King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 Class Test 1A
Textbook Sections: 4.1 to 6.2
Term 153
Time Allowed: 80 Minutes

Student's Name: …………… ………………………………………………
ID #:……………………….. **Section:** ……. **Serial Number:** …….

Provide neat and complete solutions.
Show all necessary steps for full credit and write the answer in simplest form.

No Calculators, Cameras, or Mobiles are allowed during this exam.

Question	**Points**	Student's Score
1	5	
2	5	
3	4	
4	6	
5	5	
6	5	
7	5	
8	5	
9	5	
10	5	
Total	**50**	___ / 50
		___ / 100

Q1. (5 points): Given $f(x)=\frac{1}{5}x^2-\frac{4}{25}x+k$ where $x\geq 2$. If $f^{-1}(2)=5$, then $k=?$

Solution:

21. Given $f(x)=\frac{1}{5}x^2-\frac{4}{25}x+k$, where $x\geq 2$. If $f^{-1}(2)=5$, then k is equal to

(a) $-\frac{11}{5}$

(b) $-\frac{31}{5}$

(c) $\frac{31}{5}$

(d) $\frac{11}{5}$

(e) $\frac{1}{5}$

$$f^{-1}(2)=5 \Rightarrow f(5)=2$$
$$\Rightarrow \frac{1}{5}\cdot 25-\frac{4}{25}\cdot 5+k=2$$
$$\Rightarrow 5-\frac{4}{5}+k=2$$
$$\Rightarrow k=2-5+\frac{4}{5}$$
$$=-3+\frac{4}{5}$$
$$=-\frac{11}{5}$$

Q2. (5 points)(4.2 Recitation Q#1): If the function $y=4^{x+2}-5$ is written as $y=k\left(\frac{1}{2}\right)^{bx}+c$, then $k+b+c=?$

(a) 11 (b) 7 (c) 9 (d) 13 (e) 12

Solution:

$$y=4^{x+2}-5$$
$$=\left(2^2\right)^{x+2}-5$$
$$=(2)^{4+2x}-5$$
$$=(2)^4(2)^{2x}-5$$
$$=(16)\left(\frac{1}{2}\right)^{-2x}-5$$
$$=k\left(\frac{1}{2}\right)^{bx}+c$$
$$\Rightarrow \boxed{k=16},\ \boxed{b=-2},\ \boxed{c=-5} \Rightarrow k+b+c=9$$

Q3. (4 points)(4.3Textbook Exercise 96): Evaluate the following logarithmic expressions:

(I): $\log 0.0001^5=?$ **(II):** $1000^{\log 5}=?$

Solution:

(I): $\log 0.0001^5=5\log 0.0001=5\log\left(1\times 10^{-4}\right)=5\log\left(10^{-4}\right)=-20\log 10=-20$

Another Method:

$$\log 0.0001^5=\log\left(\frac{0.0001}{1}\right)^5=\log\left(\frac{1}{10000}\right)^5=\log\left(\frac{1}{10^4}\right)^5=\log\left(10^{-4}\right)^5=\log 10^{-20}=-20$$

(II): $1000^{\log 5}=\left(10^3\right)^{\log 5}=10^{3\log 5}=10^{\log 5^3}=5^3=125$

Q4. **(6 points) (4.5 Textbook Exercise 14):** Solve $3^{x-4}=7^{2x+5}$

Solution:

14.

$$3^{x-4}=7^{2x+5}$$
$$\ln\left(3^{x-4}\right)=\ln\left(7^{2x+5}\right)$$
$$(x-4)\ln 3=(2x+5)\ln 7$$
$$x\ln 3-4\ln 3=2x\ln 7+5\ln 7$$
$$x\ln 3-2x\ln 7=4\ln 3+5\ln 7$$
$$x(\ln 3-2\ln 7)=4\ln 3+5\ln 7$$
$$x=\frac{4\ln 3+5\ln 7}{\ln 3-2\ln 7}$$

Q5. **(5 points)** (5.1 Recitation Q#1):

If α is of the complement of the angle 30.56° and β is the supplement of the angle $40^\circ\ 51'\ 27''$, then find the smallest positive angle coterminal with the angle $\beta-\alpha$ and write it as DMS.

Solution:

$$\begin{aligned}\alpha&=90^\circ-30.56^\circ\\&=59.44^\circ\\&=59^\circ+(0.44\times 60)'\\&=59^\circ+26.4'\\&=59^\circ+26'+(0.4\times 60)''\\&=59^\circ\ 26'\ 24''\end{aligned}$$

$$\begin{aligned}\beta&=180^\circ-40^\circ\ 51'\ 27''\\&=179^\circ\ 59'\ 60''-40^\circ\ 51'\ 27''\\&=139^\circ\ 8'\ 33''\end{aligned}$$

$$\begin{aligned}\beta-\alpha&=139^\circ\ 8'\ 33''-59^\circ\ 26'\ 24''\\&=138^\circ\ 68'\ 33''-59^\circ\ 26'\ 24''\\&=79^\circ\ 42'\ 09''\end{aligned}$$

The smallest positive coterminal angle of $79^\circ\ 42'\ 09''$ is

$$360^\circ+79^\circ\ 42'\ 09''=439^\circ\ 42'\ 09''$$

Q6. **(5 points)** (5.2 Recitation Q#2):

If the terminal side of the angle θ in the standard position coincides with the line $3x+2y=0$, with $x\leq 0$, then find $\sec\theta$.

Solution: $3x+2y=0 \Rightarrow 2y=-3x \Rightarrow y=-\frac{3}{2}x\ ,\ x\leq 0$

Then the point $(-2,3)$ is on the terminal side of θ.

$$r=\sqrt{(-2)^2+3^2}=\sqrt{13}$$

$$\sec\theta=\frac{r}{x}=\frac{\sqrt{13}}{-2}$$

Q7. (5 points) (5.3 Recitation Q#1):

If α is the reference angle of 675° and β is the least positive coterminal angle of -240°,

then find $\alpha+\beta=?$

Solution: The coterminal angle of 675° is $675^\circ-360^\circ=315^\circ$

Then 675° and 315° have the same reference angle:

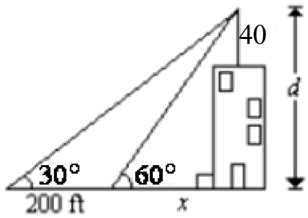
$\alpha=360^\circ-315^\circ=45^\circ$

$\beta=-240+360^\circ=120^\circ \quad \Rightarrow \quad \alpha+\beta=45^\circ+120^\circ=165^\circ$

Q8. (5 points)(Additional Exercise 13):

The angle of elevation to the top of a radio antenna on the top of a building is 30°. After moving 200 feet closer to the building, the angle of elevation is 60°. Find the height of the building if the height of the antenna is 40 feet.

Solution: $\tan 30^\circ=\dfrac{d}{200+x} \qquad \tan 60^\circ=\dfrac{d}{x} \quad \Rightarrow \quad d=x\sqrt{3}$



$$\frac{1}{\sqrt{3}}=\frac{x\sqrt{3}}{200+x}$$

$$200+x=3x \quad \Rightarrow \quad 200=2x \quad \Rightarrow \quad x=100ft$$

$$\tan 60^\circ=\frac{d}{x} \quad \Rightarrow \quad \sqrt{3}=\frac{d}{100} \quad \Rightarrow \quad d=100\sqrt{3}$$

Height of the building is $100\sqrt{3}-40=20(5\sqrt{3}-2)\,ft$

Q9. (5 points) (Additional Exercise 16):

If the arc length $\dfrac{4\pi}{3}$ cm subtends a central angle θ in a circle with diameter 12 cm, then find the degree measure of the angle θ.

Solution:

$s=r\theta \quad , \quad s=\frac{4\pi}{3}\text{ cm} \quad , \quad r=6\text{ cm}$

$\theta=\dfrac{s}{r}=\dfrac{\frac{4\pi}{3}\text{ cm}}{6\text{ cm}}=\dfrac{4\pi}{3}\cdot\dfrac{1}{6}$ radians

$=\dfrac{2\pi}{9}$ radians

$=\dfrac{2\pi}{9}\cdot\dfrac{180^\circ}{\pi}$ degrees

$=40^\circ$

Q10. (5 points) (6.2 Additional Exercise 9):

$$\csc\left(\frac{23}{6}\pi\right)\cdot\tan\left(\frac{13}{3}\pi\right)-\cos\left(\frac{7}{4}\pi\right)=?$$

Solution:

19) The exact value of $\csc\left[\frac{23}{6}\pi\right]\cdot\tan\left[\frac{13}{3}\pi\right]-\cos\left[\frac{7}{4}\pi\right]$ is equal to

A) $-\dfrac{4\sqrt{3}+\sqrt{2}}{2}$

B) $\dfrac{4\sqrt{3}+\sqrt{2}}{2}$

C) $\dfrac{\sqrt{2}-4\sqrt{3}}{2}$

D) $\dfrac{\sqrt{2}-4}{2}$

E) $-\dfrac{4+\sqrt{2}}{2}$

$=\csc\left(\frac{23\pi}{6}-4\pi\right)\cdot\tan\left(\frac{13\pi}{3}-4\pi\right)-\cos\frac{\pi}{4}$

$=\csc\left(-\frac{\pi}{6}\right)\cdot\tan\frac{\pi}{3}-\frac{\sqrt{2}}{2}$

$=(-2)\cdot\sqrt{3}-\frac{\sqrt{2}}{2}$

$=\dfrac{-4\sqrt{3}-\sqrt{2}}{2}$

$=-\dfrac{4\sqrt{3}+\sqrt{2}}{2}$