Serial #: ________ ID ______________ NAME __________________________________________

**Show all necessary steps for full marks.**

**Question 1: (5 points):** Given $F(x) = -2x^2 - 12x - 10$ for $x \le -3$.

**(a):** Sketch the graph of $F$ and $F^{-1}$

**(b):** State the domain and range of $F$ and $F^{-1}$.

**(c):** $F^{-1}(x) = ?$

## Solution:

**(a):** $F(x) = -2x^2 - 12x - 10$

$$= -2(x^2 + 6x) - 10$$

$$= -2[x^2 + 6x + 9 - 9] - 10$$

$$= -2[(x+3)^2 - 9] - 10$$

$$= -2(x+3)^2 + 18 - 10$$

$$= -2(x+3)^2 + 8 = a(x-h)^2 + k \quad \text{where } vertex = (h,k) = (-3, 8)$$

**(b):** Given $D_F = (-\infty, -3]$ then $R_F = (-\infty, 8]$

So, $D_{F^{-1}} = R_F = (-\infty, 8]$ and $R_{F^{-1}} = D_F = (-\infty, -3]$

From the graph we know that $F$ is 0ne-to-one.

Now, we find the inverse of $F(x) = -2(x+3)^2 + 8$.

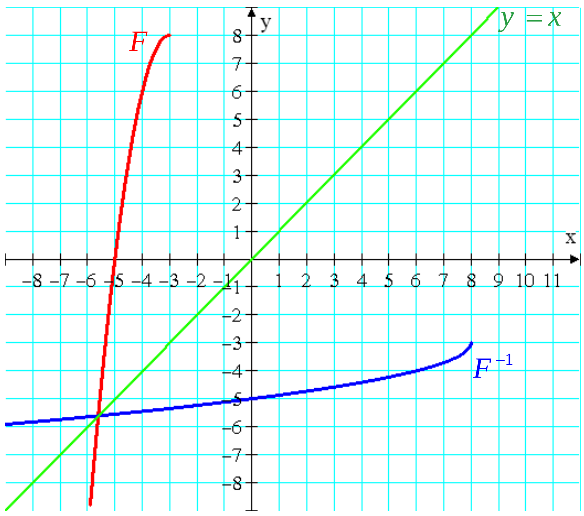
Substitute $y$ for $F(x)$:

$y = -2(x+3)^2 + 8$, $x \le -3$, $y \le 8$

Interchange $x$ and $y$:

$x = -2(y+3)^2 + 8$, $y \le -3$, $x \le 8$

Solve for $y$: to find the inverse:




**(c):** $F^{-1}(x) = ?$

First check that $F$ is a one-to-function:

$F(x) = -2x^2 - 12x - 10$, $x \le -3$

$y = -2(x+3)^2 + 8$, $x \le -3$

$x = -2(y+3)^2 + 8$, $y \le -3$

$2(y+3)^2 = -x + 8$, $-x + 8 \ge 0$, $y \le -3$

$(y+3)^2 = -\frac{1}{2}x + 4$, $-x \ge -8$, $y \le -3$

$\sqrt{(y+3)^2} = \sqrt{-\frac{1}{2}x + 4}$, $x \le 8$, $y \le -3$

$|y+3| = \sqrt{-\frac{1}{2}x + 4}$, $x \le 8$, $y \le -3$

$-(y+3) = \sqrt{-\frac{1}{2}x + 4}$ because $y \le -3$

$y + 3 = -\sqrt{-\frac{1}{2}x + 4}$, $x \le 8$, $y \le -3$

$y = -3 - \sqrt{-\frac{1}{2}x + 4}$, $x \le 8$, $y \le -3$

Substitute $F^{-1}(x)$ for $y$: $F^{-1}(x) = -3 - \sqrt{-\frac{1}{2}x + 4}$, $x \le 8$, $y \le -3$

**Question 2:** **(5 points):** Consider the function $f(x) = -2^{-x+3} + 4$

**(a):** Find the *y*-intercepts, if any.

**(b):** Find the *x*-intercepts, if any.

**(c):** Find the horizontal asymptote of $f$ , if any.

**(d):** Find the domain of $f$ in interval notation.

**(e):** Find the range of $f$ in interval notation.

**(f):** Sketch the graph of $f(x) = -2^{-x+3} + 4$.

**(g):** Sketch the graph of $G(x) = \left| -2^{-x+3} + 4 \right|$.

## Solution:

**(a):** Let $x = 0$, then $f(0) = -2^{-0+3} + 4 = -2^3 + 4 = -4$

The y-intercept is $(0, -4)$

**(b):** Let $f(x) = 0$, then $0 = -2^{-x+3} + 4 \Rightarrow 2^{-x+3} = 2^2 \Rightarrow -x + 3 = 2 \Rightarrow x = 1$

The x-intercept is $(1, 0)$

**(c):** As $x \to \infty$ , $f(x) \to -0 + 4$

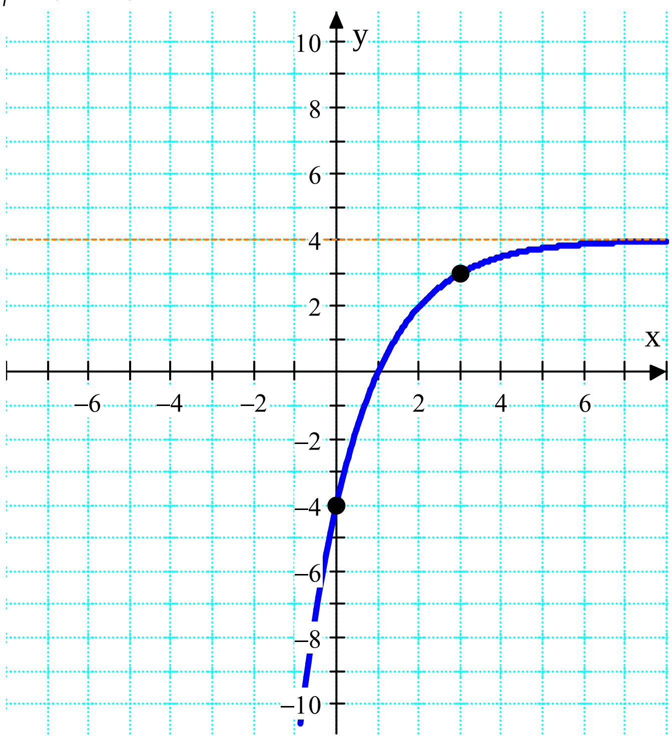
$\Rightarrow$ The line $y = 4$ is the horizontal asymptote.

**(d):** $D_f = (-\infty, \infty)$

**(e):** $-2^{-x+3} < 0 \Rightarrow -2^{-x+3} + 4 < 4 \Rightarrow y < 4 \Rightarrow R_f = (-\infty, 4)$

**(f):**

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $y = f(x)$ | $-4$ | 0 | 2 |




**(g):**