

Show all necessary steps for full marks.

Question 1: (6 points):

(a): Sketch the graph of $y = -3\cos\frac{3x}{4}$, where $-2\pi \leq x \leq 4\pi$.

(b): Find the intervals where the function is decreasing.

Solution: (a): $y = -3\cos\frac{3x}{4} \Rightarrow 0 \leq \frac{3x}{4} \leq 2\pi \Rightarrow 0 \leq 3x \leq 8\pi \Rightarrow 0 \leq x \leq \frac{8}{3}$

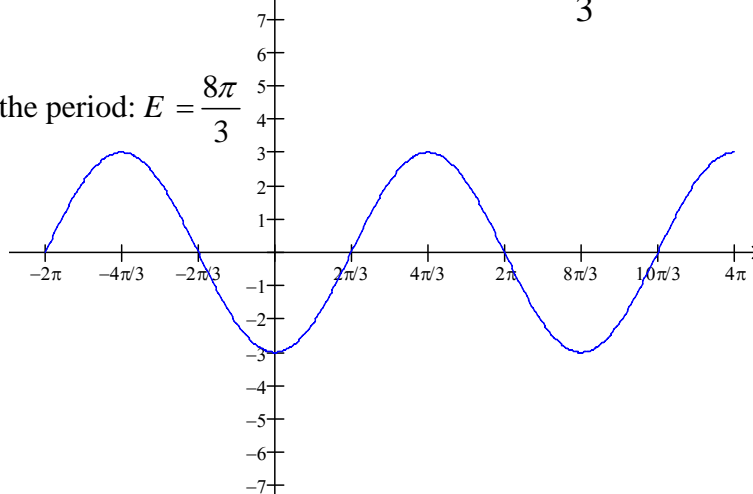
$P = \frac{2\pi}{b} = \frac{2\pi}{\frac{3}{4}} = 2\pi \cdot \frac{4}{3} = \frac{8\pi}{3}$. End Point of the period: $E = \frac{8\pi}{3}$

Beginning Point of the period: $B = 0$.

Middle Point of the period: $M = \frac{4\pi}{3}$

First Quart Point of the period: $\frac{1}{4}P = \frac{2\pi}{3}$.

Third Quarter Point of the period: $\frac{3}{4}P = 2\pi$



(b): The function is decreasing is decreasing on $\left[-\frac{4\pi}{3}, 0\right]$, $\left[\frac{4\pi}{3}, \frac{8\pi}{3}\right]$

Question 2: (7 points):

(a): Graph $y = 1 - \frac{1}{2}\csc\left(x - \frac{3\pi}{4}\right)$, $\frac{3\pi}{4} \leq x \leq \frac{11\pi}{4}$

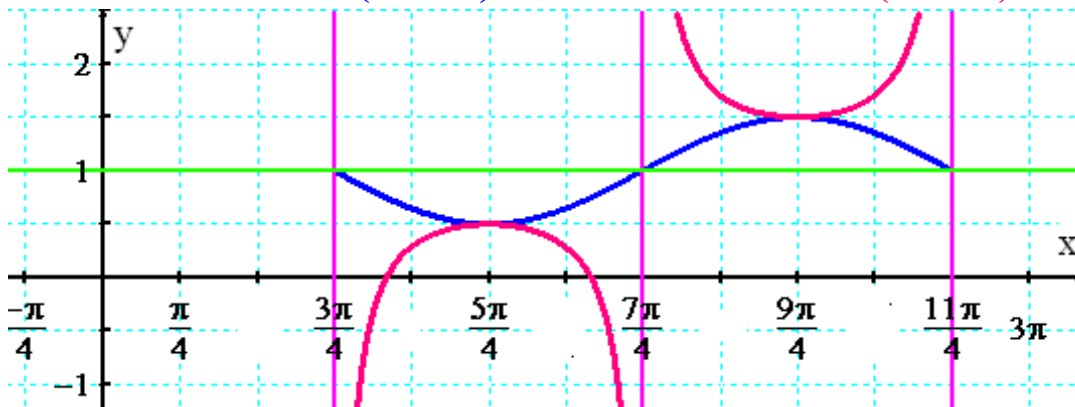
(b): Find the equations of vertical asymptote over the given interval.

(c): Find the intervals where the function is increasing.

(d): Find the intervals where the function is decreasing creasing.

Solution (a): $0 \leq x - \frac{3\pi}{4} \leq 2\pi \Rightarrow \frac{3\pi}{4} \leq x \leq 2\pi + \frac{3\pi}{4} \Rightarrow \frac{3\pi}{4} \leq x \leq \frac{11\pi}{4}$

First, graph $y = 1 - \frac{1}{2}\sin\left(x - \frac{3\pi}{4}\right)$ then graph $y = 1 - \frac{1}{2}\csc\left(x - \frac{3\pi}{4}\right)$



(b): $x = \frac{3\pi}{4}$, $x = \frac{7\pi}{4}$, $x = \frac{11\pi}{4}$

(c): $\left(\frac{3\pi}{4}, \frac{5\pi}{4}\right]$, $\left[\frac{9\pi}{4}, \frac{11\pi}{4}\right)$

(d): $\left[\frac{5\pi}{4}, \frac{7\pi}{4}\right) \cup \left(\frac{7\pi}{4}, \frac{9\pi}{4}\right]$

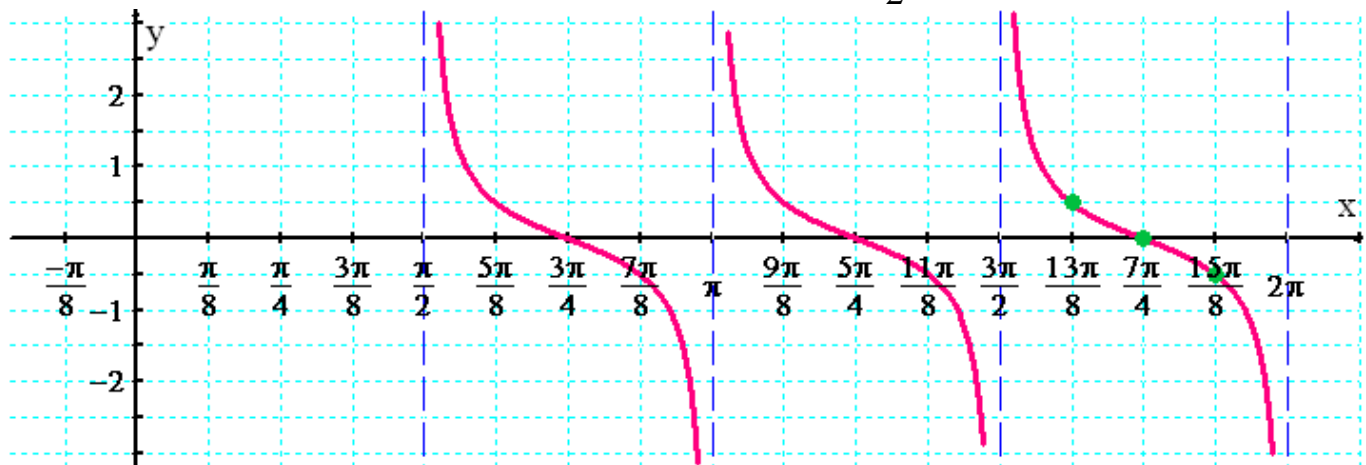
Question 3: (7 points): Given $y = \frac{1}{2} \cot(2x - 3\pi)$ where $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$

(a): Graph $y = \frac{1}{2} \cot(2x - 3\pi)$ over the given interval.

(b): Find the intervals where the graph of the function the given interval is above the x-axis.

(c): Find the equations of vertical asymptote over the given interval.

Solution (a): $0 < 2x - 3\pi < \pi \Rightarrow 3\pi < 2x < 4\pi \Rightarrow \frac{3\pi}{2} < x < 2\pi$



(b): The graph is above the x-axis over $\left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$, $\left(\pi, \frac{5\pi}{4}\right)$.

(c): The equations of vertical asymptote are: $x = \frac{\pi}{2}$, $x = \pi$, $x = \frac{3\pi}{2}$, $x = 2\pi$