha}{1+\sin\alpha}$

$= \frac{(1+\sin\alpha)^2}{1-\sin^2\alpha} = \frac{1+2\sin\alpha+\sin^2\alpha}{\cos^2\alpha} = RHS$

Another Method:

▶ EXAMPLE 5 **VERIFYING AN IDENTITY (WORKING WITH BOTH SIDES)**

Verify that the following equation is an identity.

$$\frac{\sec\alpha + \tan\alpha}{\sec\alpha - \tan\alpha} = \frac{1 + 2\sin\alpha + \sin^2\alpha}{\cos^2\alpha}$$

$$\frac{\sec\alpha + \tan\alpha}{\sec\alpha - \tan\alpha} = \frac{(\sec\alpha + \tan\alpha)\cos\alpha}{(\sec\alpha - \tan\alpha)\cos\alpha}$$ Multiply by 1 in the form $\frac{\cos\alpha}{\cos\alpha}$.

$$= \frac{\sec\alpha\cos\alpha + \tan\alpha\cos\alpha}{\sec\alpha\cos\alpha - \tan\alpha\cos\alpha}$$ Distributive property

$$= \frac{1 + \tan\alpha\cos\alpha}{1 - \tan\alpha\cos\alpha}$$ $\sec\alpha\cos\alpha = 1$

$$= \frac{1 + \frac{\sin\alpha}{\cos\alpha}\cdot\cos\alpha}{1 - \frac{\sin\alpha}{\cos\alpha}\cdot\cos\alpha}$$ $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$

$$= \frac{1 + \sin\alpha}{1 - \sin\alpha}$$ Simplify.

On the right side of the original equation, begin by factoring.

$$\frac{1+2\sin\alpha+\sin^2\alpha}{\cos^2\alpha}=\frac{(1+\sin\alpha)^2}{\cos^2\alpha}\qquad x^2+2xy+y^2=(x+y)^2 \text{ (Section R.4)}$$

$$=\frac{(1+\sin\alpha)^2}{1-\sin^2\alpha}\qquad \cos^2\alpha=1-\sin^2\alpha$$

$$=\frac{(1+\sin\alpha)^2}{(1+\sin\alpha)(1-\sin\alpha)}\qquad \text{Factor the denominator; } x^2-y^2=(x+y)(x-y).$$

$$=\frac{1+\sin\alpha}{1-\sin\alpha}\qquad \text{Lowest terms}$$

We have shown that

$$\underbrace{\frac{\sec\alpha+\tan\alpha}{\sec\alpha-\tan\alpha}}_{\text{Left side of given equation}}=\underbrace{\frac{1+\sin\alpha}{1-\sin\alpha}}_{\text{Common third expression}}=\underbrace{\frac{1+2\sin\alpha+\sin^2\alpha}{\cos^2\alpha}}_{\text{Right side of given equation}},$$

verifying that the given equation is an identity.

Question 3: (5 points) (7.2 Textbook Exercise 77): Verify

$$\sec x-\cos x+\csc x-\sin x-\sin x\tan x=\cos x\cot x$$

Solution:

$$LHS=\sec x-\cos x+\csc x-\sin x-\sin x\tan x$$

$$=\frac{1}{\cos x}-\cos x+\frac{1}{\sin x}-\sin x-\sin x\frac{\sin x}{\cos x}$$

$$=\frac{1-\cos^2 x}{\cos x}+\frac{1-\sin^2 x}{\sin x}-\frac{\sin^2 x}{\cos x}$$

$$=\frac{\sin^2 x}{\cos x}+\frac{\cos^2 x}{\sin x}-\frac{\sin^2 x}{\cos x}$$

$$=\cos x\cdot\frac{\cos x}{\sin x}$$

$$=\cos x\cot x=RHS$$

Question 4: (5 points) (7.3 Textbook Exercise 92): Suppose that s and t are angles in standard position, with $\sin s=\frac{3}{5}$ and $\sin t=-\frac{12}{13}$, s is in quadrant I and t is in quadrant III.. Find the following:

(a): $\sin(s+t)=?$ **(b):** $\tan(s+t)=?$ **(c):** Find the quadrant of $s+t$

Solution:

92. $\sin s=\frac{3}{5}$ and $\sin t=-\frac{12}{13}$, s is in quadrant I and t is in quadrant III.

First find the values of $\cos s$, $\cos t$, $\tan s$, and $\tan t$. Because s is in quadrant I and t is in quadrant III, the values of $\cos s$, $\tan s$, and $\tan t$ will be positive, while $\cos t$ will be negative.

$$\cos s=\sqrt{1-\left(\frac{3}{5}\right)^2}=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$$

$$\cos t=-\sqrt{1-\left(-\frac{12}{13}\right)^2}=-\sqrt{1-\frac{144}{169}}$$

$$=-\sqrt{\frac{25}{169}}=-\frac{5}{13}$$

$$\tan s=\frac{\sin s}{\cos s}=\frac{\frac{3}{5}}{\frac{4}{5}}=\frac{3}{4}$$

$$\tan t=\frac{\sin t}{\cos t}=\frac{-\frac{12}{13}}{-\frac{5}{13}}=\frac{12}{5}$$

(a) $$\sin(s+t)=\sin s\cos t+\cos s\sin t$$

$$=\left(\frac{3}{5}\right)\left(-\frac{5}{13}\right)+\left(\frac{4}{5}\right)\left(-\frac{12}{13}\right)$$

$$=-\frac{15}{65}+\left(-\frac{48}{65}\right)=-\frac{63}{65}$$

(b) $$\tan(s+t)=\frac{\frac{3}{4}+\frac{12}{5}}{1-\left(\frac{3}{4}\right)\left(\frac{12}{5}\right)}=\frac{15+48}{20-36}$$

$$=\frac{63}{-16}=-\frac{63}{16}$$

(c) From parts (a) and (b), $\sin(s+t)<0$ and $\tan(s+t)<0$. The only quadrant in which the values of sine and tangent are both negative is quadrant IV, so $s+t$ is in quadrant IV.