

Serial #: _____ ID _____ NAME _____

Show all necessary steps for full marks.

Question 1: (4 points) (R.3 Textbook Exercise 94):

If $\frac{k^4 - 4k^2 + 2k + 5}{k^2 + 1} = Q(x) + \frac{R(x)}{k^2 + 1}$, find $Q(x) = ?$ and $R(x) = ?$

Solution:

$$\begin{array}{r}
 k^2 - 5 \\
 k^2 + 1 \overline{) k^4 - 4k^2 + 2k + 5} \\
 \underline{k^4 + k^2} \\
 -5k^2 + 2k + 5 \\
 \underline{-5k^2 - 5} \\
 2k + 10
 \end{array}$$

$Q(x) = k^2 - 5$ and $R(x) = 2k + 10$

$$\frac{k^4 - 4k^2 + 2k + 5}{k^2 + 1} = k^2 - 5 + \frac{2k + 10}{k^2 + 1}$$

Question 2: (6 points): Given the polynomial $(3x^2 - 2)^2 - (2x^2 - x - 3)(2x^2 - x + 3)$

(a): Write the polynomial in standard form.

(b): Write down the following :

The leading coefficient	The constant Term	The coefficient of x^2	Degree

Solution (a):

$$\begin{aligned}
 f(x) &= (3x^2 - 2)^2 - (2x^2 - x - 3)(2x^2 - x + 3) \\
 &= 9x^4 - 12x^2 + 4 - [(2x^2 - x) - 3][(2x^2 - x) + 3] \\
 &= 9x^4 - 12x^2 + 4 - [(2x^2 - x)^2 - 3^2] \\
 &= 9x^4 - 12x^2 + 4 - [4x^4 - 4x^3 + x^2 - 9] \\
 &= 5x^4 + 4x^3 - 13x^2 + 13
 \end{aligned}$$

(b):

The leading coefficient	The constant Term	The coefficient of x^2	Degree
5	13	-13	4

Question 3: (5 points) (R.4 Textbook Exercise 97): Factor $64 + (3x + 2)^3$

Solution:

$$\begin{aligned}
 97. \quad & 64 + (3x + 2)^3 \\
 &= 4^3 + (3x + 2)^3 \\
 &= [4 + (3x + 2)] \left[4^2 - (4)(3x + 2) + (3x + 2)^2 \right] \\
 &= [4 + (3x + 2)] \left[\begin{array}{l} 4^2 - (4)(3x + 2) \\ + 9x^2 + 12x + 4 \end{array} \right] \\
 &= (4 + 3x + 2)(16 - 12x - 8 + 9x^2 + 12x + 4) \\
 &= (3x + 6)(9x^2 + 12) \\
 &= 3(x + 2)(3)(3x^2 + 4) = 9(x + 2)(3x^2 + 4)
 \end{aligned}$$

Question 4: (5 points) (R.4 Textbook Exercise 65): Factor $27y^9 + 125z^6$

Solution:

$$\begin{aligned}
 65. \quad & 27y^9 + 125z^6 \\
 &= (3y^3)^3 + (5z^2)^3 \\
 &= (3y^3 + 5z^2) \left[(3y^3)^2 - (3y^3)(5z^2) + (5z^2)^2 \right] \\
 &= (3y^3 + 5z^2)(9y^6 - 15y^3z^2 + 25z^4)
 \end{aligned}$$