Serial #: ________ ID ______________ NAME ____

Show all necessary steps for full marks.

Question 1: (7 points): Solve the system of equations $\begin{cases} y = x^2 - 4x + 2 \\ x^2 - xy - 2y^2 = 0 \end{cases}$ for rational number ordered pair.

Solution: $\begin{cases} y = x^2 - 4x + 2 & \text{(I)} \\ x^2 - xy - 2y^2 = 0 & \text{(II)} \end{cases}$

Factor the second equation. (II) $\Rightarrow$

x y

x $-2y$

$\Rightarrow (x - 2y)(x + y) = 0$

$\Rightarrow x - 2y = 0$ or $x + y = 0$

$\Rightarrow y = \frac{1}{2}x$ or $y = -x$

If $y = \frac{1}{2}x \overset{\text{(I)}}{\Rightarrow} \frac{1}{2}x = x^2 - 4x + 2 \Rightarrow x = 2x^2 - 8x + 4 \Rightarrow 2x^2 - 9x + 4 = 0$

2x -1

x -4

$\Rightarrow (2x - 1)(x - 4) = 0 \Rightarrow x = \frac{1}{2}, x = 4 \overset{y = \frac{1}{2}x}{\Rightarrow} \left(\frac{1}{2}, \frac{1}{4}\right), (4, 2) \in SS$

If $y = -x \overset{\text{(I)}}{\Rightarrow} -x = x^2 - 4x + 2 \Rightarrow x^2 - 3x + 2 = 0 \Rightarrow (x - 1)(x - 2) = 0$

$\Rightarrow x = 1$ or $x = 2 \Rightarrow \overset{y = -x}{\Rightarrow} (1, -1), (2, -2) \in SS$

$SS = \left\{\left(\frac{1}{2}, \frac{1}{4}\right), (4, 2), (1, -1), (2, -2)\right\}$

Question 2: (7 points): If $A = \begin{bmatrix} -10 & -9 & -8 & -7 \\ -6 & -5 & -4 & -3 \\ -2 & -1 & 1 & 2 \\ 3 & 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & -1 & -2 \end{bmatrix}$, find the element in the third row and second column of BA.

Solution: $c_{32} = \begin{bmatrix} 9 & 10 & -1 & -2 \end{bmatrix} \begin{bmatrix} -9 \\ -5 \\ -1 \\ 4 \end{bmatrix} = -81 + (-50) + 1 - 8 = -138$

Question 3: (6 points): If the augmented matrix of a system of linear equations is

$\left[\begin{array}{ccc|c} 1 & 3 & -a^2 & a^2 \\ 0 & 3 & -2a^2 - a & 2a^2 - 2 \\ 0 & 0 & a^2 + 5a - 6 & -a^2 + 1 \end{array}\right]$, then

(i): Find all values of a for which the system of equations has a unique solution.

(ii): Find all values of a for which the system of equations has infinitely many solutions.

(iii): Find all values of a for which the system of equations has no solution.

Solution: We notice that the elements below the main diagonal are zero.

From row three, we have $\left(a^2+5a-6\right)z=1-a^2 \Rightarrow (a+6)(a-1)z=(1-a)(1+a)$

(i): If $a^2+5a-6\neq 0$ then the system has a unique solution. $(a+6)(a-1)\neq 0 \Rightarrow \boxed{a\neq -6 \text{ and } a\neq 1}$.

(ii): If $a^2+5a-6=0$ and $1-a^2=0$ then the system has infinitely many solutions.
$(a+6)(a-1)=0$ and $1-a=0,\ 1+a=0$

$\Rightarrow \boxed{a=1}$

(iii): If $a^2+5a-6=0$ and $1-a^2\neq 0 \Rightarrow (a+6)(a-1)=0$ and $(1-a)(1+a)\neq 0 \Rightarrow \boxed{a=-6}$