King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 001 Class Test II
Textbook Sections: 1.1 to 2.5
Term 161
Time Allowed: 90 Minutes

Time: 5:30 pm – 7:00 pm

Student's Name: …………… ………………………………………………
ID #:……………………….. **Section:** ……. **Serial Number:** …….

Provide neat and complete solutions.
Show all necessary steps for full credit and write the answer in simplest form.

No Calculators, Cameras, or Mobiles are allowed during this exam.

Question	**Points**	Student's Score
1	4	
2	4	
3	4	
4	4	
5	4	
6	4	
7	4	
8	4	
9	6	
10	4	
11	4	
12	4	
Total	**50**	____ / 50
		____ / 100

Q1. **(4 points) (1.1 Textbook Exercise 58):** If $-x = (5x+3)(3k+1)$, then $x = ?$

Solution:

58.
$$-x = (5x+3)(3k+1)$$
$$-x = 15xk + 5x + 9k + 3$$
$$-6x - 15xk = 9k + 3$$
$$(-6-15k)x = 9k + 3$$
$$x = \frac{9k+3}{-6-15k} = \frac{3(3k+1)}{-3(2+5k)}$$
$$= -\frac{3k+1}{5k+2}$$

Q2. **(4 points):** Determine whether each of following equations is an **identity**, a **conditional** equation, or a **contradiction**. Show your work.

a) $(x-2)^2 = x^2 - 4$ Answer: [ ]

b) $(2x-3)^2 - 3x = (4x-3)(x-3)$ Answer: [ ]

c) $4(x+7) = 2(x+12) + 2(x+1)$ Answer: [ ]

d) $\frac{3x}{x-2} = \frac{6}{x-2}$ Answer: [ ]

Solution: **a)** $(x-2)^2 = x^2 - 4$ [conditional equation] Because:
$$(x-2)^2 = x^2 - 4$$
$$x^2 - 4x + 4 = x^2 - 4$$
$$-4x = -8$$
$$x = 2$$

b) $(2x-3)^2 - 3x = (4x-3)(x-3)$ [identity] Because:
$$(2x-3)^2 - 3x = (4x-3)(x-3)$$
$$4x^2 - 12x + 9 - 3x = 4x^2 - 15x + 9$$
$$4x^2 - 15x + 9 = 4x^2 - 15x + 9$$
$$0 = 0$$

c) $4(x+7) = 2(x+12) + 2(x+1)$ [contradiction] Because:

35.
$$4(x+7) = 2(x+12) + 2(x+1)$$
$$4x + 28 = 2x + 24 + 2x + 2$$
$$4x + 28 = 4x + 26$$
$$28 = 26$$
contradiction; $\varnothing$

d) $\frac{3x}{x-2} = \frac{6}{x-2}$ [contradiction] Because:

$\frac{3x}{x-2}(x-2) = \frac{6}{x-2}(x-2) \Rightarrow 3x = 6 \Rightarrow x = 2$ is rejected $\Rightarrow SS = \varnothing$

Q3. (4 points): If the equation $4[3(x-5)+a]=(a+5)x-32$ is an identity, then find the value of a.

Solution:

$$4[3(x-5)+a]=(a+5)x-32$$

$$4[3x-15+a]=(a+5)x-32$$

$$12x-60+4a=(a+5)x-32$$

$$12x+4a=(a+5)x-32+60$$

$$12x+4a=(a+5)x+28 \quad \Rightarrow \quad 12=a+5 \text{ and } 4a=28 \quad \Rightarrow \quad \boxed{a=7}$$

Q4. (4 points) (1.2 Recitation Q#5): If the length of each side of the original square is decreased by 4 inches, the perimeter of the new square is 14 inches more than half the perimeter of the original square. What are the dimensions of the original square?

Solution: $l=$ Length of the original rectangle in inches

Original square

Side is decreased 4

$$P_{new}=14+\frac{1}{2}P_{original}$$

$$4(l-4)=14+\frac{1}{2}(4l)$$

$$4l-16=14+2l$$

$$2l=30 \quad \Rightarrow \quad l=15 \text{ inches}$$

The original square is 15 by 15 inches.

Q5. (4 points) (1.4 Textbook Exercise 79b): Solve $4x^2-2xy+3y^2=2$ for $y=?$

Solution:

$$3y^2-(2x)y+(4x^2-2)=0$$

$$a=3,\ b=-2x, \text{ and } c=4x^2-2$$

$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-2x)\pm\sqrt{(-2x)^2-4(3)(4x^2-2)}}{2(3)}$$

$$=\frac{2x\pm\sqrt{4x^2-12(4x^2-2)}}{6}$$

$$=\frac{2x\pm\sqrt{4x^2-48x^2+24}}{6}$$

$$=\frac{2x\pm\sqrt{24-44x^2}}{6}=\frac{2x\pm\sqrt{4(6-11x^2)}}{6}$$

$$=\frac{2x\pm2\sqrt{6-11x^2}}{6}=\frac{x\pm\sqrt{6-11x^2}}{3}$$

Q6. (4 points) (1.6 Textbook Exercise 51): Find the solution set of $8(x-4)^4 - 10(x-4)^2 = -3$

Solution:

86. $8(x-4)^4 - 10(x-4)^2 = -3$

$8(x-4)^4 - 10(x-4)^2 + 3 = 0$

Let $u = (x-4)^2$; then $u^2 = (x-4)^4$.

$8u^2 - 10u + 3 = 0 \Rightarrow (2u-1)(4u-3) = 0$

$u = \frac{1}{2}$ or $u = \frac{3}{4}$

$(x-4)^2 = \frac{1}{2} \Rightarrow x - 4 = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$ or

$x = 4 \pm \frac{\sqrt{2}}{2} = \frac{8}{2} \pm \frac{\sqrt{2}}{2} = \frac{8 \pm \sqrt{2}}{2}$

$(x-4)^2 = \frac{3}{4} \Rightarrow x - 4 = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

$x = 4 \pm \frac{\sqrt{3}}{2} = \frac{8}{2} \pm \frac{\sqrt{3}}{2} = \frac{8 \pm \sqrt{3}}{2}$

Solution set: $\left\{\frac{8 \pm \sqrt{2}}{2}, \frac{8 \pm \sqrt{3}}{2}\right\}$

Q7. (4 points) (1.7 Recitation Q#5): Solve the following nonlinear inequality and express the solution using interval notation. $\frac{x}{2} \geq \frac{5}{x+1} + 4$

Solution: $\frac{x}{2} - \frac{5}{x+1} - 4 \geq 0$

$$x - \frac{10}{x+1} - 8 \geq 0$$

$$\frac{x^2 + x - 10 - 8x - 8}{x+1} \geq 0$$

$$\frac{x^2 - 7x - 18}{x+1} \geq 0$$

$$\frac{(x-9)(x+2)}{x+1} \geq 0$$

$CRV: -2, -1, 9$

$\frac{(x-9)(x+2)}{(x+1)} \geq 0$ − − − − − 0 + + + + + 0 − − − − − 0 + + + +

x $-\infty$ -2 -1 9 ∞

Answer: $SS = [-2, -1) \cup [9, \infty)$

Q8. (4 points) (1.8 Textbook Exercise 76): Solve $\left|\frac{x^2 + 2}{x}\right| - \frac{11}{3} = 0$

Solution:

76. $\left|\dfrac{x^2+2}{x}\right| - \dfrac{11}{3} = 0$

$\left|\dfrac{x^2+2}{x}\right| - \dfrac{11}{3} = 0 \Rightarrow \left|\dfrac{x^2+2}{x}\right| = \dfrac{11}{3}$

$\dfrac{x^2+2}{x} = \dfrac{11}{3} \Rightarrow 3x\left(\dfrac{x^2+2}{x}\right) = 3x\left(\dfrac{11}{3}\right)$

$3\left(x^2+2\right) = 11x \Rightarrow 3x^2+6 = 11x \Rightarrow$

$3x^2 - 11x + 6 = 0 \Rightarrow (3x-2)(x-3) = 0$

$3x - 2 = 0 \Rightarrow x = \frac{2}{3}$ or $x - 3 = 0 \Rightarrow x = 3$

$\dfrac{x^2+2}{x} = -\dfrac{11}{3}$

$3x\left(\dfrac{x^2+2}{x}\right) = 3x\left(-\dfrac{11}{3}\right)$

$3\left(x^2+2\right) = -11x \Rightarrow 3x^2+6 = -11x \Rightarrow$

$3x^2 + 11x + 6 = 0 \Rightarrow (3x+2)(x+3) = 0$

$3x + 2 = 0 \Rightarrow x = -\frac{2}{3}$ or $x + 3 = 0 \Rightarrow x = -3$

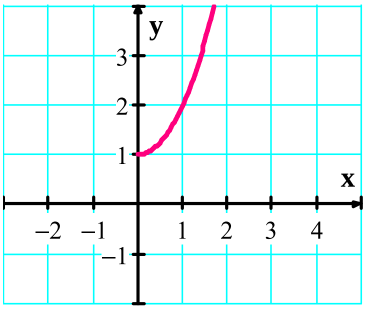
Solution set: $\left\{-3, -\frac{2}{3}, \frac{2}{3}, 3\right\}$

Q9. (6 points) (2.1 Recitation Q#3): Plot the following equations and find the domain and range:

(a): $x = \sqrt{y-1}$ **(b):** $y = -|x+4|$ **(c):** $y = x^2 + 1$

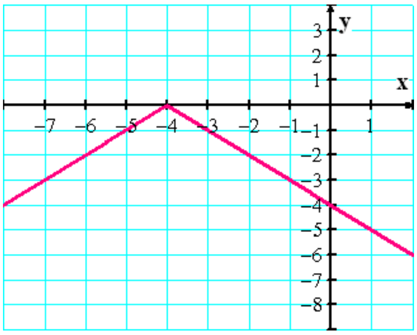
Solution (i):

(a):



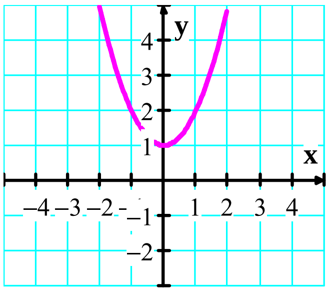
$D = [0, \infty)$ $R = [1, \infty)$

(b):



$D = (-\infty, \infty)$ $R = (-\infty, 0]$

(c):



$D = (-\infty, \infty)$ $R = [1, \infty)$

Q10. (4 points) (2.2 Textbook Exercise 23): Sketch the graph of the circle $4x^2 + 4y^2 + 4x - 16y - 19 = 0$.

Solution: Dividing the equation by 4:

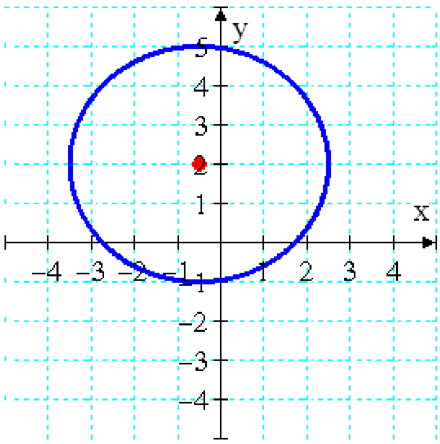
$$x^2 + y^2 + x - 4y - \frac{19}{4} = 0$$

$$x^2 + x + \left(\frac{1}{2}\right)^2 + y^2 - 4y + 2^2 = \frac{19}{4} + \frac{1}{4} + 4$$

$$\left(x + \frac{1}{2}\right)^2 + (y-2)^2 = \frac{20}{4} + 4$$

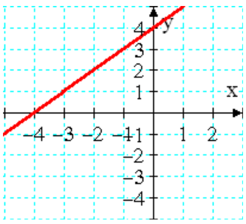
$$\left(x + \frac{1}{2}\right)^2 + (y-2)^2 = 9$$

The center of the circle is $\left(-\frac{1}{2}, 2\right)$ and the radius is 3.



Q11. (4 points) (2.3 Textbook Exercise): Decide whether each relation **defines a function** and give the **domain** and **range**.

(a): $y = x + 4$ **(b):** $y = \sqrt{2x - 1}$ **(c):** $y^2 = x$ **(d):** $y \le x - 1$ **(e):** $y = \dfrac{5}{x-1}$



Solution

(a) In the defining equation (or rule), $y = x + 4$, y is always found by adding 4 to x. Thus, each value of x corresponds to just one value of y and the relation defines a function; x can be any real number, so the domain is $\{x \mid x \text{ is a real number}\}$ or $(-\infty, \infty)$. Since y is always 4 more than x, y also may be any real number, and so the range is $(-\infty, \infty)$.

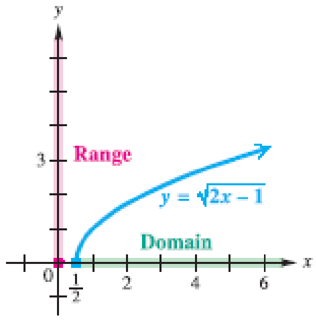


Figure 22

(b) For any choice of x in the domain of $y = \sqrt{2x - 1}$, there is exactly one corresponding value for y (the radical is a nonnegative number), so this equation defines a function. Refer to the agreement on domain stated previously. Since the equation involves a square root, the quantity under the radical sign cannot be negative. Thus,

$$2x - 1 \ge 0 \quad \text{Solve the inequality. (Section 1.7)}$$

$$2x \ge 1 \quad \text{Add 1.}$$

$$x \ge \frac{1}{2}, \quad \text{Divide by 2.}$$

and the domain of the function is $\left[\frac{1}{2}, \infty\right)$. Because the radical is a nonnegative number, as x takes values greater than or equal to $\frac{1}{2}$, the range is $y \ge 0$, that is, $[0, \infty)$. See Figure 22.

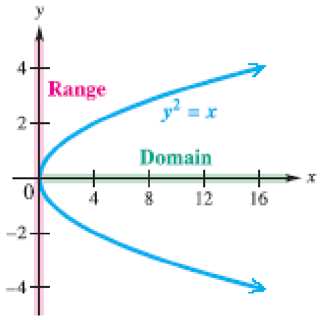
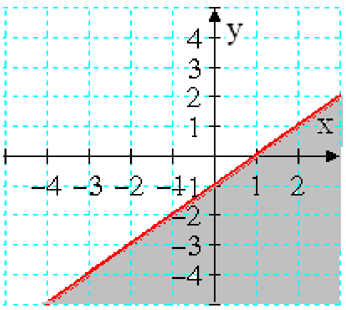


Figure 23

(c) The ordered pairs $(16, 4)$ and $(16, -4)$ both satisfy the equation $y^2 = x$. Since one value of x, 16, corresponds to two values of y, 4 and -4, this equation does not define a function. Because x is equal to the square of y, the values of x must always be nonnegative. The domain of the relation is $[0, \infty)$. Any real number can be squared, so the range of the relation is $(-\infty, \infty)$. See Figure 23.



(d) By definition, y is a function of x if every value of x leads to exactly one value of y. Substituting a particular value of x, say 1, into $y \le x - 1$, corresponds to many values of y. The ordered pairs $(1, 0)$, $(1, -1)$, $(1, -2)$, $(1, -3)$, and so on, all satisfy the inequality. For this reason, *an inequality rarely defines a function.* Any number can be used for x or for y, so the domain and the range of this relation are both the set of real numbers, $(-\infty, \infty)$.

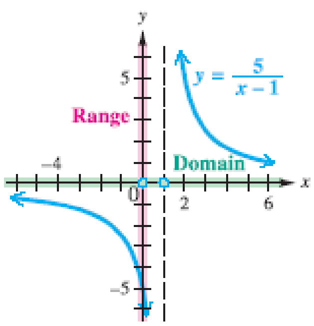


Figure 24

(e) Given any value of x in the domain of

$$y = \frac{5}{x - 1},$$

we find y by subtracting 1, then dividing the result into 5. This process produces exactly one value of y for each value in the domain, so this equation defines a function. The domain includes all real numbers except those that make the denominator 0. We find these numbers by setting the denominator equal to 0 and solving for x.

$$x - 1 = 0$$

$$x = 1 \quad \text{Add 1. (Section 1.1)}$$

Thus, the domain includes all real numbers except 1, written as the interval $(-\infty, 1) \cup (1, \infty)$. Values of y can be positive or negative, but never 0, because a fraction cannot equal 0 unless its numerator is 0. Therefore, the range is the interval $(-\infty, 0) \cup (0, \infty)$, as shown in Figure 24.

Q12. **(4 points):** Find the value of k so that the line through the points $(4,-1)$ and $(k,2)$ is perpendicular to the line $2y-5x=1$.

Solution:

$$m_1 = \frac{2-(-1)}{k-4} = \frac{3}{k-4}$$

$$2y-5x=1 \Rightarrow m_2 = \frac{5}{2}$$

$$m_1 m_2 = -1$$

$$\frac{3}{k-4}\cdot\frac{5}{2} = -1$$

$$15 = (-1)2(k-4)$$

$$15 = -2k+8$$

$$2k = -7$$

$$\Rightarrow k = -\frac{7}{2}$$