Serial #: **ID:** **Name:**

Question #	1	2	3	4	5	6	7	8	9	
Points	5	5	8	5	6	6	5	5	5	
Student's Score										**Total:** ___/50 ___/100

Q1. (5 points): Given $f(x)=\frac{5x+1}{x-2}$, then $f^{-1}\left(\frac{3}{2}\right)$ is equal to

Solution: $f^{-1}\left(\frac{3}{2}\right)=a \Rightarrow f(a)=\frac{3}{2} \Rightarrow \frac{5a+1}{a-2}=\frac{3}{2} \Rightarrow 3a-6=10a+2 \Rightarrow -8=7a \Rightarrow \boxed{a=-\frac{8}{7}}$

$\boxed{f^{-1}\left(\frac{3}{2}\right)=-\frac{8}{7}}$

Q2. (5 points): If $f(x)=\left(\frac{2}{3}\right)^{2-3x}$ is written as $f(x)=ka^x$, then $8a-27k=?$

Solution: $f(x)=\left(\frac{2}{3}\right)^{2-3x}=\left(\frac{2}{3}\right)^2\cdot\left(\frac{2}{3}\right)^{-3x}=\frac{4}{9}\left(\frac{3}{2}\right)^{3x}=\frac{4}{9}\cdot\left(\frac{3^3}{2^3}\right)^x=\frac{4}{9}\cdot\left(\frac{27}{8}\right)^x=ka^x$

$\boxed{k=\frac{4}{9}}$ and $\boxed{a=\frac{27}{8}}$

$8a-27k=8\left(\frac{27}{8}\right)-27\left(\frac{4}{9}\right)=27-12=15$

Q3. (8 points): For functions $f(x)=-\log_{1/2}(x+2)$

(a): find the asymptote

(b): find , if any, the $x-$intercept and the $y-$intercept

(c): find the domain

(d): sketch the graph of $f(x)$

(e): find the inverse function $f^{-1}(x)$

Solution: **(a):** Vertical Asymptote: $x+2=0 \Rightarrow x=-2$

(b): To find the x-intercept, put $y=0$:

$0=-\log_{1/2}(x+2)$

$0=\log_{1/2}(x+2)$

$\left(\frac{1}{2}\right)^0=x+2$

$1=x+2$

$x=-1$

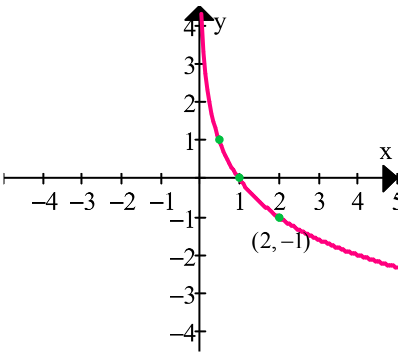
x-intercept $=(-1,0)$

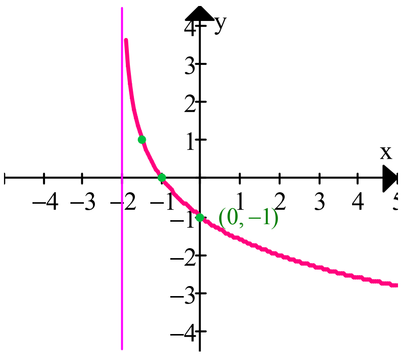
To find the y-intercept, put $x=0$:

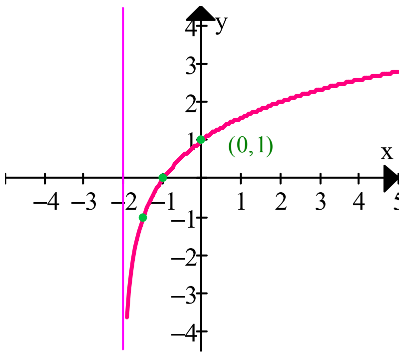
$y=-\log_{1/2}(0+2)=-\log_{1/2}2=\log_{1/2}2^{-1}=1 \Rightarrow y-\text{intercept}=(0,1)$

(c): $x+2>0 \Rightarrow x>-2 \Rightarrow D_f=(-2,\infty)$

(d): $f_1(x)=\log_{\frac{1}{2}} x$ $\qquad f_2(x)=\log_{\frac{1}{2}}(x+2)$ $\qquad f(x)=-\log_{1/2}(x+2)$







(e):

$$y=-\log_{1/2}(x+2)$$
$$-y=\log_{1/2}(x+2)$$
$$\left(\frac{1}{2}\right)^{-y}=x+2 \quad \text{(By definition of logarithm)}$$
$$\left(\frac{1}{2}\right)^{-x}=y+2 \quad \text{(Interchange variables)}$$
$$y=\left(\frac{1}{2}\right)^{-x}-2 \quad \Rightarrow \quad y=2^x-2 \quad \Rightarrow \quad f^{-1}(x)=2^x-2$$

Q4. (5 points): $A=2^{\log_8 125}$ and $B=\left(\log_{\sqrt{2}} 9\right)\cdot\left(\log_3 \sqrt{8}\right)$, find $B+A=?$ (Show your work)

Solution: $A=2^{\log_8 125}=2^{\frac{\log_2 125}{\log_2 2^3}}=2^{\frac{1}{3}\log_2 125}=2^{\log_2\left(5^3\right)^{1/3}}=5$

$$\log_{\sqrt{2}} 9=\frac{\log_2 9}{\log_2 \sqrt{2}}=\frac{2\log_2 3}{1/2}=4\log_2 3$$

$$\log_3 \sqrt{8}=\frac{\log_2 \sqrt{8}}{\log_2 3}=\frac{\log_2\left(2^3\right)^{1/2}}{\log_2 3}=\frac{3/2}{\log_2 3}$$

$$B=\left(\log_{\sqrt{2}} 9\right)\cdot\left(\log_3 \sqrt{8}\right)=4\log_2 3\cdot\frac{3/2}{\log_2 3}=6$$

$$B+A=6+5=11$$

Q5. (5 points): Solve $\log(x+6)-\log(x+2)=\log x$

Solution: Textbook 4.5 Example 6, Page 442:

$\log(x+6)-\log(x+2)=\log x$

$\log\frac{x+6}{x+2}=\log x$ — Quotient property

$\frac{x+6}{x+2}=x$ — Property of logarithms

$x+6=x(x+2)$ — Multiply by $x+2$. (Section 1.6)

$x+6=x^2+2x$ — Distributive property

$x^2+x-6=0$ — Standard form (Section 1.4)

$(x+3)(x-2)=0$ — Factor. (Section R.4)

$x=-3$ or $x=2$ — Zero-factor property (Section 1.4)

The proposed negative solution $(x=-3)$ is not in the domain of $\log x$ in the original equation, so the only valid solution is the positive number 2, giving the solution set $\{2\}$.

Q6. (6 points): Find the angles of least positive measure coterminal with each angle.

(a) $908°$ (b) $-75°$ (c) $-800°$

Solution: Textbook 5.1 Example 5:

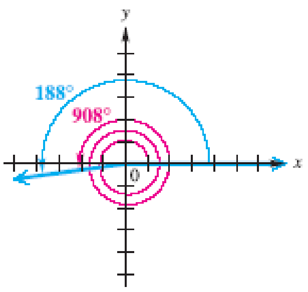
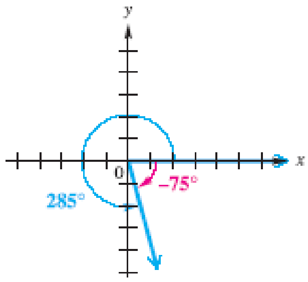


Figure 11



▶ EXAMPLE 5 FINDING MEASURES OF COTERMINAL ANGLES

Find the angles of least possible positive measure coterminal with each angle.

(a) $908°$ **(b)** $-75°$ **(c)** $-800°$

Solution

(a) Add or subtract $360°$ as many times as needed to obtain an angle with measure greater than $0°$ but less than $360°$. Since

$$908° - 2 \cdot 360° = 188°,$$

an angle of $188°$ is coterminal with an angle of $908°$. See Figure 11.

(b) Use a rotation of $360° + (-75°) = 285°$. See Figure 12.

(c) The least integer multiple of $360°$ greater than $800°$ is

$$360° \cdot 3 = 1080°.$$

Add $1080°$ to $-800°$ to obtain

$$1080° + (-800°) = 280°.$$

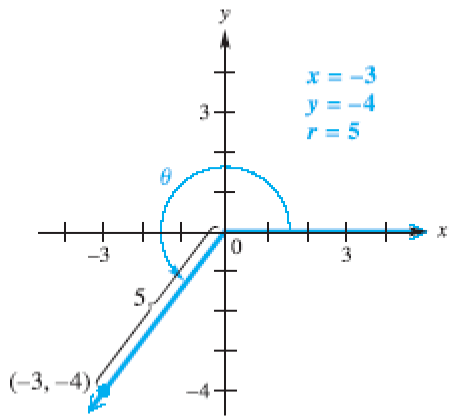
Q7. (5 points): Find $\sin\theta$ and $\cos\theta$. Given $\tan\theta = \dfrac{4}{3}$ and θ is in quadrant III.

Solution: Textbook 5.2 Example 11, page 495

Find $\sin\theta$ and $\cos\theta$, given that $\tan\theta = \frac{4}{3}$ and θ is in quadrant III.

Solution Since θ is in quadrant III, $\sin\theta$ and $\cos\theta$ will both be negative. It is tempting to say that since $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\tan\theta = \frac{4}{3}$, then $\sin\theta = -4$ and $\cos\theta = -3$. This is *incorrect*, however, since both $\sin\theta$ and $\cos\theta$ must be in the interval $[-1, 1]$.

We use the Pythagorean identity $\tan^2\theta + 1 = \sec^2\theta$ to find $\sec\theta$, and then the reciprocal identity $\cos\theta = \frac{1}{\sec\theta}$ to find $\cos\theta$.



$$\tan^2\theta + 1 = \sec^2\theta$$

$$\left(\frac{4}{3}\right)^2 + 1 = \sec^2\theta \qquad \tan\theta = \frac{4}{3}$$

$$\frac{16}{9} + 1 = \sec^2\theta$$

$$\frac{25}{9} = \sec^2\theta$$

Be careful to choose the correct sign here.

$$-\frac{5}{3} = \sec\theta$$ Choose the negative square root since $\sec\theta$ is negative when θ is in quadrant III.

$$-\frac{3}{5} = \cos\theta$$ Secant and cosine are reciprocals.

Since $\sin^2\theta = 1 - \cos^2\theta$,

$$\sin^2\theta = 1 - \left(-\frac{3}{5}\right)^2 \qquad \cos\theta = -\frac{3}{5}$$

$$\sin^2\theta = 1 - \frac{9}{25}$$

$$\sin^2\theta = \frac{16}{25}$$

Again, be careful.

$$\sin\theta = -\frac{4}{5}.$$ Choose the negative square root.

Q8. (5 points): Find all values of angle θ that has the given function value, if θ is in the interval $[0°,360°)$.

(a): $\cos\theta = -\frac{\sqrt{2}}{2}$ **(b):** $\sin\theta = \frac{\sqrt{3}}{2}$ **(c):** $\tan\theta = -1$ **(d):** $\sec^2\theta = 2$

Solution:

(a): $\cos\theta = -\frac{\sqrt{2}}{2} \Rightarrow \theta = 135°,\ 225°$

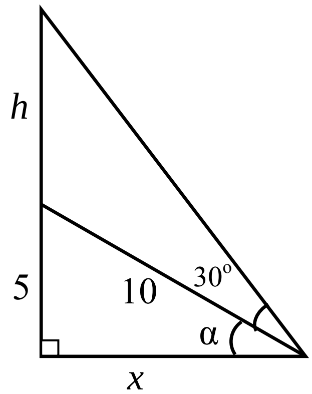
(b): $\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 60°,\ 120°$

(c): $\tan\theta = -1 \Rightarrow \theta = 135°,\ 315°$

(d): $\sec^2\theta = 2 \Rightarrow \sec\theta = \pm\sqrt{2} \Rightarrow \theta = 45°,\ 135°, 225°,\ 315°$

Q9. (5 points): Find the distance h in the following figure is

A) 10
B) $5\sqrt{3}$
C) 5
D) 15
E) $\frac{10}{3}$



Solution:

$\sin\alpha = \frac{5}{10} = \frac{1}{2} \Rightarrow \boxed{\alpha = 30°}$

$\cos\alpha = \frac{x}{10}$

$x = (10)\cos 30° = 10\left(\frac{\sqrt{3}}{2}\right) = 5\sqrt{3} \Rightarrow \boxed{x = 5\sqrt{3}}$

$\tan 60° = \frac{5+h}{x}$

$\sqrt{3} = \frac{5+h}{5\sqrt{3}}$

$5 + h = 15 \Rightarrow \boxed{h = 10}$